Origin and Consequences of
Soft Supersymmetry Breaking

G.F. Giudice

Theory Division, CERN
Geneva, Switzerland

The naturalness (or hierarchy) problem [1, 2] is considered to be the most serious theoretical argument against the validity of the Standard Model of elementary particle interactions beyond the TeV energy scale. In this respect, it can be viewed as the ultimate motivation for pushing the experimental research to higher energies. The naturalness problem arises from the difficulty, in field theory, in keeping fundamental scalar particles much lighter than the highest energy within the range of validity of the theory, \( \Lambda \). This difficulty is a consequence of the lack of a symmetry prohibiting a scalar mass term or, in more technical terms, of the presence of quadratic divergences in the quantum corrections to scalar masses.

The Standard Model Lagrangian contains a single dimensionful parameter, in the mass term for the Higgs field, which determines the size of the electroweak scale. A reasonable criterion for "naturalness" limits the validity cut-off scale \( \Lambda \) to be at most a loop factor larger than the mass scale of fundamental scalars. We are then led to the conclusion that the Standard Model can be valid only up to the TeV scale, if the naturalness criterion is

\[1\] On leave of absence from INFN, Sezione di Padova, Padua, Italy.
satisfied. New physics should appear at this scale, modifying the high-energy behaviour and divorcing the Higgs mass parameter from its ultraviolet sensitivity.

An elegant solution to the naturalness problem is provided by supersymmetry [3]–[5]. Since this relates bosons to fermions, a scalar mass term is never generated by quantum corrections in a supersymmetric theory, if the corresponding fermion mass is forbidden by a chiral symmetry. Moreover, a supersymmetric theory is free from quadratic divergences [6]–[9].

In the real world, supersymmetry must be broken. However, if supersymmetry provides a solution to the naturalness problem, it must then be an approximate symmetry of the theory above the TeV scale. This is possible when supersymmetry is broken only softly [10]–[12], i.e. by terms that do not introduce quadratic divergences. These terms always have dimensionful couplings, and the naturalness criterion implies that the corresponding mass scale cannot exceed the TeV region. The soft terms provide gauge-invariant masses to all supersymmetric partners of the known Standard Model particles. These masses give a precise physical meaning to the Standard Model ultraviolet cut-off $\Lambda$.

As the soft terms determine the mass spectrum of the new particles, the mechanism of supersymmetry breaking is the key element for understanding the low-energy aspects of supersymmetric theories. However, given our present lack of knowledge of the supersymmetry-breaking dynamics, we can start by following an agnostic approach in which we will not try to explain how supersymmetry is broken, but just include all supersymmetry-breaking terms that do not introduce quadratic divergences.

The simplest way to derive the possible soft terms is to introduce a background supersymmetry-breaking field $X = m_s \theta^2$, where $m_s$ is the typical scale of mass splitting inside the supermultiplets, and $\theta$ is the Grassmann superspace coordinate. The soft terms are then obtained by coupling $X$ to the usual matter and gauge superfields, recall-
ing the following three properties: (i) $X$ has vanishing canonical dimension; (ii) $X^n = 0$ whenever $n > 1$; (iii) $X^t$ cannot appear in a $\int d^2 \theta$ integral. Let me denote a generic matter chiral superfield by $\Phi$ (with scalar component $\phi$), the superpotential by $f(\Phi)$, a generic gauge vector superfield by $V$ (with fermionic component $\lambda$), and the chiral gauge field strength by $W$. The most general soft terms are given by

\begin{align}
\int d^2 \theta XWW &= m_s \lambda \lambda \quad \text{gaugino mass term} \\
\int d^4 \theta X^d X^d \Phi^* \Phi &= m_s^2 \phi^* \phi \quad \text{squark mass term} \\
\int d^4 \theta X^d X^d \Phi^* \Phi &= m_s \phi \frac{\partial f}{\partial \phi} \quad \text{A-terms} \\
\int d^2 \theta X f(\Phi) &= m_s f(\phi) \quad \text{A-terms.}
\end{align}

It is apparent that the soft-breaking parameter $m_s$ sets the mass scale for the partners of the ordinary particles. Therefore, to understand the origin of the soft terms is a central experimental and theoretical question. Actually, present experimental information already significantly constrains the allowed structure of the soft terms. The constraints essentially arise from the clash between two concepts, relevant to all attempts to construct theories beyond the Standard Model: the hierarchy problem and the "extrinsic" symmetries.

