COSMOLOGY AND ASTROPHYSICS

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Abstract
We review in this series of lectures the basics of cosmology. The main topics covered are: Friedmann equations for expanding Universe, physical processes in the early Universe, nucleosynthesis, baryogenesis, inflation and cosmological parameters, dark matter.

1 INTRODUCTION
At first sight, cosmology and particle physics seem to be completely unrelated branches of physics. The goal of particle physics is to describe elementary particles and fundamental interactions between them at small scales, say, $l < 10^{-14}$ cm. On the contrary, the goal of astronomy and cosmology is to describe the structure of the Universe at very large length scales, $l > 10$Kpc $\simeq 10^{22}$ cm. Can we learn anything from cosmology for particle physics? Can we learn anything from particle physics for cosmology? The bridge between particle physics and cosmology is provided by the evolution of the Universe. Observations of the present Universe give us information about the early Universe. Evolution of the early Universe depends crucially on the properties of elementary particles and interactions between them. This fact provides some constraints on particle physics theories, in some cases superior to those coming from terrestrial experiments. The list of traditional cosmological bounds involves neutrino masses and numbers of neutrino species, properties of hypothetical particles, physics at the Planck scale, testing of different conservation laws, etc. On the other hand, progress in particle physics has led to a number of advances in modern cosmology. The non-conservation of baryon number, arising naturally in unified theories of strong, weak and electromagnetic interactions and in the electroweak theory itself, has led to qualitative understanding of the absence of antimatter in the Universe; new stable particles, predicted by supersymmetric theories, may play the role of dark matter in the Universe; consideration of phase transitions in particle-physics models has led to the suggestion of a new paradigm in cosmology - inflation. So “simple” a thing as the dynamics of a free quantum scalar field in the expanding Universe proposes a solution of a number of outstanding problems in cosmology, such as flatness, horizon, homogeneity and structure formation. There is a number of excellent textbooks on cosmology (e.g. [1, 2, 3]) which a reader can consult for a thorough study of the subject.

The plan of the lectures is as follows. First, we are going to note the basic facts about the Universe: Hubble expansion, existence of a cosmic microwave background (CMB), large-scale isotropy and homogeneity. We shall consider different elements of standard cosmology: Friedman equations, the evolution of the Universe with dominance of radiation, matter or cosmological constant. Turning to the study of the early universe, we shall consider photon and neutrino decoupling, nucleosynthesis and elements of baryo- and leptogenesis. Another topic is inflation: we shall discuss the problems of the standard cosmological model and their solutions with inflation and study the dynamics of the scalar field that provides a simple particle-physics model for the inflationary Universe. The last topic is the determination of cosmological parameters (accelerating Universe and power spectra of CMB), the problems of dark matter and of the cosmological constant. Unless otherwise specified, we are going to use the natural system of units, in which $\hbar = c = 1$ and energy is measured in GeV.
2 BASIC FACTS ABOUT THE UNIVERSE

2.1 Expansion

There are several types of objects in the Universe that can be considered as “standard candles”. This means that we know their total luminosity $L$ and the spectrum of emitted light at the position they find themselves. Examples of such objects include supernovae of the type Ia, first-ranked E galaxies in nearby groups and clusters, first-ranked cluster galaxies in rich clusters, etc. [4]. With this knowledge, we can find for the number of objects the red shifts $z$, defined as:

$$z = \frac{\lambda_{\text{rec}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}}$$

(1)

where $\lambda_{\text{emit}}$ and $\lambda_{\text{rec}}$ are the wavelengths of the emitted and received light respectively. Simultaneously, one can find the distance $r$ to the corresponding object, by measuring the energy flux $f$ (apparent brightness) from the star or galaxy, through the equation

$$f = \frac{L}{4\pi r^2}$$

(2)

In astronomy, the so-called apparent magnitude $m$ is often used instead of distance:

$$m = -2.5 \log_{10} f + \text{const} \times 5 \log_{10} r + \text{const}.$$  

(3)

Very interestingly, the dependence of red shift on the apparent magnitude has a universal character and does not depend on the type of the object, on the frequency of the emitted light or on the direction in the sky, see Fig. 1.

The universal character of this dependence indicates that it is a property of the universe as a whole rather than of a particular object in it. An explanation of the red shift can be provided by the Doppler
effect, as the relation between the frequencies of emitted and received light is given by

\[ z = \frac{\sqrt{1 + v}}{\sqrt{1 - v}} - 1 \simeq v, \tag{4} \]

for \( v \ll 1 \), where \( v \) is the relative velocity of the emitter to the receiver. The slope of the curve in Fig. 1 happens to be such that we get a linear relation between the speed of runaway galaxies and the distances between them,

\[ \ddot{v} = \dot{r} = Hr, \tag{5} \]

which is the famous Hubble law, telling us that the universe expands in a homogeneous and isotropic way. The isotropy of the expansion is obvious since both sides of this equation are vectors: homogeneity of the expansion is a consequence of the fact that the parameter \( H \) (called the Hubble constant) does not depend on space coordinates (but may depend on time). A way to understand the expansion of the Universe is to imagine that the unit of length increases in time. One should stress, however, that for gravitationally bounded systems (e.g. the Solar system or a galaxy) this law is not applicable.

Since the universal expansion does not depend on the direction, one can introduce an overall scale factor \( a \) and write \( H = \frac{\dot{a}}{a} \).

The speed of light being finite, observation of the sources at large distances means that we observe them in the past. Another way to write the red shift is

\[ z = \frac{a_{\text{now}}}{a_{\text{emit}}} - 1, \tag{6} \]

where \( a_{\text{now}} \) and \( a_{\text{emit}} \) are the scale factors of the Universe at the present time and at the time the light was emitted. This follows from the fact that the frequency \( \omega \) of light changes in an expanding universe in such a way that \( \omega a = \text{const} \), which is easy to understand because the product \( \omega a \) just shows the number of wavelengths in a box of the size \( a \) and this number does not change if the size of the box changes.

2.2 Cosmic microwave background radiation

In 1965, cosmic microwave background radiation was discovered by Penzias and Wilson. It is isotropic and has a thermal equilibrium Planck spectrum with temperature \( T = 2.73 \text{ K} \), see Fig. 2,

\[ dI_\nu \sim \frac{\nu^3 d\nu}{\exp(\frac{\nu}{T}) - 1}, \tag{7} \]

where \( I \) is the energy density.

The CMB was theoretically predicted by Gamow back in 1946. The logic is in fact quite simple. We know that the Universe expands and that it is isotropic and homogeneous at large scales. Therefore it was dense in the past and looked like a uniform soup of different elementary particles which were close to each other. Hence, reactions between particles were rapid enough, which meant that the system was driven to a state of thermal equilibrium. Thermal equilibrium is characterized by a specific spectrum, namely the Plank distribution for photons. Thus, the CMB we observe today is simply the equilibrium spectrum of relic photons, red-shifted to the present time.

2.3 Large-scale isotropy and homogeneity

The Universe is believed to be isotropic and homogeneous at large scales, say at \( l > 100 \text{ Mps} \). The best limit is based on the observations of isotropy of the CMB for which the variation of the CMB temperature is \( \frac{\Delta T}{T} < 10^{-5} \). Other evidence stems from the isotropy of the diffusive \( \gamma \)-ray background and from distributions of galaxies. Of course, the distribution of matter on smaller scales is lumpy, as structures such as clusters and superclusters, galaxies, the Solar system, stars and planets are known to exist.
3 STANDARD COSMOLOGY

3.1 Non-relativistic matter

The aim of this section is to derive the equations that describe the evolution of the Universe. We will start from an approach based on Newtonian gravity (valid for local properties and for non-relativistic equations of state, \( p \ll \epsilon \), where \( p \) is the pressure and \( \epsilon \) is the energy density).

