Mass Segregation in Star Clusters

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Abstract. Star clusters – open and globulars – experience dynamical evolution on time scales shorter than their age. Consequently, open and globular clusters provide us with unique dynamical laboratories for learning about two-body relaxation, mass segregation from equipartition of energy, and core collapse. We review briefly, in the framework of star clusters, some elements related to the theoretical expectation of mass segregation, the results from N-body and other computer simulations, as well as the now substantial clear observational evidence.

1. Three Characteristic Time Scales

The dynamics of any stellar system may be characterized by the following three dynamical time scales: (i) the crossing time $t_{\text{cr}}$, which is the time needed by a star to move across the system; (ii) the two-body relaxation time $t_{\text{rlx}}$, which is the time needed by the stellar encounters to redistribute energies, setting up a near-maxwellian velocity distribution; (iii) the evolution time $t_{\text{ev}}$, which is the time during which energy-changing mechanisms operate, stars escape, while the size and profile of the system change.

Several (different and precise) definitions exist for the relaxation time. The most commonly used is the half-mass relaxation time $t_{\text{rh}}$ of Spitzer (1987, Eq. 2-62), where the values for the mass-weighted mean square velocity of the stars and the mass density are those evaluated at the half-mass radius of the system (see Meylan & Heggie 1997 for a review).

In the case of globular clusters, $t_{\text{cr}} \sim 10^6$ yr, $t_{\text{rlx}} \sim 100 \times 10^6$ yr, and $t_{\text{ev}} \sim 10 \times 10^9$ yr. Table 1 displays, for open clusters, globular clusters, and galaxies, some interesting relations between the above three time scales. For open clusters, crossing time $t_{\text{cr}}$ and relaxation time $t_{\text{rlx}}$ are more or less equivalent, both being significantly smaller than the evolution time $t_{\text{ev}}$. This means that most open clusters dissolve within a few gigayears. For galaxies, the relaxation time $t_{\text{rlx}}$ and the evolution time $t_{\text{ev}}$ are more or less equivalent, both being significantly larger than the crossing time $t_{\text{cr}}$. This means that galaxies are not relaxed, i.e., not dynamically evolved. It is only for globular clusters that all three time scales are significantly different, implying plenty of time for a clear dynamical evolution in these stellar systems, although avoiding quick evaporation altering open clusters.

Consequently, star clusters – open and globular – represent interesting classes of dynamical stellar systems in which some dynamical processes take
Table 1. Dynamical time scales for open clusters, globular clusters and galaxies

<table>
<thead>
<tr>
<th>System</th>
<th>$t_{cr}$</th>
<th>$t_{rlx}$</th>
<th>$t_{ev}$</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>open clusters</td>
<td>$t_{cr}$</td>
<td>$t_{rlx}$</td>
<td>$t_{ev}$</td>
<td>quickly dissolved</td>
</tr>
<tr>
<td>globular clusters</td>
<td>$t_{cr}$</td>
<td>$t_{rlx}$</td>
<td>$t_{ev}$</td>
<td>relaxed</td>
</tr>
<tr>
<td>galaxies</td>
<td>$t_{cr}$</td>
<td>$t_{rlx}$</td>
<td>$t_{ev}$</td>
<td>not relaxed</td>
</tr>
</tbody>
</table>

place on time scales shorter than their age, i.e., shorter than the Hubble time, providing us with unique dynamical laboratories for learning about two-body relaxation, mass segregation from equipartition of energy, stellar collisions, stellar mergers, and core collapse. All these dynamical phenomena are related to the internal dynamical evolution only, and would also happen in isolated globular clusters. The external dynamical disturbances — tidal stripping by the galactic gravitational field — influence equally strongly the dynamical evolution of globular clusters.

2. Observed in all N-body Calculations

Mass segregation was one of the early important results to emerge from computer N-body simulations of star clusters. See, e.g., von Hoerner (1960) who made the first N-body calculations with $N = 4, 8, 12, \text{ and } 16$ bodies. The heavier stars would gradually settle towards the center, increasing their negative binding energy, while the lighter stars would preferentially populate the cluster halo, with reduced binding energy. Later, direct integrations using many hundreds of stars showed the same tendency. Soon it was also realized that computation of individual stellar motions could be replaced by statistical methods. The same mass segregation was observed in models which integrated the Fokker-Planck equation for many thousands of stars (e.g., Spitzer & Shull 1975).