We will refer to "extrinsic" symmetries as to accidental or approximate symmetries of the Standard Model that are not a fundamental requirement of the theory, as the gauge symmetries. Nevertheless, they are of great phenomenological importance, since the experimental confirmation of their (approximate) validity has been the key of the Standard Model success. Since the "extrinsic" symmetries are not fundamental, we expect them to be broken above the Standard Model ultraviolet cut-off $\Lambda$, where the new theory sets in. At low energies, the violations of the "extrinsic" symmetries can be described by higher-dimensional operators, suppressed by powers of the mass scale $\Lambda$. The experimental bounds on such violations then provide us with information on the new-physics scale $\Lambda$. 
The simplest example of an "extrinsic" symmetry of the Standard Model is that of the baryon number. The symmetry is violated by dimension-six operators involving one lepton and three quark fields

\[
\frac{1}{\Lambda^2} g q q \ell. \tag{5}
\]

The present limit on proton decay requires \( \Lambda \gtrsim 10^{15} \) GeV. Another example is given by lepton number, which can be violated by a dimension-five operator involving the Higgs field and the lepton weak doublet

\[
\frac{1}{\Lambda} H H \ell \ell. \tag{6}
\]

The limit on the electron-neutrino mass requires \( \Lambda \gtrsim 10^{12} \) GeV. Individual lepton number can be violated by the operator

\[
\frac{1}{\Lambda^2} H L \ell \sigma^{\mu\nu} \ell R F_{\mu\nu}. \tag{7}
\]

The limit on the decay branching ratio for \( \mu \to e \gamma \) requires \( \Lambda \gtrsim 10^7 \) GeV. Quark flavour symmetry is violated by the operator

\[
\frac{1}{\Lambda^2} s_L \gamma^\mu d_L s_L \gamma^\mu d_L. \tag{8}
\]

The measurement of \( K^0 - \bar{K}^0 \) mixing requires \( \Lambda \gtrsim 10^6 \) GeV. We will not further enumerate limits on "extrinsic" symmetries, but it should be clear by now that in general they require strong constraints on \( \Lambda \). New-physics effects violating such symmetries can only appear at energies much larger than the TeV range, the Standard-Model cut-off suggested by the hierarchy problem. Therefore, a theory aiming at solving the hierarchy problem has to respect (at least approximately) the "extrinsic" symmetries of the Standard Model. We can now wonder whether a supersymmetric theory with generic soft terms satisfies this property.

In supersymmetry, baryon and lepton number can be violated at the renormalizable level, because of the presence of scalar partners of quarks and leptons. This is undoubtedly
an unpleasant feature. However baryon- and lepton-number conservation in supersymmetry could be an automatic consequence of an extended gauge symmetry valid at very high energies.

A more subtle difficulty arises from the violation of the flavour symmetry. In the limit of vanishing Yukawa couplings, the Standard Model Lagrangian is invariant under a global $U(3)^5$ symmetry, with each $U(3)$ acting on the generation indices of the five irreducible fermionic representations of the gauge group $(q_L, u^c_R, d^c_R, \ell_L, e^c_R)_i$. This symmetry, called flavour (or family) symmetry, follows from the property that gauge interactions do not distinguish between the three generations of quarks and leptons. This symmetry is explicitly broken by the Yukawa couplings, but in such a way that, with the minimal Higgs structure, flavour-changing neutral currents are not generated at tree level, and are suppressed by the GIM mechanism at the loop level.