Suppose that we have a uniform distribution of matter. Let us select a sufficiently large, spherically symmetric, portion of space, with radius \( R \) (\( R \) must be larger than the scale of a typical inhomogeneity) and total mass \( M \), Fig. 3. The energy density then can be written as

\[
\epsilon = \frac{M}{\frac{4}{3} \pi R^3} .
\]

In an expanding Universe the radius of this sphere changes and

\[
\frac{\dot{R}}{R} = H ,
\]
whereas the total mass $M$ is independent of time. Differentiating eq. (8) with respect to time and using eq. (9) gives

$$\frac{d\varepsilon}{dt} = -3H\varepsilon.$$  

(10)

This equation is nothing but energy conservation. We should now determine how the Hibble constant depends on time. To this end we may use a dynamical equation for $R$, which is Newton’s second law plus a law of gravity:

$$\frac{d^2 R}{dt^2} = -\frac{GM}{R^2},$$  

(11)

where $G$ is the Newton constant. Using eqns. (8,9),

$$\frac{dH}{dt} = -H^2 - \frac{4\pi G\varepsilon}{3}.$$  

(12)

Equations (12) and (10) do not depend on the intermediate parameters $\varepsilon$ and $\varrho$ and can be used for the determination of all local properties of a homogeneous and isotropic universe containing nothing but non-relativistic matter.

To analyse these equations from a qualitative point of view, we rewrite eq. (11) in the form of energy conservation by multiplying it by $W\varepsilon$ and integrating over $\varrho$:

$$\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G\varepsilon_0 R_0^3}{3R} - \frac{8\pi G R_0^2}{3} (\varepsilon_0 - \rho_c),$$  

(13)

where index 0 refers to the present time and the parameter $\rho_c$, called the critical density, is given by

$$\rho_c = \frac{3H_0^2}{8\pi G}.$$  

(14)

The constant $H_0$ is usually parametrized as $H_0 = 100h \frac{km}{s\cdot Mpc}$ where $h$ is taken from observations, $0.5 < h < 0.8$. Smaller error bars can be found, for example, in [4]: $h = 0.58 \pm 0.063$. Numerically, the critical density is $\rho_c = 1.88h^2 \cdot 10^{-29}$ g/cm$^3$.

Consider first what happened in the past, assuming the validity of eq. (13). Since the Universe expands, the first term of eq. (13) dominates and for $t \to 0$ we can write

$$\left(\frac{dR}{dt}\right)^2 \sim \frac{8\pi G\varepsilon_0 R_0^3}{3R}.$$  

(15)

This equation can be easily integrated with a solution $R \sim t^{2/3}$. Thus, one expects to have a “singularity” in the past, when for $t \to 0$ we have $R \to 0$ and $\dot{R} \to \infty$. We should stress, however, that the starting equations are not correct in this limit: in the first place, the equation of state is ultrarelativistic near $t = 0$; in the second, classical physics with classical gravity is hardly likely to be a correct theory near the singularity, where quantum gravity effects must be important.

Let us see now what will happen in the future, depending on parameter $\Omega = \frac{\rho_\varphi}{\rho_c}$. Three different cases can be considered.

(i) $\Omega > 1$. With the expansion of the Universe the first term of eq. (13) decreases, and the right-hand side of (13) eventually becomes equal to zero at some moment of time. After that, the expansion of the Universe changes to a contraction $\dot{R} < 0$ and Universe collapses eventually, see Fig. 4. As we will discuss later, in this case the universe is spatially closed, i.e. it has a finite volume at every moment of time and its spatial curvature is positive.

(ii) $\Omega < 1$. In this case the right-hand side of eq. (13) is always positive, and the Universe expands forever, see Fig. 5. The Universe is said to be spatially open and it has an infinite volume at any moment of time; its spatial curvature is negative.
(iii) $\Omega = 1$. In this case the solution to eq. (13) can be readily found in the closed form,

$$R = R_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}},$$  \hspace{1cm} (16)

where the age of the Universe $t_0$ is related to the Hubble constant as $t_0 = \frac{2}{3H_0}$.

### 3.2 General relativity analysis

In the previous subsection we considered non-relativistic matter with equation of state $p = 0$. There are two other important equations of state that appear in relativistic analysis. The first one is that for relativistic matter,

$$p = \epsilon/3.$$  \hspace{1cm} (17)

This is adequate for photons, for massless neutrinos and, in general, for any type of particle if its kinetic energy is much greater than its rest mass. The second one is related to the vacuum energy or, equivalently, to the cosmological constant. It has the equation of state

$$p = -\epsilon.$$  \hspace{1cm} (18)

This is quite a peculiar equation of state which is in fact Lorentz-invariant. This can be seen by consideration of the energy-momentum tensor. For a uniform isotropic substance it has a diagonal form

$$T_{\mu\nu} = \text{diag}(\epsilon, p, p, p).$$  \hspace{1cm} (19)

The Lorentz-invariant form of $T_{\mu\nu}$, specific for the vacuum state, is

$$T_{\mu\nu} = \text{const} \cdot g_{\mu\nu} = \text{const} \cdot \text{diag}(1, -1, -1, -1),$$  \hspace{1cm} (20)

which gives exactly the equation of state (18).
The modifications of our previous evolution equations (12,10) are written as

\[
\frac{d\varepsilon}{dt} = -3H(\varepsilon + p), \tag{21}
\]
\[
\frac{dH}{dt} = -H^2 - \frac{4\pi G(\varepsilon + 3p)}{3}. \tag{22}
\]

For \( p = 0 \), they coincide with (12,10). As for the non-relativistic case, we can integrate one of the equations (22,21) to bring it into the form of energy conservation,

\[
\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G\varepsilon}{3}, \tag{23}
\]

and define, as previously, the parameter \( \Omega \) and the critical density. Now, depending on the sign of the integration constant \( k \) (in fact, by rescaling of the scale factor \( a \) the value of \( k \) can always be chosen to be \( k = 1, -1 \) or 0) we get the closed, open or spatially flat Universe, respectively, with finite or infinite volumes.

A way to derive (21,22) is to write the Einstein equations

\[
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu}, \tag{24}
\]

where \( R_{\mu\nu} \) is the Ricci tensor, \( R \) is the scalar curvature, and the energy momentum tensor is

\[
T_{\mu\nu} = -p g_{\mu\nu} + (\varepsilon + p) u_{\mu}u_{\nu}, \tag{25}
\]

with \( u_{\mu} \) being the four-velocity of the medium. The corresponding metric is that of Friedman-Robertson-Walker. In spherical coordinates \( r, \theta, \phi \),

\[
ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right). \tag{26}
\]

We take \( k = 0 \) for simplicity and consider several important cases.

