WHERE ARE THE FRAGMENTS OF THE VIRTUAL PHOTON?  *)

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ABSTRACT

The inclusive cross-section for the production of a single hadron in deep inelastic electroproduction is studied in a dual resonance model. The Bjorken scaling behaviour in the virtual photon fragmentation region for finite $x(=2p_L^c.m./s)$ is

$$\frac{1}{\sigma_{\gamma p}} \frac{d^3\sigma_{\gamma p}}{d^2p/E} \sim \frac{1}{q^2} F(x, p_t^2/q^2)$$

and thus the transverse momentum grows like $q^2$, whereas in the parton model

$$\frac{1}{\sigma_{\gamma p}} \frac{d^3\sigma_{\gamma p}}{d^3p/E} \sim F(x, p_1^2) .$$

A related effect is the absence of two-jet structure in $e^+e^-$ annihilation. We believe that dual model results may give a more reliable indication of the deep inelastic behaviour for composite hadrons than the parton model.

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The nature of the individual hadronic final states in electroproduction and their relationship to the Bjorken scaling of the total cross-section is one of the most intriguing questions of deep inelastic scattering. This question perhaps arises most simply in the single particle inclusive cross-section for

\[ \gamma^* + h'(p') \rightarrow h(p) + \text{anything}, \]  

(1)

since a partial sum over the final states is taken. The behaviour of (1) in the virtual photon fragmentation region gives information about the photon-hadron interaction not obtainable from the total cross-section.

The kinematics for the above reaction is shown in Fig. 1. Let \( (d^3 \sigma^{T,L}/d^3 p/E) \) be the cross-section for (1), (T) and (L) referring to transverse and longitudinal photons, respectively (*), and \( \sigma^{T,L} \) the corresponding total cross-section. We shall be especially interested in large \(-q^2\) and large \(Q = s/q^2\), e.g., the combined Bjorken 1) \((-q^2 \rightarrow \infty)\) and Mueller-Regge 2) \((Q \rightarrow \infty)\) limits. In this regime virtual photon fragmentation corresponds to the Regge limit shown in Fig. 2a.

Here we study virtual photon fragmentation in a dual resonance model. For fragments with longitudinal momentum a finite fraction of that of the virtual photon \((x = 2p_L/s > 0)\), we find

\[ \frac{1}{\sigma^{T,L}} \frac{d^3 \sigma^{T,L}}{d^3 p/E} \left. \right|_{-q^2 \rightarrow \infty} \left. \right|_{Q \rightarrow \infty} \frac{1}{q^2} F \gamma^* \rightarrow h(x, p_L^2/q^2). \]  

(2)

The interesting feature of (2) is that the transverse momentum grows with \(q^2\) **. This is quite different behaviour than that suggested by the parton model where the scaling variable is \(p_L^2\) rather than \(p^2/q^2\).

Contributions to virtual photon fragmentation for finite \(x\) are constrained by the energy-momentum conservation sum rule

*) An average over the azimuthal angle of \(h\) is always understood.

**) The possibility of the behaviour (2) has also been noted in Refs. 3), 4) and 5).
\[ \int \frac{d^3 p}{E} \frac{(E+p_z)}{\sqrt{s}} \frac{1}{\sigma_{T,L}} \frac{d^3 \sigma_{T,L}}{d^3 p/E} = \int_0^1 dx \int_0^\infty dP_{L} \frac{1}{\sigma_{T,L}} \frac{d^3 \sigma_{T,L}}{d^3 p/E} = 1. \quad (3) \]

Scaling behaviour of the form (2) is clearly compatible with (3), but there are obviously many other possible scaling behaviours. For example, the usual parton model is assumed to have the behaviour \[ \frac{1}{\sigma_{T,L}} \frac{d^3 \sigma_{T,L}}{d^3 p/E} \sim \frac{1}{x^2} \to \infty \quad F_{T,L} \to h(x, P_{L}^2) \quad . \quad (4) \]

This behaviour is found in some explicit formulations of the parton model when the parton struck by the virtual photon decays into a group of final state hadrons - for a recent careful discussion, see Ref. 7. If the struck parton emerges directly in the final state (i.e., h is the parton),

\[ \frac{1}{\sigma_{T,L}} \frac{d^3 \sigma_{T,L}}{d^3 p/E} \sim q^2 \to \infty \quad q^2 F_{T,L} \to h(q^2(1-x), P_{L}^2) \quad . \quad (5) \]

which is also consistent with (3). Therefore, even with the constraint of energy momentum conservation (and also the constraint of consistency with Mueller behaviour of Fig. 2a if one wishes) the behaviour for finite x is very model-dependent and hence a particularly good place to look for clues as to the nature of the virtual photon interaction \(^*\). Thus the picture of virtual photon interaction in the dual model with scaling behaviour (2) is quite different from the picture in the parton model with behaviour (4) or (5).

