The Master Gauge String

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Abstract A string background, which is in some precise sense universal (i.e.,
incorporating all orders in the Feynman diagram expansion), is proposed to represent
pure gauge theories.

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Vacuum configurations for the closed bosonic string are determined by the sigma model beta function equations [8], [9]. These are the equations that ensure world sheet conformal invariance. At tree level in string perturbation theory these equations, for vanishing tachyon expectation value, are given by:

\[ R_{\mu\nu} + \nabla_\mu \nabla_\nu \Phi = 0 \]  
\[ (\nabla\Phi)^2 - \nabla^2 \Phi = C \]  

With \( \Phi \) the dilaton field and the constant \( C \) being determined by the target space-time dimension as follows:

\[ C = \frac{26 - D}{3} \]  

The renormalization group approach to the string representation of gauge theories [3], [4],[2] is based on:

i) Identifying the string coupling constant with the Yang Mills coupling:

\[ g = e^{\frac{\Phi}{2}} \]  

ii) Identifying one space-time coordinate with the renormalization group variable \( \mu \), and

iii) Looking for vacuum solution to equations (1)(2) and to the renormalization group equation:

\[ \mu \frac{dg}{d\mu} = \beta(g(\mu)) \]  

for \( \beta \) the Yang Mills beta function.

In this letter we will present a critical, i.e \( C = 0 \), universal solution to equations (1),(2),(6) for \( \beta(g) \) a generic formal power series of \( g \) of the type:

\[ \beta(g) = \sum_i b_i g^i \]  

In terms of the dilaton field \( \Phi(\mu) \) defined by equations (4) and (6) the universal metric is given by:

\[ ds^2 = e^{\Phi} d\bar{x}^2_d + (de^\Phi)^2 + d\bar{y}_D^2 \]
Before going into a brief discussion on the physical meaning of this string metric let us prove that metric (7) with $D = 21$ and a dilaton field given be $\Phi(\mu)$ is in fact solution to equations (1) and (2) for $C = 0$.

The proof of our previous statement follows directly from the expression for the Ricci tensor for the universal metric

$$
R_{\mu\nu} = -\frac{1}{4} \frac{2a''b - a'b'a + 2a''ba}{b^2 a} \eta_{\mu\nu}
$$

$$
R_{44} = -\frac{a''b + 2a''ba - a'b'a}{a^2 b}
$$

(8)

(where $a \equiv e^{2\Phi}$ and $b \equiv e^{2\Phi (\phi^2)}$). The most illustrative condition comes, however, from the beta function of the dilaton, $\beta_\Phi = (\nabla \Phi)^2 - \nabla^2 \Phi$, which can be easily shown to be equal to $e^{-2\Phi}(2 - d/2))$.

It is important to notice that the metric (7) is solution of the sigma model beta function equations only if $D = 21$ with the other five coordinates having the meaning of the four dimensional space-time coordinates and the renormalization group scale. Moreover the universal metric is solution independently of the sign of the beta function i.e both for asymptotically or infrared free theories. The difference between them is related to the meaning of the singularity of the metric (7) at $e^\Phi = 0$.

At the level of the metric the $S$-duality transformation can be implemented as:

$$
\Phi \to -\Phi
$$

(9)

Hence we can define the $S$-dual metric by performing the transformation (9) on the universal metric. From a physical point of view this S dual metric could correspond to the same gauge theory but described in terms of 't Hooft’s loop variables.

It would be convenient for future discussion to define the universal metric as the five dimensional piece of the metric (7) and to consider this metric as solution to the sigma model beta function equations with $C = 0$. An important property of this five dimensional metric is that it is conformally flat. Introducing new coordinates $x' = 2x$ and $z^2 = e^\Phi$ the five dimensional metric becomes:

$$
\text{d}s^2 = 4z^2 (\text{d}x'^2_a + \text{d}z^2)
$$

(10)
with a dilaton field

$$\Phi(z) = -\ln z^2$$  \hfill (11)

Different input Yang Mills beta functions correspond to changes of variables

$$z \rightarrow z(\mu)$$  \hfill (12)

with $\mu$ the renormalization group coordinate.

