Contents

Introduction and Survey

I.1 Maxwell Equations in Vacuum, Fields, and Sources 2
I.2 Inverse Square Law, or the Mass of the Photon 5
I.3 Linear Superposition 9
I.4 Maxwell Equations in Macroscopic Media 13
I.5 Boundary Conditions at Interfaces Between Different Media 16
I.6 Some Remarks on Idealizations in Electromagnetism 19

References and Suggested Reading 22

Chapter 1 / Introduction to Electrostatics 24

1.1 Coulomb’s Law 24
1.2 Electric Field 24
1.3 Gauss’s Law 27
1.4 Differential Form of Gauss’s Law 28
1.5 Another Equation of Electrostatics and the Scalar Potential 29
1.6 Surface Distributions of Charges and Dipoles and Discontinuities in the Electric Field and Potential 31
1.7 Poisson and Laplace Equations 34
1.8 Green’s Theorem 35
1.9 Uniqueness of the Solution with Dirichlet or Neumann Boundary Conditions 37
1.10 Formal Solution of Electrostatic Boundary-Value Problem with Green Function 38
1.11 Electrostatic Potential Energy and Energy Density; Capacitance 40
1.12 Variational Approach to the Solution of the Laplace and Poisson Equations 43
1.13 Relaxation Method for Two-Dimensional Electrostatic Problems 47

References and Suggested Reading 50
Problems 50

Chapter 2 / Boundary-Value Problems in Electrostatics: I 57

2.1 Method of Images 57
2.2 Point Charge in the Presence of a Grounded Conducting Sphere 58
2.3 Point Charge in the Presence of a Charged, Insulated, Conducting Sphere 60
2.4 Point Charge Near a Conducting Sphere at Fixed Potential 61
2.5 Conducting Sphere in a Uniform Electric Field by Method of Images 62
2.6 Green Function for the Sphere; General Solution for the Potential 64
2.7 Conducting Sphere with Hemispheres at Different Potentials 65
Chapter 3 / Boundary-Value Problems in Electrostatics: II

3.1 Laplace Equation in Spherical Coordinates 95
3.2 Legendre Equation and Legendre Polynomials 96
3.3 Boundary-Value Problems with Azimuthal Symmetry 101
3.4 Behavior of Fields in a Conical Hole or Near a Sharp Point 104
3.5 Associated Legendre Functions and the Spherical Harmonics $Y_{lm}(\theta, \phi)$ 107
3.6 Addition Theorem for Spherical Harmonics 110
3.7 Laplace Equation in Cylindrical Coordinates; Bessel Functions 111
3.8 Boundary-Value Problems in Cylindrical Coordinates 117
3.9 Expansion of Green Functions in Spherical Coordinates 119
3.10 Solution of Potential Problems with the Spherical Green Function Expansion 112
3.11 Expansion of Green Functions in Cylindrical Coordinates 125
3.12 Eigenfunction Expansions for Green Functions 127
3.13 Mixed Boundary Conditions, Conducting Plane with a Circular Hole 129
References and Suggested Reading 135
Problems 135

Chapter 4 / Multipoles, Electrostatics of Macroscopic Media, Dielectrics

4.1 Multipole Expansion 145
4.2 Multipole Expansion of the Energy of a Charge Distribution in an External Field 150
4.3 Elementary Treatment of Electrostatics with Ponderable Media 151
4.4 Boundary-Value Problems with Dielectrics 154
4.5 Molecular Polarizability and Electric Susceptibility 159
4.6 Models for Electric Polarizability 162
4.7 Electrostatic Energy in Dielectric Media 165
References and Suggested Reading 169
Problems 169

Chapter 5 / Magnetostatics, Faraday's Law, Quasi-Static Fields

5.1 Introduction and Definitions 174
5.2 Biot and Savart Law 175
Chapter 6 / Maxwell Equations, Macroscopic Electromagnetism, Conservation Laws

6.1 Maxwell’s Displacement Current; Maxwell Equations 237
6.2 Vector and Scalar Potentials 239
6.3 Gauge Transformations, Lorenz Gauge, Coulomb Gauge 240
6.4 Green Functions for the Wave Equation 243
6.5 Retarded Solutions for the Fields: Jefimenko’s Generalizations of the Coulomb and Biot–Savart Laws; Heaviside–Feynman Expressions for Fields of Point Charge 246
6.6 Derivation of the Equations of Macroscopic Electromagnetism 248
6.7 Poynting’s Theorem and Conservation of Energy and Momentum for a System of Charged Particles and Electromagnetic Fields 258
6.8 Poynting’s Theorem in Linear Dissipative Media with Losses 262
6.9 Poynting’s Theorem for Harmonic Fields; Field Definitions of Impedance and Admittance 264
6.10 Transformation Properties of Electromagnetic Fields and Sources Under Rotations, Spatial Reflections, and Time Reversal 267
6.11 On the Question of Magnetic Monopoles 273
6.12 Discussion of the Dirac Quantization Condition 275
6.13 Polarization Potentials (Hertz Vectors) 280