In supersymmetric theories, soft terms give an additional source of flavour breaking. These contributions are very dangerous [11, 13, 14]. The mismatch between the diagonalization matrices for quarks and squarks (and analogously for leptons and sleptons) leads to flavour-violating gaugino vertices, and eventually to large contributions to flavour-changing neutral-current processes. Studies of the $\bar{K}^0 - K^0$ mass difference, $\mu \rightarrow e\gamma$ and similar processes set very stringent bounds on the relative splittings among different generations of squarks and sleptons [15, 16]. Only very specific structures of soft terms can pass the test of the flavour symmetries: the soft terms are either almost flavour-universal, or there is an approximate alignment between particle and sparticle mass matrices [17]–[19].

An important theoretical issue is to understand the underlying dynamics that leads to these very specific structures of soft terms. This seems to require knowledge of the supersymmetry-breaking origin. However, the mechanism of communicating the original supersymmetry breaking to the ordinary particle supermultiplets plays an equally or even more important rôle. As an analogy, one can think of the case of the Standard Model, in
which the Higgs vacuum expectation value determines the scale of electroweak breaking, but the detailed mass spectrum of bosons and fermions is dictated by the coupling constants of the forces that communicate the information of electroweak breaking, i.e. gauge and Yukawa interactions, respectively.

A major difference between the case of supersymmetry breaking and this Standard Model analogy is that the supertrace theorem [20] essentially rules out the possibility of constructing simple models in which supersymmetry breaking is communicated to ordinary supermultiplets by tree-level renormalizable couplings. Indeed, in a globally supersymmetric theory with a gauge group free from gravitational anomalies [21], the sum of the particle tree-level squared masses, weighted by the corresponding number of degrees of freedom, is equal in the bosonic and fermionic sectors [20]:

\[ \text{STr} \mathcal{M}^2 = \sum_J (-1)^{2J}(2J + 1)M_J^2 = 0. \] \hspace{1cm} (9)

Here \( \mathcal{M}_J \) denotes the tree-level mass of a particle with spin \( J \). Rather generically, this theorem implies, in cases of tree-level communication, the existence of a supersymmetric particle lighter than its ordinary partner.

As a consequence of this difficulty, the paradigm for constructing realistic supersymmetric theories is to assume that the sector responsible for supersymmetry breaking (the analogue of the Higgs sector) has no renormalizable tree-level couplings with the "observable sector", which contains the ordinary particles and their supersymmetric partners. Moreover, the effective theory describing the observable sector, obtained by integrating out the heavy particles in the supersymmetry-breaking sector, should have a non-vanishing supertrace.

One possibility is to consider a theory that is altogether non-renormalizable, and such that the supertrace over the whole spectrum is non-vanishing. The best-motivated example is given by gravity. Indeed the most general supergravity Lagrangian, in the
presence of supersymmetry breaking, leads to an effective theory for the low-energy modes containing the desired soft terms [22]–[27]. This is the scenario most commonly considered in phenomenological applications (for reviews, see for instance refs. [28]–[30]), and it is certainly a very attractive one as, for the first time, gravity ventures to play an active rôle in electroweak physics.

Another possibility is that the relevant dynamics is described by a renormalizable Lagrangian, and, at tree level, the theory has a vanishing supertrace and no mass splittings inside the observable supermultiplets. However the low-energy modes are described by an effective Lagrangian, which has non-renormalizable kinetic terms (and non-vanishing supertrace) at the quantum level, induced by known gauge interactions. This is the case of theories with gauge-mediated supersymmetry breaking [31]–[33] (for a review, see ref. [34]), which I want to discuss in more detail in these lectures.

The fundamental difference between the two approaches is related to the problem of flavour. We ignore the dynamical origin of the Yukawa couplings or, ultimately, of the flavour-symmetry breaking, but let me just define \( \Lambda_F \) as the relevant energy scale of the corresponding new physics. Above \( \Lambda_F \) lie some unknown dynamics responsible for flavour breaking. Below \( \Lambda_F \) these dynamics are frozen, leaving their scars on the flavour-breaking structure of Yukawa couplings.