Mass segregation is expected from the basic properties of two-body relaxation. The time scale for dynamical friction to significantly decrease the energy of a massive star of mass $M$ is less than the relaxation time scale for lighter stars of mass $m$ by a factor $m/M$ (see, e.g., Eq. 14.65 in Saslaw 1985). As massive stars in the outer regions of a cluster lose energy to the lighter ones, they fall toward the center and increase their velocity. The massive stars continue to lose the kinetic energy they gain by falling and continue to fall. The lighter stars, on the other hand, increase their average total energy and move into the halo. As light stars rise through the system, their velocity decreases, altering the local relaxation time for remaining massive stars.

Will this mass segregation process have an end, i.e. will the system reach an equilibrium? Two conditions would have to be satisfied: mechanical equilibrium determined by the scalar virial theorem:

$$2\langle T \rangle + \langle W \rangle = 0$$

(1)
Figure 1. Mass stratification: the change, as a function of time, of the median radius $r_h$ for each component (heavy, medium, and light stars) of a three-subpopulation model, as obtained from a Monte-Carlo simulation by Spitzer & Shull (1975).

and thermal equilibrium determined by equipartition of energy among components of different mass $m_i$:

$$m_i \langle v_i^2 \rangle = 3kT$$  \hspace{1cm} (2)

All species must have the same temperature, so there is no energy exchange among the different species.

3. Mass Segregation from Photometric Observations

From a pure observational point of view, mass segregation has now been observed clearly in quite a few open and globular clusters. These observational constraints are essentially photometric: different families of stars, located in different areas of the color-magnitude diagram (CMD), exhibit different radial cumulative distribution functions. Such an effect, when observed between binaries and main sequence stars or between blue stragglers and turn-off stars, is generally interpreted as an indication of mass segregation between subpopulations of stars with different individual masses.

We present hereafter examples of observations of mass segregation in three different kinds of star clusters: (i) in the very young star cluster R136, (ii) in a few open clusters, and (iii) in a few globular clusters.

3.1. In the Very Young Star Cluster R136

The Large Magellanic Cloud star cluster NGC 2070 is embedded in the 30 Doradus nebula, the largest HII region in the Local Group (see Meylan 1993 for a review). The physical size of NGC 2070, with a diameter $\sim 40$ pc, is typical of old galactic and Magellanic globular clusters and is also comparable to the
size of its nearest neighbor, the young globular cluster NGC 2100. With an age of \( \sim 4 \times 10^6 \) yr (Meylan 1993, Brandl et al. 1996), NGC 2070 appears slightly younger than NGC 2100 which has an age of \( \sim 12-16 \times 10^6 \) yr (Sagar & Richtler 1991).

Brandl et al. (1996) obtained for R136, the core of NGC 2070, near-IR imaging in \( H, K \) bands with the ESO adaptive optics system ADONIS at the ESO 3.6-m telescope. They go down to \( K = 20 \) mag with 0.15" resolution over a 12.8" \( \times \) 12.8" field containing R136 off center. They present photometric data for about 1000 individual stars of O, B, WR spectral types. There are no red giants or supergiants in their field.

Brandl et al. (1996) estimate from their total \( K \) magnitude that the total stellar mass within 20" is equal to \( 3 \times 10^4 \, M_\odot \), with an upper limit on this value equal to \( 1.5 \times 10^5 \, M_\odot \). A star cluster with a mass of this range and a typical velocity dispersion of \( \sim 5 \, \text{km} \, \text{s}^{-1} \) would be gravitationally bound, a conclusion not immediately applicable to NGC 2070 because of the important mass loss due to stellar evolution experienced by a large number of its stars (see Kennicutt & Chu 1988).