(i) The radiation-dominated Universe, \( p = \varepsilon/3 \). Here

\[
\varepsilon = \frac{3}{32\pi G} \frac{1}{t^2}, \quad a = a_0 \left( \frac{t}{t_0} \right)^{\frac{1}{2}}, \quad H = \frac{1}{2t}. \tag{27}
\]

(ii) The matter-dominated Universe, \( p = 0 \). Here

\[
\varepsilon = \frac{1}{6\pi G} \frac{1}{t^2}, \quad a = a_0 \left( \frac{t}{t_0} \right)^{\frac{2}{3}}, \quad H = \frac{2}{3t}. \tag{28}
\]

(iii) The vacuum-energy-dominated Universe, \( \varepsilon = -p, \varepsilon > 0 \). Here

\[
\varepsilon = \text{const}, \quad a = a_0 \exp(+Ht), \quad H = \text{const} = \sqrt{\frac{8\pi G\varepsilon}{3}}. \tag{29}
\]

The last equation may look counter-intuitive since, in spite of the expansion of the Universe, the energy density does not change. This is related to the fact that the vacuum pressure is negative and it performs negative work which keeps the energy density exactly constant.

A more general case is a mixture of radiation, non-relativistic matter and the vacuum-energy density. Let us introduce different densities, specific for each type of matter, \( \Omega_M = \frac{\varepsilon_M}{\rho_c}, \quad \Omega_\nu = \frac{\varepsilon_\nu}{\rho_c}, \quad \Omega_\Lambda = \)
\[ \frac{a}{\rho_c}, \] where the indices \( M, r \) and \( \Lambda \) refer to the contributions of matter, radiation and vacuum energy respectively. As the Universe expands, different components of the energy scale in the following way:

\[ \epsilon_M \sim a^{-3}, \quad \epsilon_r \sim a^{-4}, \quad \epsilon_\Lambda \sim \text{const}, \tag{30} \]

which follows from eqns. (27,28,29) and is easy to understand. The equation for matter tells us that the total energy of non-relativistic matter is conserved, the equation for radiation shows that the total number of photons and other light particles is conserved, while their energy is redshifted. Thus, eq. (23) can be written as

\[ H^2 = H_0^2 \left( \Omega_r \frac{a_0^4}{a^4} + \Omega_M \frac{a_0^3}{a^3} + \Omega_\Lambda + \Omega_k \frac{a_0^2}{a^2} \right), \tag{31} \]

where the curvature contribution, \( \Omega_k = \frac{k}{a_0^2 H_0^2} \) has been introduced for uniformity of notation. As before, the index 0 refers to the present moment of the Universe expansion and we have

\[ \Omega_r + \Omega_M + \Omega_\Lambda + \Omega_k = 1. \tag{32} \]

It is clear that the dominant component of energy density in the early Universe is that related to radiation. Later on, matter dominates. The curvature contribution, potentially important for the evolution of the Universe at later time, happens to be numerically unimportant, and the \( \Lambda \) term dominates. The schematic dependence of the scale factor on time is represented in Fig. 6. The moment when the matter energy density starts to dominate the radiation, \( \Omega_M = \Omega_r \), is important for structure formation. This happens at red shift \( z_{\text{eq}} \) approximately equal to \( z_{\text{eq}} = 3.1 \cdot 10^4 \Omega_M h^2 \approx 3900 \) and corresponds to the age of the Universe \( t_{\text{eq}} = 7 \cdot 10^4 \) years.

![Fig. 6: Dependence of the scale factor on time.](image)

### 4 PHYSICAL PROCESSES IN THE EARLY UNIVERSE

As we have seen, the particle number densities in the Universe scale as \( n \sim 1/a^3 \) and are thus very high in the initial stages of the Universe expansion. The average distance between particles decreases as \( l \sim a \) if we go back in time and therefore collisions between particles happen quite often. Now, if the rate of collision is greater than the rate of expansion of the Universe, the system should be close to a state of thermal equilibrium, characterized by some temperature \( T \) and, perhaps, by a number of chemical
potentials, associated with different conserved numbers. The statement about thermal equilibrium is very powerful. It immediately allows one to express the dependence of particle distribution on momentum and to compute the rate of different reactions. Eventually, the rates can be compared with the rate of expansion of the Universe and the assumption about thermal equilibrium may be verified.

In this section we will consider the radiation-dominated Universe, although most of what follows is valid for other cases as well.

The equilibrium particle number distributions are

$$n(p) = \frac{1}{e^{\frac{E - \mu}{T}} \pm 1},$$

where the plus sign refers to fermions and the minus sign to bosons, $E$ is the energy of the particle and $\mu$ is the chemical potential. In cosmology the values of the chemical potentials, at least in the radiation-dominated epoch, are rather small and may be omitted for most purposes.

In the relativistic limit $t \gg m$, where $m$ is the mass of a particle, the energy density can be expressed as

$$\epsilon = \frac{\pi^2}{30} g_* T^4, \quad p = \frac{\epsilon}{3},$$

where $g_* = g_B + \frac{7}{8} g_f$ is the effective number of massless degrees of freedom, $g_B$ and $g_f$ are the corresponding numbers for bosons and fermions. This equation allows one to write a relation between the temperature and the expansion time, combining eqs. (27) and (34):

$$t = 0.301 \frac{M_{Pl}}{\sqrt{g_* T^2}} = \frac{M_0}{T^2},$$

where $M_{Pl}$ is the Planck mass, related to Newton’s gravitational constant as $M_{Pl} = G^{-\frac{1}{2}} \approx 1.2 \cdot 10^{19}$ GeV. To appreciate the orders of magnitude, here is an equation to remember: $t[s] = 1/T[\text{MeV}]^2$.

In the non-relativistic limit, $T \ll m$, the particle-number densities are

$$n = g_{B,f} \left( \frac{m T}{2 \pi} \right)^\frac{3}{2} e^{-\frac{m^2 \mu}{2T^2}}.$$

This leads to the energy density $\epsilon = mn$ and to the pressure $p \sim nT \ll \epsilon$.

The assumption about thermal equilibrium is not valid if the Universe expands faster than the reactions can equilibrate. To find what happens with particle densities in this case, one should write kinetic equations, taking into account the expansion of the Universe and particle collisions. We are going to omit technical details here and use instead the so-called freeze-out approximation. Consider, for example, collisions of stable particles. To some level of accuracy one would expect the particle concentrations to follow their equilibrium values, if $\Gamma > H$, where

$$\Gamma = \frac{1}{\langle \sigma v \rangle}$$

is an average collision rate, $\sigma$ is a cross-section of the reaction, and $v$ is the relative velocity of colliding particles. Now, if $\Gamma < H$ the particles roughly stop interacting and their number does not change because of collisions so that concentration “freezes” at the value of their fluctuation at the moment of “last scattering”, when

$$\Gamma < H.$$

There are plenty of phenomena that can be associated with freezing (or decoupling) of different interactions. We are going to discuss decoupling of photons (freezing of electromagnetic interactions), of neutrino (weak interactions), nucleosynthesis and baryogenesis.
4.1 Decoupling of photons

If the temperature of the Universe is larger that the binding energy of electrons in atoms, the cosmic plasma is ionized and the mean free path of photons is rather small so that photons are in thermal equilibrium. When the temperature drops, plasma neutralizes and the photons no longer interact with matter but propagate freely. The cosmic microwave radiation, which is observed today, is a snapshot of the Universe at the moment of decoupling. Thus, by the study of CMB today we may find the temperature and matter-density fluctuations, existing at redshifts associated with the photon decoupling.