In the gravity-mediated approach, the soft terms are generated at the Planck scale, and therefore necessarily at a scale larger than or equal to \( \Lambda_F \). There is then no obvious reason why the supersymmetry-breaking masses for squarks and sleptons should be flavour-invariant. Even if at tree level, for some accidental reason, they are flavour-symmetric, loop corrections from the flavour-violating sector will still distort their structure. Even contributions from ordinary grand unified theories (GUTs) can lead to significant flavour-breaking effects in the soft terms [35]–[37]. Of course this does not mean that gravity-mediated scenarios cannot give a realistic theory, but only that at the moment we
do not have full control of the flavour symmetries relevant at the quantum-gravity scale.

On the other hand, in gauge-mediated theories, soft terms are generated at the messenger scale $M$, which is \textit{a priori} unrelated to $\Lambda_F$. If $M \ll \Lambda_F$, the soft terms feel the breaking of flavour only through Yukawa interactions. Yukawa couplings are the only relevant sources of flavour violation, as in the Standard Model. More precisely, all other sources of flavour violation at the messenger scale correspond to operators of dimension larger than 4, suppressed by the suitable powers of $1/\Lambda_F$. The contribution of these operators to soft masses is necessarily suppressed by powers of $M/\Lambda_F$. As a consequence, the GIM mechanism is fully operative and it can be generalized to a super-GIM mechanism, involving ordinary particles and their supersymmetric partners. Since it is reasonable to expect that $\Lambda_F$ is as large as the GUT or the Planck scales, in gauge-mediated theories the flavour problem is naturally decoupled, in contrast to the case of supergravity or, in a different context, of technicolour theories [38, 39].

Let me now describe the basic structure of theories with gauge-mediated supersymmetry breaking. The first ingredient of these models is an \textit{observable sector}, which contains the usual quarks, leptons, and two Higgs doublets, together with their supersymmetric partners. Then the theory contains a sector responsible for supersymmetry breaking. I will refer to it as the \textit{secluded sector}, to distinguish it from the hidden sector of theories where supersymmetry breaking is mediated by gravity. I will not specify the secluded sector, since it still lacks a standard description. For our purposes, all we need to know is that the goldstino field overlaps with a chiral superfield $X$, which acquires a vacuum expectation value along the scalar and auxiliary components

$$
\langle X \rangle = M + \theta^2 F.
$$

(10)

The parameters $M$ and $\sqrt{F}$, which are the fundamental mass scales in the theory, can vary from several tens of TeV to almost the GUT scale.
Finally the theory has a *messenger sector*, formed by some new superfields that transform under the gauge group as a real non-trivial representation and couple at tree level with the goldstino superfield \( X \). This coupling generates a supersymmetric mass of order \( M \) for the messenger fields and mass-squared splittings inside the messenger supermultiplets of order \( F \). This sector is also unknown and it is the main source of model dependence. It is fairly reasonable to expect that the secluded and messenger sectors have a common origin, and models in which these two sectors are unified have been constructed [34].

The simplest messenger sector is described by \( N_f \) flavours of chiral superfields \( \Phi_i \) and \( \Phi_i (i = 1, \ldots, N_f) \) transforming as the representation \( \text{r} + \bar{\text{r}} \) under the gauge group. In order to preserve gauge coupling-constant unification, one usually requires that the messengers form complete GUT multiplets. If this is the case, the presence of messenger fields at an intermediate scale does not modify the value of \( M_{\text{GUT}} \), but the inverse gauge coupling strength at the unification scale \( \alpha_{\text{GUT}}^{-1} \) receives an extra contribution

\[
\delta \alpha_{\text{GUT}}^{-1} = \frac{N}{2\pi} \ln \frac{M_{\text{GUT}}}{M} ,
\]

\[
N = \sum_{i=1}^{N_f} n_i .
\]