Mass segregation may have been observed in R136, the core of NGC 2070. From their luminosity function, Brandl et al. (1996) determine, for stars more massive than 12 \( M_\odot \), a mean mass-function slope \( x = 1.6 \) \([x(\text{Salpeter}) = 1.35]\), but this value increases from \( x = 1.3 \) in the inner 0.4 pc to \( x = 1.6 \) for 0.4 pc \( < r < 0.8 \) pc, and to \( x = 2.2 \) outside 0.8 pc. The fraction of massive stars is higher in the center of R136. Brandl et al. (1996) attribute these variations to the presence of mass segregation. Given the very young age of this system, which may still be experiencing from violent relaxation, the cause of this mass segregation is not immediately clear. It may be due to a spatially variable initial mass function, a delayed star formation in the core, or the result of dynamical processes that segregated an initially uniform stellar mass distribution.

### 3.2. In Young and Old Open Clusters

Obviously, the older the cluster, the clearer the mass segregation effect. One of the first such clear cases was observed by Mathieu & Latham (1986) in M67 which, with an age of about 5 Gyr, is one of the oldest galactic open clusters. They studied the radial cumulative distribution functions of the following three families of stars: single stars, binaries, and blue stragglers, the latter being possibly the results of stellar mergers. The radial cumulative distribution functions of binaries and blue stragglers are similar and significantly more concentrated than the distribution function of the single stars. In such a dynamically relaxed stellar system, this result may be explained only by mass segregation between stars of different individual masses.

In one of the most recent such studies, Raboud & Mermilliod (1998) have observed some clear presence of mass segregation (see Fig. 1) in three open clusters – NGC 6231, the Pleiades, and Praesepe – which, with ages equal to 4, 100, 800 Myr, respectively, are significantly younger than M67. The presence of mass segregation in the Pleiades and Praesepe open clusters is expected given the fact that their relaxation times are shorter than their ages. This is not the case for NGC 6231, where the presence of mass segregation may be as problematic as it is in the case of R136.
3.3. In Globular Clusters

Because of their very high stellar densities, globular clusters have been hiding for decades any clear observational evidence of mass segregation, expected to be present essentially in their crowded central regions. Differences in the radial distributions of stars of different luminosities/masses have finally been definitely observed with HST, providing conclusive observational evidence of mass segregation in the central parts of globular clusters.

One of the most serious and detailed such studies is the one by Anderson (1997) who has used HST/FOC and HST/WFPC2 data to demonstrate the presence of mass segregation in the cores of three galactic globular clusters: M92, 47 Tucanae, and ω Centauri.

Anderson has first determined the luminosity function of each cluster at two different locations in the core. Then he has compared these luminosity functions with those from King-Michie multi-mass models, in the cases with and without mass segregation between the different stellar species.

Fig. 3 displays the comparison between the observed luminosity function (dots) and the model predictions with (continuous lines) and without (dashed lines) mass segregation, at the center of 47 Tucanae (left panel) and at one core radius from the center (right panel). The two different models differ strongly over a large range in magnitude (18-26 mag). The observed luminosity function shows a very clear agreement with the model containing mass segregation, and rules out completely any model without mass segregation (Anderson 1997).
Figure 3. Mass segregation as observed in the central parts of 47 Tucanae with HST/FOC data (Anderson 1997): the observed luminosity function (dots) agrees with the multi-mass King-Michie model with mass segregation (continuous lines) and fails totally to reproduce similar models without mass segregation (dashed lines).

The globular cluster M92 displays results very similar to those obtained for 47 Tucanae. This is not surprising given the fact that both clusters have rather similar structural parameters and concentrations, providing similar central relaxation times of the order of 100 Myr. This is not the case for ω Centauri, which is the most massive galactic globular cluster and has a central relaxation time of about 6 Gyr. As expected, the two model luminosity functions (with and without mass segregation) computed at the center of ω Centauri differ only slightly, and the observed luminosity function is right between the curves of the two models. The two model luminosity functions computed at 16′ from the center (at about 5 core radii) do not differ significantly and consequently agree similarly with the observations. As expected, ω Centauri, which has had hardly any time to become dynamically relaxed, even in its center, exhibits a very small amount of mass segregation (Anderson 1997).