To estimate the temperature of decoupling one notes that the main reactions to be taken into account are the scattering of photons on electrons, \( \sigma_{\gamma e} \sim (\gamma p) \) (the cross-section of the \( \gamma p \) reaction is much smaller) and the reaction of hydrogen (\( H \)) dissociation, \( e p \leftrightarrow H \gamma \), that controls the concentration of free electrons. When the second reaction is in thermal equilibrium, concentrations of electrons (\( n_e \)), protons (\( n_p \)) and of the hydrogen atoms (\( n_H \)) are related by the Saha formula

\[
\frac{n_e n_p}{n_H} = g_{Bf} \left( \frac{m_e T}{2\pi} \right)^{3/2} \exp \left( - \frac{I}{T} \right),
\]

where \( I = 13.6 \text{ eV} = 1.58 \cdot 10^5 \text{ K} \) is the ionization energy. The decoupling moment is determined by the solution of the equation \( \sigma_{\gamma e} n_e \simeq t \), where the Compton scattering cross-section is \( \sigma_{\gamma e} = \frac{8\pi \alpha^2}{3m_e^2} \). The system of equations is closed by adding the condition of plasma neutrality (\( n_e = n_p \)) and introducing as an input parameter the ratio of baryon number (\( n_B \)) to the number of photons (\( n_\gamma \)),

\[
\frac{n_B}{n_\gamma} = \frac{n_p + n_H}{n_\gamma} = \eta_{10} \cdot 10^{-10},
\]

where \( \eta_{10} \simeq (1.5 - 6.3) \) is an observational limit coming from consideration of the abundance of light elements (see below subsection 4.3). In addition, the relation between time and temperature can be taken from eq. (35). Numerically, \( T^* = 0.25 \text{ eV} = 3000 \text{ K}, z = 1100 \), which corresponds to the age of the Universe \( t_{dec} \simeq 5 \cdot 10^5 \text{ years} \).

4.2 Decoupling of neutrinos

Let us address the question of the temperature and concentration of relic neutrinos at the present time. Neutrinos interact with other particles via charged and neutral currents, see Fig. 7, and are at thermal equilibrium up to the temperature \( T^* \) which can be found from the freezing condition (38),

\[
\langle \sigma v \rangle \sim G_F^2 T^2 \cdot T^3 \simeq \frac{T^2}{M_0},
\]

where \( G_F \) is the Fermi coupling constant. From here \( m_e \leq T^* \leq \text{few MeV} \). Thus, at \( T = T^* \) we have an equilibrium mixture of \( e^+, e^-, \gamma \), together with all types of neutrinos and antineutrinos. At \( T < m_e \) electrons and positrons annihilate into photons, but not into neutrinos, since \( \nu \) are out of thermal equilibrium. During adiabatic expansion of the Universe entropy is conserved, so that

\[
\left[ 2(\text{photons}) + \frac{7}{8} (2(\text{electrons}) + 2(\text{positrons})) \right] (T_{in}a_{in})^3 = 2(\text{photons}) (T_\gamma a_{\gamma})^3,
\]

where words in brackets refer to the corresponding particles. For neutrinos one can write:

\[
(T_{in}a_{in})^3 = (T_\nu a_{\nu})^3,
\]

\footnote{Strictly speaking, the decoupling of photons occurs in the epoch of matter dominance, shortly after the moment at which \( \Omega_M = \Omega_\gamma \). Accounting for this fact has only a slight influence on the estimate.}
and, therefore,
\[ T_\nu = \left( \frac{4}{11} \right)^{\frac{1}{3}} T_\gamma \simeq 2 \text{ K}, \]  
(43)
which gives the relic concentration of neutrinos of each kind, \( n_\nu = \frac{3 \xi_\nu T_\nu^3}{4 \pi^2} \simeq 56 \text{ cm}^{-3} \). From this one can obtain constraints on neutrino masses from cosmology, assuming that neutrinos are stable particles (or that their lifetime is greater than the age of the Universe). Indeed, the energy density of relic neutrinos should be smaller than the critical density,
\[ \epsilon_\nu = \sum (n_\nu + n_\bar{\nu}) m_\nu < \rho_{crit} \]  
(44)
giving the constraint \( \sum m_\nu < 100 h^2 \text{ eV} \simeq 40 \text{ eV} \).

### 4.3 Nucleosynthesis

If the temperature of the Universe is greater than the binding energies of protons and neutrons in nuclei, the primordial plasma consists of nucleons rather than nuclei. At smaller temperatures it is energetically more favorable to hide protons and neutrons in nuclei. The question arises whether all chemical content of the Universe can be explained by the nuclear reactions occurring at \( T \sim 1 \text{ MeV} \). If not, which elements can be created?

It is clear that deviations from thermal equilibrium coming from the expansion of the Universe play an important role in nucleosynthesis. Indeed, in thermal equilibrium all baryon number would reside in nuclei with the maximal binding energy per nucleon, which is \(^{56}\text{Fe}!\) Thus, the dynamics of decoupling of different nuclear reactions must be taken into account. Nuclear abundances are obtained from the solution of a system of kinetic equations incorporating different processes in the expanding Universe. There are various computer codes written for this purpose, which use experimental data for cross-sections of nuclear reactions, supplemented by necessary theoretical information. We shall not discuss this in any detail, see, e.g. [6].

Instead, we will make a rough estimate of \(^4\text{He} \) abundance, which can be done without complicated computations. The first step is to determine the freezing concentration of neutrons. The equilibrium ratio of neutron to proton concentration is simply
\[ \frac{n_n}{n_p} = \exp \left( -\frac{m_n - m_p}{T} \right) \]  
(45)
and is smaller than unity because neutrons are heavier than protons. The fastest reaction that keeps neutron concentration in equilibrium is \( p + e \leftrightarrow \nu + n \). It goes out of equilibrium at \( T \simeq 0.8 \) MeV. Therefore, \( \frac{n_n}{n_p} \simeq \frac{1}{4} \) for temperatures smaller than \( T^* \) but for the time \( t \ll 15 \) min, which is a neutron lifetime. Now, if one looks at the binding energies of light elements (say, with atomic number smaller than 8, the cross-sections for creation of even heavier elements are exponentially suppressed because of the Coulomb barrier) one finds that it is highest in \( \text{He}^4 \). Thus, the abundance \( Y \) of \( \text{He}^4 \) is given simply by a number of available free neutrons in the plasma,

\[
Y = \frac{\text{baryons in } \text{He}^4}{\text{total number of baryons}} = \frac{4n_n}{n_n + n_p} \simeq 0.25 .
\]

Abundances of other light elements (\( \text{He}^3 \), \( \text{D} \), and \( \text{Li} \)) can be found from kinetic equations, and theoretical predictions can be compared with cosmological observations, see Fig. 8. These are usually plotted as a function of parameter \( \eta = \frac{n_n}{n_\gamma} = \eta_{10} \cdot 10^{-10} \), showing the ratio of baryon to photon density for the case of three neutrino species\(^2\). Amazingly, all light-element abundances are in accordance with observations if \( \eta \) is in the interval \( 1.5 < \eta_{10} < 6.3 \), which may be considered as a most important confirmation of the Big Bang theory up to temperatures of the order of 1 MeV. Other elements, present in the Universe, with atomic number greater than 12 are believed to be created in massive stars, while lighter elements, such as \( \text{B} \), \( ^{10}\text{Be} \), \( \text{Li} \) are created by a cosmic ray spallation process.

![Fig. 8](image_url)

Fig. 8: Dependence of light element abundances on \( \eta_{10} \) and observational limits. From ref. [6].