We now discuss the dual model in some detail. Sugawara some time ago proposed a dual model for currents \(^9\). It has only poles in \(q^2\) and the various channels \((s, t, u, W^2, \text{etc.})\), accommodates falling form factors and Bjorken scaling, and shares a number of properties with field theory

\(^*\) "Photon pulverization" \(^8\) where the cross-section grows as a power of \(q^2\) for values of \(x\) which go to zero as \(-q^2 \to \infty\) is yet another way to satisfy (3). It is outside the Mueller picture.
models of currents for composite hadrons *). While the phenomenological features of the model are quite satisfactory, from a theoretical point of view it is deficient in a number of important respects: it does not factorize and does not satisfy current conservation. Since a dual model which does have these properties has not yet been found, we have turned to the Sugawara model for clues on the behaviour of the virtual photon fragmentation region since we feel it may well correctly represent the qualitative features of composite hadron interactions.

In this model an amplitude involving currents is represented by an ordinary dual amplitude with two "fictitious parton" lines for each current (see Fig. 3) **). The "trajectories" for "channels" containing one fictitious parton and one or more hadrons control the asymptotic behaviour of form factors. For example, from the well-known behaviour of the beta function, we have

\[ F(q^2) \sim \left( \frac{-q^2}{\alpha} \right)^{\gamma}, \]

where \( \gamma \) is the "trajectory" defined in Fig. 3a. The "trajectories" for "channels" containing fictitious partons from different currents control the behaviour in the Bjorken limit. Looking at Fig. 3b, for example, one sees that the Bjorken limit corresponds to a "Regge" limit of \( B_0 \) with the exchange of the fictitious Regge trajectory \( k \), thus

\[ S \sim \text{Disc} B_0 \sim (-q^2)^k \sim F(2). \]

The parameter \( k \) must be chosen to give the usual Bjorken scaling behaviour, e.g., \( k = -1 \) if \( B_0 \) is a model for the invariant amplitude \( W_2 \). The ordinary Regge limit \( L \to \infty \) gives

\[ \sim L^{\alpha(0)} \beta. \]

*) For a review and discussion of its properties, see Ref. 10).

**) We stress that the parton lines are fictitious in the sense that they have no momentum associated with them.
We now turn to the single particle inclusive cross-section and study the contributions of Figs. 3c and 3d. It is especially convenient to introduce the scaling variables *)

$$\mathcal{L} = 5/4 q^2, \quad \mathcal{T} = t/4 q^2, \quad \mathcal{U} = 4/4 q^2, \quad \mathcal{M} = M^2/4 q^2.$$ \hfill (9)

The momentum of $h$ in the $Y-h$ centre-of-mass system is then

$$E \approx \frac{\sqrt{s}}{2} \left[ -\frac{T}{L} - \frac{\alpha + 1}{2} \right],$$

$$p_L \approx \frac{\sqrt{s}}{2} \left[ -\frac{T}{L} + \frac{\alpha + 1}{2} \right],$$

$$m_1^2 = p_1^2 = m^2 \approx \frac{t}{4} \left[ \mathcal{U} + 1 + \frac{T}{L} \right],$$ \hfill (10)

where terms $O(1/L^2)$, $O(m^2 + m^2/t)$, and $O(m^2/u - q^2)$ have been neglected.

Again the Bjorken limit corresponds to a "Regge" limit of $B_q$. For Fig. 3c, we find **)

$$S \frac{d^3 \sigma_{T,L}}{d^3 p/E} \propto \text{Disc}_{M^2} B_q \sim (-q^2)^k \tilde{F}(M^2, t, \mathcal{U})$$

$$= (-q^2)^k \bar{F}(\mathcal{L}, t, \mathcal{U}).$$ \hfill (11)

In the $\mathcal{L} \to \infty$ limit, this term contributes in the hadronic central region (Fig. 2b) and the target fragmentation region (Fig. 2c) as well as the virtual photon fragmentation region. In the first two regions the usual Bjorken-Mueller scaling behaviour obtains since the residue of the Reggeon on the left in Fig. 2 factorizes. In the latter region, we have

$$\bar{F}(\mathcal{L}, t, \mathcal{U}) \sim \mathcal{L}^{\alpha(0)} F(t/L, \mathcal{U})$$ \hfill (12)

and hence

*) These are related to the conventional variables as follows in the Bjorken limit: $\mathcal{L} = \mathcal{U} - 1 \sim \omega - 1$, $\mathcal{U} \sim \omega - 1$, $t = 2m^2 + m^2 + m^2$.