Here \( n_i \) is twice the Dynkin index of the gauge representation \( \text{r} \) with flavour index \( i \), e.g. \( n = 1 \) or 3 for an \( SU(5) \) 5 or 10, respectively. I will refer to \( N \) as the *messenger index*, a quantity that plays an important rôle in the phenomenology of gauge-mediated theories. From eq. (11) we infer that perturbativity of gauge interactions up to the scale \( M_{\text{GUT}} \) implies

\[
N \lesssim 150 / \ln \frac{M_{\text{GUT}}}{M} .
\]

If \( M \) is as low as 100 TeV, then \( N \) can be at most equal to 5. However, this upper bound on \( N \) is relaxed for larger values of \( M \). For instance, for \( M = 10^{10} \) GeV, eq. (13) shows that \( N \) as large as 10 is allowed.
In the case under consideration the interaction between the chiral messenger superfields \( \Phi \) and \( \Phi^\dagger \) and the goldstino superfield \( X \) is given by the superpotential term

\[
W = \lambda_{ij} \Phi_i X \Phi_j .
\]  

(14)

After replacing in eq. (14) the \( X \) VEV, see eq. (10), we find that the spinor components of \( \Phi \) and \( \Phi^\dagger \) form Dirac fermions with masses \( \lambda M \), while the scalar components have a squared-mass matrix

\[
\begin{pmatrix}
\Phi^\dagger & \Phi \\
(\lambda M) \Phi^\dagger & (\lambda M)(\lambda M)^\dagger \\
(\lambda F)^\dagger & (\lambda F)
\end{pmatrix} .
\]

(15)

Here I have dropped flavour indices and, with a standard abuse of notation, I have denoted the superfields and their scalar components by the same symbols. If there is a single field \( X \), then the matrices \( \lambda M \) and \( \lambda F \) can be simultaneously made diagonal and real, and the scalar messenger mass eigenvectors are \( (\Phi + \Phi^\dagger)/\sqrt{2} \) and \( (\Phi - \Phi^\dagger)/\sqrt{2} \), with squared-mass eigenvalues \( (\lambda M)^2 \pm (\lambda F)^2 \). It is now convenient to absorb the coupling constant \( \lambda \) in the definition of \( M \) and \( F \), \( \lambda_{ii} M \rightarrow M_i, \lambda_{ii} F \rightarrow F_i \).

The mass scale \( \sqrt{F} \) is the measure of supersymmetry breaking in the messenger sector. However, we are of course mainly interested in the amount of supersymmetry breaking in the observable sector. Ordinary particle supermultiplets are degenerate at the tree level, since they do not directly couple to \( X \), but splittings arise at the quantum level because of gauge interactions between observable and messenger fields. While vector bosons and matter fermion masses are protected by gauge invariance, gauginos, squarks, and sleptons can acquire masses consistently with the gauge symmetry, once supersymmetry is broken. Gaugino masses are generated at one loop, but squark and slepton masses can only arise at two loops, since the exchange of both gauge and messenger particles is necessary.

Direct computation of the relevant Feynman diagrams leads to the following expression for the gaugino masses

\[
\tilde{M}_{\lambda_r} = k_r \frac{\alpha_r}{4\pi} \Lambda_G \quad (r = 1, 2, 3) ,
\]

(16)
\[ \Lambda_G = \sum_{i=1}^{N_f} n_i \frac{F_i}{M_i} \left[ 1 + \mathcal{O}(F_i^2/M_i^4) \right], \]  

where \( k_1 = 5/3, k_2 = k_3 = 1, \) and the gauge coupling constants are normalized such that \( k_r \alpha_r \) \((r = 1, 2, 3)\) are all equal at the GUT scale. In the simple case in which there is a single \( X \) superfield, and the ratio \( F_i/M_i \) is therefore independent of the flavour index \( i \), eq. (17) becomes

\[ \Lambda_G = N \frac{F}{M} \left[ 1 + \mathcal{O}(F^2/M^4) \right], \]

where the messenger index \( N \) is defined in eq. (12).