\(^2\)Changing the number of neutrino species changes the rate of the Universal expansion and thus predictions of Big Bang nucleosynthesis. One cannot admit more than four types of massless neutrinos in order not to spoil successful predictions of BBN, see, e.g. [6].
4.4 Baryogenesis

4.4.1 Evidence

As we discussed in the previous sections, the parameter $\eta = \frac{\Delta}{n_\gamma}$, the inverse entropy per nucleon, plays an important role in cosmology. It determines the moment of matter-radiation equality and influences primordial abundances of light elements and structure formation. As we shall see, it is related to the fact that there is no antimatter in the Universe (at least, not in amounts comparable to matter).

Antimatter in the Universe can be detected by a number of different means. First, if antigalaxies exist, we should see antinuclei in cosmic rays, precisely in the same way as we see cosmic nuclei. However, no antinuclei have been observed in cosmic rays, the recent limit on the ratio of antihelium to helium nuclei in cosmic rays comes from the AMS experiment \cite{7}, $\bar{He}/He \approx 10^{-6}$.

Positrons, antiprotons and antineutrons are observed. These antiparticles can be produced in collisions of cosmic protons or nuclei with galactic gas and with particles in the Earth’s atmosphere, and their amount is consistent with expectations \cite{8}. On the contrary, the probability of forming a complicated nucleus, such as $\bar{He}$ by collision of particles (rather than antiparticles) is extremely small, and an observation of just one antihelium nucleus would show that there must be antimatter in the Universe.

Secondly, in regions where matter and antimatter are mixed, annihilation of protons and antiprotons must take place. Annihilation will produce about 5-6 $\pi^0$ and $\pi^\pm$ mesons which, in turn, will decay into $\gamma$-quanta, electrons, positrons, neutrinos and antineutrinos. The spectrum of $\gamma$-quanta has a specific form, with energy peaked around $(2 \text{ GeV})/(5 - 6)/2 \approx 150 \text{ MeV}$. The form and normalization of annihilation $\gamma$-spectra at the present time depends on the size of antimatter clusters and on the amount of antimatter in them. Also, one should take into account the red shift, as $\gamma$-quanta we see today were created in the past. For the globally symmetric Universe one can put a strong constraint on the size $l$ of antimatter clusters \cite{9}, $l > 1000 \text{ Mps}$. This number may be compared with the visible size of the Universe, $3000 \text{ Mps} \sim 10^{10}$ light-years.

It is therefore very likely that the observable Universe is globally asymmetric and contains no antimatter. Even in the baryon-symmetric case with huge antimatter clusters, one should be able to understand why the Universe is asymmetric over cosmological distances and how this complicated structure with islands of matter and antimatter could emerge.

The parameter $\eta$ in fact gives baryon asymmetry of the early Universe, at temperatures of the order of 1 GeV ($t \sim 10^{-6}$ s from the beginning of Big Bang):

$$\Delta(t) = \left. \frac{n_B - n_\bar{B}}{n_B + n_\bar{B}} \right|_{T \sim 1 \text{ GeV}}.$$ (47)

Since the plasma was hot, with a temperature higher than the masses of light quarks, the number of quarks and antiquarks is the same as the number of photons, up to spin factors, and

$$\Delta(t) \approx \left. \frac{n_B - n_\bar{B}}{n_\gamma} \right|_{T \sim 1 \text{ GeV}}.$$ (48)

The ratio $\frac{n_B - n_\bar{B}}{n_\gamma}$ does not change much during the evolution of the Universe since then, because $(n_B - n_\bar{B})a^3$ gives (conserved) baryon number in comoving volume, and $n_\gamma a^3$ is constant up to entropy generation factors related to the annihilation of light particles, as we discussed in connection with neutrino temperature. Therefore, $\Delta(t \sim 10^{-6}) \approx \eta \sim 10^{-10}$ and the baryon asymmetry at that time is tiny. When the Universe cools down from this state, the symmetric part of the baryon-antibaryon background annihilates into photons and neutrinos, but the nucleons that do not find a pair survive, see Fig. 9. These give rise to galaxies, stars and planets.

In 1967 Sakharov suggested \cite{10} (see also the somewhat later paper by Kuzmin \cite{11}) that the Universe is asymmetric because baryon number is in fact non-conserved. In this case, the Universe
could start its expansion from a truly symmetric state, containing an equal number of particles and antiparticles. Then, in the course of the expansion, the particle physics reactions with $B$ and $CP$ non-conservation would produce an excess of particles over antiparticles.

He assumed that there exist some heavy particles $X$ - maximons, with masses of the order of the Planck scale, $10^{10}$ GeV, which can decay with baryon number non-conservation and $CP$ violation. If $CP$ is broken, an equal number of $X$ and $\bar{X}$ will, after their decay, leave a different number of baryons and antibaryons, precisely as the decays of $K^0$ and $\bar{K}^0$ mesons leave different numbers of electrons and positrons. It is sufficient to produce a small asymmetry, $\sim 10^{-9} - 10^{-10}$, which is then converted into a $100\%$ asymmetry after the annihilation of matter and antimatter.

Besides $B$ non-conservation and $CP$ violation it is required that particle reactions occur in a non-equilibrium fashion, because in thermal equilibrium the baryon number of the system must be zero: the total rate of the processes which increase baryon number is exactly compensated by the rate of the processes that decrease it, as a consequence of the CPT-theorem.

From particle physics, baryogenesis requires baryon number non-conservation and $CP$-violation. Depending on the mechanism of $B$-violation, one can consider grand unified baryogenesis, electroweak baryogenesis and leptogenesis. We will briefly review different scenarios below.

### 4.4.2 GUT baryogenesis

The source of non-conservation of baryon number in GUT baryogenesis is associated with the unification of strong, weak and electromagnetic interactions. The leptoquarks $X$ of grand unified theories can decay as $X \rightarrow q\ell$, $\bar{q}\bar{q}$ and $\bar{X} \rightarrow \bar{q}\ell, qq$, see fig 10. If $CP$ is broken, an equal number of $X$ and $\bar{X}$ will, after their decay, leave a different number of quarks and antiquarks. To find the magnitude of $CP$-violation one has to consider radiative corrections to the leptoquark decays, of the type shown on Fig. 11.
If the universe were as hot as the leptoquark mass $M_X$ (typically, $M_X \sim 10^{15}$ GeV) and in a state close to thermal equilibrium then baryon asymmetry of the Universe resulting from leptoquark decays would be of the order

$$\Delta \sim \frac{1}{N_{\text{eff}}} \delta_{CP} \cdot S_{\text{macro}}$$

(49)

where $\delta_{CP}$ is the asymmetry in leptoquark decays,

$$\delta_{CP} = \frac{\Gamma(X \rightarrow qq) - \Gamma(\bar{X} \rightarrow \bar{q}\bar{q})}{\Gamma_{\text{tot}}}$$

(50)

$\Gamma_{\text{tot}}$ is the total width, and $S_{\text{macro}}$ is a factor taking into account the kinetics of the leptoquark decays: $S_{\text{macro}} \sim M_X^2 / \Gamma_{\text{tot}} M_{Pl}$ for $M_X^2 < \Gamma_{\text{tot}} M_{Pl}$ and $S_{\text{macro}} \sim 1$ for the opposite case.