**) This diagram has also been calculated in Ref. 11).
\[ \frac{1}{\sigma_{\pi N}} \frac{d^3\pi_{\pi N}}{d^3p/E} \sim F_{\pi,\pi}(t/\Lambda^2, u). \quad (13) \]

Referring to (10), we see that finite \( t/\Lambda^2 \) and finite \( u \) correspond to finite \( p_1^2 \) and

\[ x \approx \left( \frac{t}{\Lambda^2} \right) \left( \frac{1}{2} + \frac{u+1}{2} \right) \approx \frac{1}{2}(\frac{t}{\Lambda^2} + \frac{u}{\Lambda^2}) \]

or rapidity

\[ y = \ln \frac{E+p_1}{m_\perp} = \frac{1}{2} \ln s - \ln(-q^2) + \text{finite}. \quad (14) \]

This region a distance \( \ln(-q^2) \) from the phase space boundary is called the "hole fragmentation region" in the parton model \( ^6 \). The behaviours (11) and (13) are common to all discussions of virtual photon fragmentation: light-cone \( ^{12} \), parton \( ^{6},^7 \), and Mueller \( ^{3},^{13},^{14} \). It is essential for providing a smooth connection with the hadronic central region.

The contribution of Fig. 3d has the behaviour:

\[ \text{Disc}_{\mu^2} B_8 \sim (-q^2)^{k'} \tilde{F}(\mu^2, m^2, u) = (-q^2)^{k'} F(\mu^2, \mathcal{F}, \mathcal{U}) \quad (15) \]

and contributes in the virtual photon fragmentation region as \( \mu^2 \to \infty \) as

\[ \mathcal{F}(\mu^2, \mathcal{F}, \mathcal{U}) \sim \mu^{(10)} F(\mu^2, \mathcal{U}). \quad (16) \]

Referring again to (10) we see that finite \( \mathcal{F}/\mu^2 \) and finite \( \mathcal{U} \) correspond to finite \( x \) and \( p_1^2/q^2 \) and hence the behaviour (2) if \( k' \) is taken to be \( k-1 \) for compatibility with (3). This contribution is analogous to the "parton fragmentation" contribution (4) in the parton model. We note though that, since \( p_1^2 \approx q^2 \), finite \( x \) corresponds to \( y \) differing from \( \frac{1}{2} \ln s - \frac{1}{2} \ln(-q^2) \) by a finite amount rather than from \( y \approx \frac{1}{2} \ln s \) by a finite amount.
The explicit forms of the scaling functions will not be given here. We only note the behaviour near the phase space boundary \( x \approx -\frac{t}{2} \approx 1 \),

\[
F_{\eta L}^{x} h (\frac{t}{2}, \eta L) \sim \left[ \frac{1}{2} (1-x) \right]^{1-2y}
\]

and

\[
F_{\eta L}^{x} h (\frac{t}{2}, \eta L) \sim (1-x)^{1-2y}
\]

(17)

for \((13)\) and \((2)\), respectively. The behaviour is thus correlated with the asymptotic behaviour of form factors \((6)\). (A similar correlation also holds in explicit parton models and seems to be general knowledge amongst specialists although unpublished.) There is no triple Regge-like behaviour in the virtual photon fragmentation region contrary to the suggestions of some authors \((13)\).

The dual model can, of course, be applied to other processes. The diagram in Fig. 3d leads to corresponding non-parton-like behaviours. In \(e^+e^-\) annihilation into hadrons there is no two jet structure - i.e., the cross-section for \(e^+e^- \rightarrow h + h' + \text{anything}\) is not sharply peaked about momenta of \(h\) and \(h'\) in the same or opposite directions. In \(h + h' \rightarrow \mu^+\mu^- + \text{anything}\), \(p_{\perp}^2\) of the virtual photon grows like \(q^2\) in the central region \([this\ behaviour\ is\ also\ obtained\ in\ (4)]\).

In the dual model, the virtual photon couples to the hadrons through an infinite set of vector mesons. In parton language, one might imagine this corresponds to a strong binding of the partons - they can never get too far apart and appear in the final state since they must always form resonances. Conventional parton models do not have this property.

The structure of the photon propagator in quantum electrodynamics leads to "shrinkage" of the photon, and transverse momentum distributions growing like \(q^2\) \((16)\). Perhaps the scaling behaviour \(p_{\perp}^2/q^2\) for finite \(x\) is a manifestation of this phenomenon in the interaction of photons with composite hadrons. It would be very interesting to know what the physics of such a behaviour is. This is difficult to abstract from a dual model for which we have no space-time picture. It would be interesting to know if models such as Johnson's \((17)\) in which the hadron is a composite of deeply bound partons have the same behaviour.
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FIGURE CAPTIONS

Figure 1  Kinematics for $\gamma + h' \rightarrow h + \text{anything}$.

Figure 2  Mueller diagrams for
(a) virtual photon fragmentation,
(b) hadronic central, and
(c) target fragmentation regions.

Figure 3  Dual model for
(a) form factor,
(b) virtual photon total cross-section, and
(c) and (d) single particle inclusive cross-section.
Some inessential trajectories have been suppressed in (b),
(c) and (d).
Fig. 2
Fig. 3