It is worth noticing the difference between metric (10) and the AdS metric in five dimensions

$$ds^2 = z^{-2}(dx^2 + dz^2)$$  \hfill (13)

These two conformally flat metrics are special in the following sense. In the AdS case the Ricci tensor which is determined in terms of the conformal factor becomes exactly equal to a cosmological constant term. In the case (10) the Ricci tensor becomes exactly equal to $\nabla_\mu \nabla_\nu \Phi$ for $\Phi$ the logarithm of the conformal factor in (10). We feel these two metrics are the natural candidates to describe gauge theories with vanishing \cite{14,17,13} and non vanishing beta functions respectively\footnote{An important difference between AdS metric and metric (10) is the existence of a singularity for the metric (10) in contrast with the horizon in the AdS case.} . Notice that in our framework ,based on the closed bosonic string, the case of vanishing beta function is a bit special. Namely in this case the only solution is flat Minkowski in 26 critical dimensions. In other words for $C = 0$ and constant dilaton the equations (1) and 2 do not determine in any natural way the dimension of the space-time in contrast to the case where the beta function does not vanish, where four dimensional space-time is singled out. Moreover we should expect, from physical grounds based in Wilson loop computations \cite{15,18}, that the conformally invariant situation, , corresponding to vanishing beta function will determine string metrics that are invariant under appropriated conformal rescalings of the coordinates, which is not the case for flat Minkowski metric ( as opposed to AdS$_5$, which enjoys the conformal group, $SO(2, 4)$, as its isometry group). In this sense please notice that the universal metric (10) is not invariant under rescalings of the coordinates, which is only natural owing to our non trivial dilaton background \footnote{The problem with the string representation of pure gauge theories with vanishing beta function i.e...}.
The predictive power of the renormalization group approach to the string description of pure gauge theories should be contained in those dynamical aspects of the gauge theory that we can describe in terms of what it is properly speaking the string output of our construction, namely the string space-time metric. The first prediction in this sense is related with the so called Zamolodchikov’s c-theorem. It was proved in reference [19], for two dimensional quantum field theories, the existence of a c-function, roughly counting the number of degrees of freedom, that monotonically decreases along the renormalization group flow. In reference [3] (see also [7],[11],[1],[16],[12]) it was suggested, in the holographic scheme, a generalization of this theorem to four dimensional quantum field theories admitting a five dimensional gravitational description. The idea is of course to relate the function c with some geometrical property of the string metric and to use the sigma model beta function equations to derive the desired behavior for c. The most natural candidate for c is the expansion parameter θ for a congruence of null geodesics. In terms of the gauge beta function we get for the universal metric the following behavior of θ

$$\mu \frac{d\theta}{d\mu} = F_U(\mu) \frac{1}{g^2} \beta(g)$$

(14)

with \( F_U(\mu) \) a universal function independent of beta given by

$$F_U(\mu) = \frac{3}{2} (\ln \mu)^2$$

(15)

As it is clear from (14) the sign of variation of θ depends on the sign of the beta function.

To conclude just a comment on possible string corrections to the sigma model beta function equations. We know that tadpoles beyond tree level will in general modify the form of the background string metric by a Fishler-Susskind [10] type of mechanism. The difference between it and the RGA stems precisely from the interpretation of the renormalization group scale as a spacetime coordinate. This in particular implies that the effect of tadpoles is absorbed in the dilaton dependence on this coordinate; but we have shown that this dependence can be always integrated in the universal metric. The problem of tachyon with pure Maxwell theory of free photons, could be probably be related to a similar problem appearing in the AdS approach, concerning the physical interpretation of the near horizon geometry of just one D-brane (cf. [5]). In heuristic physical terms the problem has its origin in trying to represent in terms of gravity a purely linear theory [17]
stability of the string metric worked out in this letter is analyzed in a different paper see ref [6].

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References


[16] V. Sahakian, *Holography, a covariant c-function, and the geometry of the renormalization group* hep-th/9910099


[18] E. Witten, *Anti-de-Sitter space, thermal phase transition and confinement in gauge theories*, hep-th/9803131