The factor $S_{\text{macro}}$ can be understood in the following way. Besides the decay of $X$-particles, there are inverse processes (inverse decays) and four-fermion scattering of quarks and leptons. In a state of thermal equilibrium no baryon asymmetry is produced because of complete compensation of the rate of different reactions. Largest deviations are expected in concentrations of the heavy particles, in this case leptoquarks. Indeed, $X$-particles start to decay at a temperature $T_d = \sqrt{M_X / \Gamma}$ determined from the condition $M_X^2 / \Gamma = \Gamma^{-1}$ so, for $T > T_d$, the number of $X$-particles is effectively conserved and is equal to the number of photons, up to the spin factors, see Fig. 12. If $T_d < M_X$ then the number of $X$-particles is substantially greater than their equilibrium value, which is Boltzmann suppressed in this case, $\exp (-M_X / T)$. Thus, practically all leptoquarks decay in an out-of-equilibrium way, giving $S_{\text{macro}} \sim 1$. If, on the contrary, $T_d > M_X$, the processes equilibrating $X$-particles are in thermal equilibrium at $T < T_d$ and leptoquark concentration follows its equilibrium value, with small deviations of order $O(M_X^2 / T_d^2)$, giving the suppression factor defined above.

Under the assumption that the universe had temperatures of the requisite order of magnitude, many grand unified theories give rise to baryon asymmetry of the required order of magnitude. Some
care should be taken over the equilibrium character of anomalous electroweak reactions with B-non-conservation, see below. Basically, the requirement for successful GUT baryogenesis is that asymmetry in $B - L$ must be generated.

Perhaps the only drawback of GUT baryogenesis is that it is hardly compatible with inflation. In inflationary cosmology there are several constraints on the temperature of the Universe after reheating. It should not be larger than about $10^{10}$ GeV [12], otherwise a lot of gravitinos would be produced. However, the typical mass of leptoquarks in grand unified theories is of the order of $10^{15}$ GeV, which is substantially larger than the reheating temperature. Thus, there simply cannot be any leptoquarks to decay and produce the asymmetry.

There is a possible way out from this situation, related to the so-called pre-heating stage of the expansion of the universe [13]. At this time classical inflaton dynamics allows for a non-thermal production of heavy particles because of parametric resonance [14, 15]. Still, according to [16], the effective leptoquark concentration $n_X/T^3$ is typically quite small, $n_X/T^3 \sim 10^{-6}$, which would require very large CP asymmetry in the leptoquark decays, $\delta_{CP} \sim 1$.

### 4.4.3 Electroweak baryogenesis

Electroweak baryogenesis is based on the observation that the rate of B non-conservation in the electroweak theory is large at high temperatures [17].

On a perturbative level the electroweak theory has four conserved fermionic numbers: $B$ - baryon number, and $L_\nu$, $L_\tau$ - leptonic numbers. Quantum anomaly

$$\partial_\mu J_\mu^B = \partial_\mu J_\mu^L = \frac{n_f}{32\pi^2} \text{Tr} \left( F_{\mu\nu} \tilde{F}_{\mu\nu} \right) + U(1) \text{ part}$$

leads to anomalous processes with non-conservation of baryon and lepton number [18]:

$$\text{bosons} \leftrightarrow \text{bosons} + 9q + 3l.$$ 

Here $J^B$ and $J^L$ are baryon and lepton currents, $F_{\mu\nu}$ is the SU(2) field strength, $n_f$ is the number of fermion generations, $q$ and $l$ are quarks and leptons.

The rate of anomalous fermion-number non-conservation at zero and non-zero temperatures is of the order of (see [18, 17, 19, 20] and reviews [21, 22]):

$$\Gamma \sim \begin{cases} \exp\left(-\frac{4\pi}{\alpha_W}\right) \sim 10^{-160}, & T = 0 \\ (\alpha_W T)^4 \left(\frac{M_{\text{ph}}}{T}\right)^7 \exp\left(-\frac{M_{\text{ph}}}{T}\right), & T < T_c \\ (\alpha_W)^5 T^4, & T > T_c \end{cases}$$ (51)

where $M_{\text{ph}} \sim M_W/\alpha_W$ is the sphaleron mass and $T_c$ is the temperature of the electroweak (EW) phase transition.

Other ingredients of baryogenesis, C and CP violations, are also present in the standard model or its extensions. The requirement of non-equilibrium happened to be quite non-trivial in the standard model. At the typical scale of electroweak theory, $T \sim 100$ GeV, all electroweak reactions are rapid enough to keep concentrations of different particles close to thermal equilibrium. Large deviations from thermal equilibrium may arise at the first-order phase transition with the breaking of SU(2)×U(1) group. For detailed discussion of the mechanism of electroweak baryogenesis see refs. [21, 22, 23]. Here we just mention the EW baryogenesis constraints on the particle spectrum of the SM and its SUSY extensions.

The nature of the electroweak phase transition depends crucially on the Higgs mass. If $M > M_{\text{crit}}$ then there is no phase transition at all, so that during the cooling of the Universe the system gradually
changes from the so-called symmetric phase into the Higgs phase (in fact, there are no distinct phases but just one – the Higgs-confinement phase – which explains the possibility of the absence of phase transition). Lattice computations carried out in refs. [24, 25] have shown that the critical value of the Higgs mass in the MSM is

$$M_{\text{crit}} = (72.3 \pm 0.7) \text{ GeV}. \quad (52)$$

In view of the experimental lower limit on the Higgs mass $m_H > 114 \text{ GeV}$ it is clear that the MSM does not have any phase transition at all, so that the deviations from thermal equilibrium at the electroweak scale are only associated with the expansion of the Universe. The baryon asymmetry that can appear in this situation is much smaller than the observed value, so that new physics is required for its explanation.

A popular extension of the standard model is the MSSM, in which the strength of the electroweak phase transition depends on a number of parameters, the most important being the mass of the Higgs and the mass of the right-handed stop (the scalar superpartner of the top quark). There can be a strong first-order phase transition, sufficient for electroweak baryogenesis, if the lightest Higgs mass is somewhere between 75 and 120 GeV and the right scalar top mass $100 \text{ GeV} < m_{R} < 160 \text{ GeV}$ [26], for a recent review see [27]. This set of parameters is constrained by existing experiments and this interval of Higgs masses has been partially covered at the electron–positron collider at CERN. The baryogenesis-carrying version of the SUSY extension of the standard model has a number of consequences for phenomenology as it requires a specific spectrum of SUSY particles and a particular pattern of CP-violation.

4.4.4 Leptogenesis

There is strong experimental evidence in favor of neutrino oscillations [28, 29]. If neutrino oscillates, it has a mass. Theoretically, a lowest-order SU(2)$\times$ U(1) gauge-invariant operator that can be added to the SM Lagrangian has the form:

$$\Delta L = f_{ab}(\bar{\nu}_{a}^{\nu} \phi) (\phi^{\dagger} \nu_{b}), \quad (53)$$

where $\phi$ is the Higgs doublet, $M$ is some high-energy scale, and $\nu$ is left-handed neutrino. This term gives Majorana neutrino masses and a lepton number violation. The simplest way to obtain this effective interaction from renormalizable field theory is to have right-handed neutrino $\nu_{R}$ with a large Majorana mass $M_{R}$. Then (53) comes from the see-saw mechanism [30, 31]. A heavy right-handed neutrino can decay and produce lepton asymmetry in the early universe, in precisely the same way as leptoquarks produce baryon asymmetry in GUTs. There the lepton number is converted into baryon asymmetry by sphalerons [32] (for a recent review see [33]). The resulting baryon asymmetry is just a numerical factor of order one smaller than the lepton asymmetry.