References and Suggested Reading 282
Problems 283
## Chapter 7 / Plane Electromagnetic Waves and Wave Propagation 295

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>Plane Waves in a Nonconducting Medium</td>
<td>295</td>
</tr>
<tr>
<td>7.2</td>
<td>Linear and Circular Polarization; Stokes Parameters</td>
<td>299</td>
</tr>
<tr>
<td>7.3</td>
<td>Reflection and Refraction of Electromagnetic Waves at a Plane Interface Between Two Dielectrics</td>
<td>302</td>
</tr>
<tr>
<td>7.4</td>
<td>Polarization by Reflection, Total Internal Reflection; Goos–Hänchen Effect</td>
<td>306</td>
</tr>
<tr>
<td>7.5</td>
<td>Frequency Dispersion Characteristics of Dielectrics, Conductors, and Plasmas</td>
<td>309</td>
</tr>
<tr>
<td>7.6</td>
<td>Simplified Model of Propagation in the Ionosphere and Magnetosphere</td>
<td>316</td>
</tr>
<tr>
<td>7.7</td>
<td>Magnetohydrodynamic Waves</td>
<td>319</td>
</tr>
<tr>
<td>7.8</td>
<td>Superposition of Waves in One Dimension; Group Velocity</td>
<td>322</td>
</tr>
<tr>
<td>7.9</td>
<td>Illustration of the Spreading of a Pulse As It Propagates in a Dispersive Medium</td>
<td>326</td>
</tr>
<tr>
<td>7.10</td>
<td>Causality in the Connection Between D and E; Kramers–Kronig Relations</td>
<td>330</td>
</tr>
<tr>
<td>7.11</td>
<td>Arrival of a Signal After Propagation Through a Dispersive Medium</td>
<td>335</td>
</tr>
<tr>
<td>References and Suggested Reading</td>
<td>339</td>
<td></td>
</tr>
<tr>
<td>Problems</td>
<td>340</td>
<td></td>
</tr>
</tbody>
</table>

## Chapter 8 / Waveguides, Resonant Cavities, and Optical Fibers 352

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1</td>
<td>Fields at the Surface of and Within a Conductor</td>
<td>352</td>
</tr>
<tr>
<td>8.2</td>
<td>Cylindrical Cavities and Waveguides</td>
<td>356</td>
</tr>
<tr>
<td>8.3</td>
<td>Waveguides</td>
<td>359</td>
</tr>
<tr>
<td>8.4</td>
<td>Modes in a Rectangular Waveguide</td>
<td>361</td>
</tr>
<tr>
<td>8.5</td>
<td>Energy Flow and Attenuation in Waveguides</td>
<td>363</td>
</tr>
<tr>
<td>8.6</td>
<td>Perturbation of Boundary Conditions</td>
<td>366</td>
</tr>
<tr>
<td>8.7</td>
<td>Resonant Cavities</td>
<td>368</td>
</tr>
<tr>
<td>8.8</td>
<td>Power Losses in a Cavity; $Q$ of a Cavity</td>
<td>371</td>
</tr>
<tr>
<td>8.9</td>
<td>Earth and Ionosphere as a Resonant Cavity: Schumann Resonances</td>
<td>374</td>
</tr>
<tr>
<td>8.10</td>
<td>Multimode Propagation in Optical Fibers</td>
<td>378</td>
</tr>
<tr>
<td>8.11</td>
<td>Modes in Dielectric Waveguides</td>
<td>385</td>
</tr>
<tr>
<td>8.12</td>
<td>Expansion in Normal Modes; Fields Generated by a Localized Source in a Hollow Metallic Guide</td>
<td>389</td>
</tr>
<tr>
<td>References and Suggested Reading</td>
<td>395</td>
<td></td>
</tr>
<tr>
<td>Problems</td>
<td>396</td>
<td></td>
</tr>
</tbody>
</table>

## Chapter 9 / Radiating Systems, Multipole Fields and Radiation 407

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.1</td>
<td>Fields and Radiation of a Localized Oscillating Source</td>
<td>407</td>
</tr>
<tr>
<td>9.2</td>
<td>Electric Dipole Fields and Radiation</td>
<td>410</td>
</tr>
<tr>
<td>9.3</td>
<td>Magnetic Dipole and Electric Quadrupole Fields</td>
<td>413</td>
</tr>
<tr>
<td>9.4</td>
<td>Center-Fed Linear Antenna</td>
<td>416</td>
</tr>
<tr>
<td>9.5</td>
<td>Multipole Expansion for Localized Source or Aperture in Waveguide</td>
<td>419</td>
</tr>
</tbody>
</table>
## Contents