4.1. Spitzer’s Equipartition Instability

As seen above, the various stellar species of a star cluster must have the same temperature in order to have equipartition of energy. Spitzer (1969) derived a criterion for equipartition between stars of two different masses $m_1$ and $m_2$. Let us consider the analytically tractable case where the total mass of the heavy stars, $M_2$, is much smaller than the core mass of the system of the lighter stars, $\rho_c V_{c1}^3$, and the individual heavy stars are more massive than the light stars, $m_2 > m_1$. In such a case, equipartition will cause the heavy stars (e.g., binaries
and/or neutron stars) to form a small subsystem in the center of the core of the system formed by the light (e.g., main sequence) stars.

In equipartition, \( m_2 \langle v_2^2 \rangle = m_1 \langle v_1^2 \rangle = 3m_1 \sigma^2 \), where \( \sigma \) represents the central one-dimensional dispersion of the light stars. It can be easily seen (e.g., Binney & Tremaine 1987) that equipartition cannot be satisfied unless the following inequality holds:

\[
\frac{M_2}{\rho c_1 r_{c1}^3} \leq \frac{1.61}{fg} \left( \frac{m_1}{m_2} \right)^{3/2}
\]  

(3)

where \( f \) and \( g \) are dimensionless constants. When \( M_2 \) become too large, the inequality is violated, there is the “equipartition instability” (Spitzer 1969), which has a simple physical explanation: when the mass in heavy stars is too large, these stars form an independent high-temperature self-gravitating system at the center of the core of light stars.

In a realistic system with a distribution of stellar masses, the chief effect of the equipartition instability is to produce a dense central core of heavy stars, which contracts independently from the rest of the core. However, as this core becomes denser and denser, the gravothermal instability dominates over the equipartition instability (Antonov 1962, Lynden-Bell & Wood 1968) and the cluster experiences core collapse (Makino 1996).

From an internal point of view, the dynamical evolution of star clusters is driven by two-body relaxation, mass segregation, equipartition instability, and core collapse. From an external point of view, the dynamical evolution of star clusters is driven by the dynamical disturbances due to the crossing of the galactic plane, which create tidal tails. In whatever location, these stellar systems are dynamically never at rest.

4.2. Mass Segregation in M15, a prototypical core-collapse cluster

The globular cluster M15 has long been considered as a prototype of the collapsed-core star clusters. High-resolution imaging of the center of M15 has resolved the luminosity cusp into essentially three bright stars. Post-refurbishment Hubble Space Telescope star-count data confirm that the 2.2” core radius observed by Lauer et al. (1991) and questioned by Yanny et al. (1994), is observed neither by Guhathakurta et al. (1996) with HST/WFPC2 data nor by Sosin & King (1996, 1997) with HST/FOC data. This surface-density profile clearly continues to climb steadily within 2”. It is not possible to distinguish at present between a pure power-law profile and a very small core (Sosin & King 1996, 1997). Consequently, among the galactic globular clusters, M15 displays one of the best cases of clusters caught in a state of deep core collapse.

Sosin & King (1997) have estimated the amount of mass segregation in the core of M15 from their HST/FOC data: the mass functions at 20” and 5’ from the center clearly show substantial mass segregation for all stars with masses between 0.55 and 0.80 \( M_\odot \).

- the MF at \( r = 20” \) is best fit by a power-law with slope \( x = -0.75 \pm 0.26 \),
- the MF at \( r = 5’ \) is best fit by a power-law with slope \( x = +1.00 \pm 0.25 \).

These two slopes differ at the 5-\( \sigma \) level. Once compared with models, the amount of mass segregation is somewhat less than predicted by a King-Michie
5. Mass Segregation from Kinematical Observations

Mass segregation is also present in kinematical data, i.e., in the radial velocities and proper motions of individual stars. So far, radial velocities have been obtained essentially only for the brightest stars, giants and subgiants, which have very similar masses.