This mechanism for baryogenesis requires sufficient concentration of right-handed neutrinos at the moment of their decay. If $m_{R} \sim 10^{10} \text{ GeV}$ or less, right-handed neutrinos could be thermally produced at the end of the inflationary period; the reheating temperature is sufficiently low to prevent the overproduction of gravitino [12]. Right-handed neutrinos may also be produced non-thermally at preheating [34]. A detailed study of this mechanism can be found in [35].

4.4.5 Affleck-Dine baryogenesis

The Affleck-Dine mechanism [36] takes advantage of supersymmetry. Supersymmetric theories contain scalar fields that carry lepton or baryon numbers and the effective potential for squarks and sleptons has flat directions, i.e. the energy of the static scalar field configuration at large $\phi$ is much smaller than $\phi^{4}$. In this scenario, a combination of squarks and sleptons, or some other fields carrying a baryon or lepton number, has a large expectation value along some flat direction of the potential at the end of inflation. At large VEV, the baryon number can be strongly violated by the high-scale physics. As a result of the baryon number non-conservation, along with the CP violation, the scalar condensate acquires a baryon number and the complex scalar field is characterized by the time-dependent phase,
\(\phi = |\phi(t)|\exp(\Omega(t))\). The subsequent evolution leads it into the domain of conserved baryon number because the field amplitude \(\phi\) decreases with time. Finally, squarks decay into ordinary quarks and release baryon number stored in the scalar condensate. A study of this scenario in different models was made in refs. [36, 37, 38] with the result that baryon asymmetry of the universe can be explained by this mechanism.

5 INFLATION

5.1 Problems of standard cosmology

To explain what kind of problems faced standard cosmology before the invention of inflation, we will introduce the notion of particle horizons. In a static Universe, if two events are separated by distance \(\Delta l\) and time \(\Delta t\), they are causally independent, provided \(\Delta l > \Delta t\). What is the analogue of this statement in an expanding Universe? To answer this, let us write the equation describing the propagation of light, taking into account the fact that the speed of light is \(c\) in the natural system of units:

\[
\frac{dl}{dt} = 1 + \frac{\dot{a}}{a}. \tag{54}
\]

The solution of this equation is

\[
l(t) = \int_{t_0}^t \frac{a(t)}{a(t')} dt'. \tag{55}
\]

For both radiation- and matter-dominated Universes, with \(a(t) \sim t^{1/2}\) and \(a(t) \sim t^{2/3}\) respectively, the integral in eq. (55) converges even if \(t_0 = 0\). The distance light travels since \(t_0\) is called the particle horizon, \(l_H(t)\). For different epochs:

\[
l_H(t) = \begin{cases} 2t, & \text{radiation-dominated epoch,} \\ 3t, & \text{matter-dominated epoch.} \end{cases} \tag{56}
\]

If the distance between two points is greater than \(l_H(t)\), the points were not in causal contact in the past and thus we should expect that the parameters of the Universe may be different there.

5.1.1 The horizon and homogeneity problem

Assuming that there were only radiation- and matter-dominated epochs in the past, then, as already discussed, the photons decoupled from the plasma at some moment \(t_d \simeq 5 \cdot 10^5\) y. Thus while looking at different points of the sky separated by some angle \(\theta > \theta_H\) we observe CMB emitted from regions that were never in causal contact and so should have different temperatures. To estimate \(\theta_H\), one should find the present size \(L\) of the region that was the horizon at the decoupling time,

\[
L \sim 3l_d \frac{a_{\text{now}}}{a_d} \sim 3l_d \left(\frac{t_{\text{now}}}{t_d}\right)^{\frac{2}{3}}, \tag{57}
\]

where \(3l_d\) is the horizon scale at \(t_d\). The angle \(\theta_H\) is simply the ratio of this scale to the present size of the horizon,

\[
\theta_H \simeq \frac{3l_d}{3l_{\text{now}}} \left(\frac{t_{\text{now}}}{t_d}\right)^{\frac{2}{3}} \simeq \left(\frac{t_d}{t_{\text{now}}}\right)^{\frac{2}{3}}, \tag{58}
\]

which corresponds to \(\theta_H \sim 1^\circ\). This means that the present horizon contains \(O \left(\frac{t_{\text{now}}}{t_d}\right) \sim 10^4\) domains that were not in causal contact before recombination, see Fig. 13. However, observations show that the cosmic microwave background is isotropic for these angles, with accuracy better than \(10^{-4}\). This is the essence of the horizon and homogeneity problem.
5.1.2 The flatness problem

Let us consider the relationship (23) in somewhat different form,

$$\Omega - 1 = \frac{k}{a^2 H^2},$$

where $\Omega$ is the ratio of the total energy density to the critical density. For both matter- and radiation-dominated epochs, $H \sim \frac{1}{t}$ and $a \sim t^\alpha$, with $\alpha = \frac{1}{2}$ or $\alpha = \frac{2}{3}$. Thus, $\omega$ increases with $t$ as $\Omega - 1 \sim t^{2(1-\alpha)}$. Therefore, to have $\Omega \approx 1$ at the present time, as follows from observation, $\Omega$ must have been finely tuned to one with huge accuracy in the past. For example, at the nucleosynthesis time, $|\Omega - 1|$ must be of the order of $10^{-15}$. It is unclear why the Universe should have been so flat in the past.

5.2 Inflation as a solution of cosmological problems

In fact, the two problems described above are related to each other. The inflationary paradigm, introduced in [39, 40, 41], provides a simultaneous solution to both of them.

Supposing that, for some reason, the dependence of $a$ on $t$ before recombination were such that the integral in (55) is very large and the factor $aH$ increases with time rather than decreases, both problems would be solved simultaneously. For example, if the dependence of $a$ on time has a power-law behavior with exponent $\alpha$ such that $\alpha > 1$ during some time after an initial singularity, the integral in (55) diverges (i.e., formally, the size of the particle horizon is infinite at the recombination time), while the factor $aH$ increases with time.

For a physical picture of how this might happen, we suppose that vacuum energy density $\epsilon_{vac}$ dominated the Universe expansion for some time $t_0 < t < t_1 < t_d$. The Universe therefore expands exponentially, $a \sim \exp(+H(t - t_0))$ with $H$ defined in (29). Thus, the horizon at recombination is at least

$$l_H \approx \frac{1}{H} \exp(+H(t_1 - t_0)), \quad (60)$$

and is much greater than the horizon would be in a radiation- or matter-dominated epoch, provided $H(t_1 - t_0) \gg 1$. For example, if we take a typical Grand-Unified scale for $\epsilon_{vac} \sim (M_X)^4 \sim (10^{15} \text{ GeV})^4$, it is sufficient to have $H(t_1 - t_0) > 65$ for the horizon problem to be solved, see Fig. 14. The flatness problem finds its solution as well, since after the period of exponential inflation $\Omega - 1 \sim \exp(-2H(t_1 - t_0))$, which gives an exponentially small deviation from the flat Universe.
Fig. 14: An observer at point $r = 0$ sees just one particle horizon corresponding to photon decoupling.

5.2.1 A particle-physics model of inflation

There are many different particle-physics models of inflation. Practically all of them are associated with the dynamics of single or multiple scalar fields. We refer to a comprehensive review [42, 43] and describe here just one possibility which is called “chaotic inflation” [44].

Consider a theory of a single scalar field in a curved background with an action

$$S = \int d^4x \sqrt{\text{g}} \left( \frac{1}{2} \text{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) \right) ,$$

where the potential is

$$U(\phi) = \frac{1}{2} \dot{\phi}^2 + \frac{\lambda}{4} \phi^4 .$$

We will assume that $m^2 \ll M_{Pl}^2$ and $\lambda \ll 1$.