9.6 Spherical Wave Solutions of the Scalar Wave Equation 425  
9.7 Multipole Expansion of the Electromagnetic Fields 429  
9.8 Properties of Multipole Fields, Energy and Angular Momentum of Multipole Radiation 432  
9.9 Angular Distribution of Multipole Radiation 437  
9.10 Sources of Multipole Radiation; Multipole Moments 439  
9.11 Multipole Radiation in Atoms and Nuclei 442  
9.12 Multipole Radiation from a Linear, Center-Fed Antenna 444  

References and Suggested Reading 448  
Problems 449  

Chapter 10 / Scattering and Diffraction 456  

10.1 Scattering at Long Wavelengths 456  
10.2 Perturbation Theory of Scattering, Rayleigh’s Explanation of the Blue Sky, Scattering by Gases and Liquids, Attenuation in Optical Fibers 462  
10.3 Spherical Wave Expansion of a Vector Plane Wave 471  
10.4 Scattering of Electromagnetic Waves by a Sphere 473  
10.5 Scalar Diffraction Theory 478  
10.6 Vector Equivalents of the Kirchhoff Integral 482  
10.7 Vectorial Diffraction Theory 485  
10.8 Babinet’s Principle of Complementary Screens 488  
10.9 Diffraction by a Circular Aperture; Remarks on Small Apertures 490  
10.10 Scattering in the Short-Wavelength Limit 495  
10.11 Optical Theorem and Related Matters 500  

References and Suggested Reading 506  
Problems 507  

Chapter 11 / Special Theory of Relativity 514  

11.1 The Situation Before 1900, Einstein’s Two Postulates 515  
11.2 Some Recent Experiments 518  
11.3 Lorentz Transformations and Basic Kinematic Results of Special Relativity 524  
11.4 Addition of Velocities; 4-Velocity 530  
11.5 Relativistic Momentum and Energy of a Particle 533  
11.6 Mathematical Properties of the Space-Time of Special Relativity 539  
11.7 Matrix Representation of Lorentz Transformations, Infinitesimal Generators 543  
11.8 Thomas Precession 548  
11.9 Invariance of Electric Charge; Covariance of Electrodynamics 553  
11.10 Transformation of Electromagnetic Fields 558  
11.11 Relativistic Equation of Motion for Spin in Uniform or Slowly Varying External Fields 561  
11.12 Note on Notation and Units in Relativistic Kinematics 565  

References and Suggested Reading 566  
Problems 568
Chapter 12 / Dynamics of Relativistic Particles and Electromagnetic Fields

12.1 Lagrangian and Hamiltonian for a Relativistic Charged Particle in External Electromagnetic Fields 579
12.2 Motion in a Uniform, Static Magnetic Field 585
12.3 Motion in Combined, Uniform, Static Electric and Magnetic Fields 586
12.4 Particle Drifts in Nonuniform, Static Magnetic Fields 588
12.5 Adiabatic Invariance of Flux Through Orbit of Particle 592
12.6 Lowest Order Relativistic Corrections to the Lagrangian for Interacting Charged Particles: The Darwin Lagrangian 596
12.7 Lagrangian for the Electromagnetic Field 598
12.8 Proca Lagrangian; Photon Mass Effects 600
12.9 Effective "Photon" Mass in Superconductivity; London Penetration Depth 603
12.10 Canonical and Symmetric Stress Tensors; Conservation Laws 605
12.11 Solution of the Wave Equation in Covariant Form; Invariant Green Functions 612

References and Suggested Reading 615
Problems 617

Chapter 13 / Collisions, Energy Loss, and Scattering of Charged Particles, Cherenkov and Transition Radiation

13.1 Energy Transfer in Coulomb Collision Between Heavy Incident Particle and Free Electron; Energy Loss in Hard Collisions 625
13.2 Energy Loss from Soft Collisions; Total Energy Loss 627
13.3 Density Effect in Collisional Energy Loss 631
13.4 Cherenkov Radiation 637
13.5 Elastic Scattering of Fast Charged Particles by Atoms 640
13.6 Mean Square Angle of Scattering; Angular Distribution of Multiple Scattering 643
13.7 Transition Radiation 646

References and Suggested Reading 654
Problems 655

Chapter 14 / Radiation by Moving Charges

14.1 Liénard–Wiechert Potentials and Fields for a Point Charge 661
14.2 Total Power Radiated by an Accelerated Charge: Larmor's Formula and Its Relativistic Generalization 665
14.3 Angular Distribution of Radiation Emitted by an Accelerated Charge 668
14.4 Radiation Emitted by a Charge in Arbitrary, Extremely Relativistic Motion 671
14.5 Distribution in Frequency and Angle of Energy Radiated by Accelerated Charges: Basic Results 673