It is only recently that internal proper motions of individual stars have been obtained in globular clusters. The following team (PI. G. Meylan, with CoIs. D. Minniti, C. Pryor, E.S. Phinney, B. Sams, C.G. Tinney, joined later by J. Anderson, I.R. King, and W. Van Altena) have acquired HST/WFPC2 images in the core of 47 Tucanae in three different epochs (Oct. 1995 - Nov. 1997 - Oct. 1999) defining a total time baseline of 4 years. The choice of the $U = FW300$ filter prevents saturation for the brightest stars and allows simultaneous measurement of proper motions for the brightest stars as well as for stars more than two magnitudes below the turn-off (Meylan et al. 1996).

For each epoch we have 15 images with careful dithering. Each measurement of the position of a star has a different bias since in each pointing the star is measured at a different pixel phase. We use an iterative process on positions and local PSF determinations. We achieve a position accuracy of 0.020 pixel for a single image, amounting to 0.006 pixel for the mean of 15 images. This corresponds to 0.3 mas in the PC frame and 0.6 mas in the WF2, WF3, and WF4 frames, for about 14,000 stars in the core of 47 Tucanae (Anderson & King in preparation).

Preliminary results show a clear difference between the proper motions of blue stragglers and stars of similar magnitudes: the former are significantly slower than the latter. Since blue stragglers are either binaries or mergers, with masses higher than the turn-off mass, the above difference unveils the first kinematical observation of mass segregation in a globular cluster (Meylan et al. in preparation).

References

Meylan G., Heggie D.C., 1997, A&AR, 8, 1-143

Questions - Answers - Comments

Comment by S. Portegies Zwart  From a theoretical point of view, it is not always clear what observers considered as the center of a star cluster and what theorists should use as the center. One can use, for example, the geometric center, the area with the highest luminosity density, number density, mass density.

Comment by G.M. From an observational point of view, the determination of the center of a globular cluster is also difficult and uncertain. Ideally, the algorithm used should determine the barycenter of the stars, not of the light. In the case of a collapsed globular cluster like M15, which has a very small unresolved core, the task is difficult because of the very small number of stars detectable in such a small area. The uncertainty in the position of the center is of the order of the core radius value, i.e., about 0.2". In the case of a globular cluster like 47 Tucanae, various methods give positions differing by 2-3". Such a large uncertainty is nevertheless acceptable, given the core radius value of about 25".
**Question by H. Zinnecker**  Is the mass segregation observed in the 30 Doradus cluster due to dynamical evolution or due to preferential birth of the more massive stars near the cluster center? Can we distinguish between these two possibilities?

**Answer by G.M.**  It is not known if the mass segregation is the consequence of dynamical evolution or of a flatter IMF in the center. I fail to see any reliable way to distinguish between these two possibilities.

**Question by P. Kroupa**  The globular clusters 47 Tucanae and ω Centauri do not appear to have a pronounced binary sequence in color-magnitude diagrams, whereas other globulars have pronounced binary sequences. Does this imply different dynamical histories?

**Answer by G.M.**  It would be interesting to compare the locations where these various color-magnitude diagrams have been obtained. In the case of 47 Tucanae, the excellent photometry we obtained is for stars right in the center, where encounters and collisions operate and probably decrease the fraction of binaries. It would be interesting to make a precise comparative study, for a few globular clusters, for which we would have data from the same instrument and reduced with the same software, in fields at the same relative distance from the center.

**Question by D. Calzetti**  About studies which find flatter IMFs in the centers of clusters: do you think that these studies may suffer from effects of crowding towards the cluster center, and therefore, find a flatter IMF because of this?

**Answer by G.M.**  Yes, definitely! Crowding is always present in photometry of globular clusters, especially in observations from the ground. Nevertheless, I think that some careful photometric studies using HST data have provided reliable results in relation to IMF slope (see, e.g., King et al., 1998, ApJ, 492, L37).

**Question by C. Boily**  Core motion: has the relative position of core to outer envelope been studied for candidate core-collapse clusters, e.g., 47 Tuc?

**Answer by G.M.**  It would be an interesting study, which, as far as I know, has not been done yet. It is partly due to the difficulties in determining precisely the center of the core and to the difficulties in determining precisely outer isophotes which suffer from very low star counts and are strongly polluted by foreground stars and background galaxies.