Neglecting Yukawa-coupling effects, the supersymmetry-breaking scalar masses are

\[ m^2_i = 2 \sum_{r=1}^{3} C^I_r k_r \alpha^2 \frac{1}{(4\pi)^2} \Lambda^2_S. \]  

In the case of a single \( X \) superfield,

\[ \Lambda^2_S = N \frac{F^2}{M^2} \left[ 1 + \mathcal{O}(F^2/M^4) \right]. \]

In eq. (19) \( C^I_r \) is the quadratic Casimir of the \( \tilde{f} \) particle, \( C = \frac{N_2 - 1}{2N} \) for the \( N \)-dimensional representation of \( SU(N) \), and \( C = Y^2 = (Q - T_3)^2 \) for the \( U(1) \) factor. The physical scalar squared mass is obtained by adding to eq. (19) the \( D \)-term contribution \( M_2^2 \cos 2\beta(T_3^I - Q^I \sin^2 \theta_W) \). Moreover, eqs. (16) and (19) are valid at the messenger mass scale and one should include the familiar renormalization-group evolution.

The most important feature of the gauge-mediation mass formula in eq. (19) is of course flavour universality, which is guaranteed by the symmetry of gauge interactions. Another attractive feature of the gauge-mediated mass formulae is that gaugino mass terms are generated at one loop, and (positive) squark mass terms are generated at two loops. Since fermion and boson bilinears have different canonical dimensions, all supersymmetry-breaking mass parameters have the same scaling property \( \tilde{m} \sim (\alpha/\pi)F/M \). However, the Higgs mass parameters \( \mu \) and \( B_\mu \) do not satisfy this property [40].
Furthermore the gauge-mediation mass formulae allow a high degree of predictivity. The whole supersymmetric spectrum is determined by the effective supersymmetry-breaking scale $\Lambda = F/M$, the messenger index $N$, the messenger mass $M$, and $\tan \beta$. Another remarkable success of the gauge-mediated mass spectrum is that radiative electroweak-symmetry breaking [41] can be achieved.

The phenomenological aspects of gauge-mediated theories can be quite different from those in the gravity-mediated scenario. The main reason is that the gravitino mass,

$$m_{3/2} = \frac{F}{\sqrt{3} M_P} = \left( \frac{\sqrt{F}}{100 \text{ TeV}} \right)^2 2.4 \text{ eV},$$

is not necessarily of the order of the weak scale, but can be much lighter. In contrast to the case of ordinary R-parity conserving supersymmetry, the lightest partner of the Standard Model particles (called the NLSP) can decay. The experimental consequences are very diverse, depending on the parameter choice. However, they can roughly be divided into two regimes. Models with low values of $F$, and therefore comparable values of $M$, have the characteristic that the NLSP decays promptly (for $\sqrt{F}$ roughly less than $10^6$ GeV) and that the gravitino relic density is less than the critical density (for $\sqrt{F}$ roughly less than $10^7$ GeV). Depending on whether the NLSP is a neutralino or a stau, the characteristic collider signals are anomalous events characterized by missing energy and $\gamma$ or $\tau$, respectively. For larger $\sqrt{F}$, the NLSP lifetime is longer and the collider phenomenology can resemble the well-known missing-energy supersymmetric signatures (for a neutralino NLSP) or can lead to a long-lived heavy charged particle penetrating the detector (for a stau NLSP). In this regime, the gravitino or the NLSP decay can cause cosmological difficulties.

In conclusion, the origin of supersymmetry breaking and of the soft terms is a very intriguing theoretical issue, with vast experimental consequences. If supersymmetry is discovered at the next generation of colliders, it will become a central question in the future of particle physics.
References

[1] K. Wilson, as quoted in ref. [39].