We do not know how to describe the state of the Universe at the Planck scale, since the classical theory of gravity is not applicable there. Nevertheless, it is natural to assume that at Planck time there were fluctuations in the scalar field with energy density

$$\epsilon \sim \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} (\nabla \phi)^2 + U(\phi) \sim M_{Pl}^4 .$$

Suppose that there is a sufficiently large region of space where fluctuations of potential energy dominate, i.e.

$$U(\phi) \sim M_{Pl}^4 \gg (\nabla \phi)^2 \text{ and } \dot{\phi}^2 .$$

In these regions the value of $\phi$ is much larger than the Planck scale,

$$\phi \approx \frac{M_{Pl}}{m} \gg M_{Pl} ,$$

and the scalar field is nearly homogeneous, so that the equation representing its evolution is simply

$$\ddot{\phi} + 3H \dot{\phi} + \frac{dU(\phi)}{d\phi} = 0 ,$$

where

$$H^2 = \frac{8\pi G}{3} \left( \frac{\dot{\phi}^2}{2} + U(\phi) \right) .$$

This looks like an equation of motion of a non-relativistic particle with unit mass in the potential $U(\phi)$ with a friction term that depends on the position and velocity of the particle, see Fig. 15. For large values
of $\phi$ the regime is overdamped, with $H \gg \ddot{\phi}/\dot{\phi}$, where

$$H^2 \simeq \frac{4\pi m^2 \phi^2}{3M_{Pl}^2}. \quad (68)$$

Thus, eq. (66) has the form

$$\sqrt{12\pi m} \phi \frac{\ddot{\phi}}{M_{Pl}} + m^2 \phi = 0 \quad (69)$$

and has a solution

$$\phi \simeq \phi_0 - \frac{m M_{Pl} t}{\sqrt{12\pi}} \simeq \phi_0 \left(1 - O\left(\frac{m^2 t}{M_{Pl}}\right)\right). \quad (70)$$

The “slow-roll-down” approximation breaks down at $\phi^2 \simeq V(\phi)$, where $\phi \simeq M_{Pl}, \ t \simeq M_{Pl}/m^2$. Before this time the Universe expands exponentially and the scale factor changes by

$$\exp(Ht) \simeq \exp\left(\frac{M_{Pl}^2}{m^2}\right) \gg 1. \quad (71)$$

During exponential expansion the non-homogeneities are red-shifted away and the Universe becomes practically uniform at cosmological distances.

After time $t \simeq M_{Pl}/m^2$ inflaton oscillates near the origin, transferring its energy to other particles. This process is usually called reheating: if the energy is converted through the parametric resonance, it is called preheating [13]. For $m \ll M_{Pl}$ this simple model of scalar field solves horizon, homogeneity and flatness problems. Moreover, quantum fluctuations of the scalar field, existing on the De-Sitter (exponential) part of the expansion of the Universe, give rise to the scale-invariant spectrum of primordial density perturbations, which eventually lead to structure formation in the Universe.

The most important and generic predictions of inflation are that the universe should be spatially flat, $\Omega_{tot} = 1 + O(10^{-5})$ and that the spectrum of density perturbations should be scale-invariant. Both predictions can be verified by cosmological observations.
6 CONTENT OF THE UNIVERSE OR COSMOLOGICAL OBSERVATIONS VERSUS THE STANDARD MODEL

6.1 Cosmological parameters

As we have seen, the amount of each of the different substances in the universe is measured as the ratio of the corresponding density $\rho_A$ of the substance to the critical energy density $\rho_c$. The simplest cosmological model is obtained by considering the following contributions to the total energy density:

(i) matter: $\Omega_M$ with the equation of state $p = 0$ where $p$ is the pressure. This counts all non-relativistic particles in the universe (protons, neutrons, electrons and, possibly, massive neutrinos).

(ii) radiation: $\Omega_r$ with $p = \epsilon/3$, where $\epsilon$ is the energy density. Radiation is associated with photons and very light or massless neutrinos.

(iii) The standard model also admits a cosmological constant with $p = -\epsilon$. The corresponding parameter is $\Omega_\Lambda$.

In the Universe at the present time the contribution of radiation is not numerically essential, $\Omega_r \ll \Omega_M$. Thus, there are essentially two parameters $\Omega_M$ and $\Omega_\Lambda$ that determine the global structure of the Universe and its future. The “phase structure” of the Universe as a function of these parameters is shown in Fig. 16. It is more complicated than for the case $\Omega_\Lambda = 0$.

\[ \begin{array}{c|c}
\Omega_\Lambda & \Omega_M \\
\hline
-1 & 0 \\
0 & 1 \\
1 & 2 \\
2 & 3 \\
\end{array} \]

Fig. 16: Schematic phase structure of the Universe depending on parameters $\Omega_M$ and $\Omega_\Lambda$. In regions I and II the Universe is accelerating now, and in regions III – VI the Universe is decelerating. In the regions I, IV, VI the Universe is closed, and in II, III, V it is open. In regions I – IV the Universe expands forever, while in regions V and VI it will collapse in the future.

The parameters $\Omega_M$ and $\Omega_\Lambda$ should be determined from observations. There are several independent ways to estimate them (for a review see [45, 46]).

6.1.1 Clusters and the dynamics of galaxies

The dynamics of clusters offers several ways of estimating the matter contribution. A cluster mass $M_{cl}$ can be defined by the consideration of galaxy motion within the cluster and/or by gravitational lensing.
by a cluster-gravitational potential. An estimate of the mass of matter in the Universe would then be

\[ M_{\text{matter}} = \frac{M_d}{L_{cl}} L_U , \]  

(72)

where \( L_{cl} \) and \( L_U \) are respectively the luminosities of a cluster and of the Universe as a whole. This gives the estimate [47, 48, 49, 50]

\[ \Omega_M = 0.2 \pm 0.1 . \]  

(73)

Incidentally, the fraction of energy in luminous matter is known to be much smaller,

\[ \Omega_{\text{lum}} \simeq 0.003 - 0.01 . \]  

(74)

Another estimate comes from the baryon fraction in matter. Part of the baryonic matter in clusters is luminous, while another part corresponds to a gas whose mass can be estimated from X-ray emission. Assuming that the baryon fraction in clusters is the same as the average in the Universe, and taking \( \Omega_B \simeq 0.045 \pm 0.0025 \) from nucleosynthesis [51, 52], then [53, 54, 55]

\[ \frac{M_{\text{baryons}}}{M_{\text{total}}} \simeq 0.15 , \]  

(75)

which gives

\[ \Omega_M = 0.3 \pm 0.1 . \]  

(76)

Yet another estimate comes from consideration of the cluster abundance [56]

\[ \Omega_M = 0.25^{+0.15}_{-0.10} . \]  

(77)

These estimates show that the amount of matter in the Universe that can cluster is larger than that of baryonic matter. This indicates that the Universe contains dark matter of unknown nature. Another piece of evidence in favor of this conclusion comes from the consideration of rotational curves of spiral galaxies.

Orbital velocities of stars or of a gas far from the galactic center can be measured. The dependence of velocity on the distance from the center follows from the virial theorem, telling that for a gravitationally bounded system the kinetic energy of a body \( \frac{1}{2}mv^2 \) is proportional to its potential energy \( \frac{GMm}{r} \), where \( m \) is the mass inside a sphere with radius \( r \). Thus,

\[ v \sim \sqrt{\frac{GM}{r}} . \]  

(78)

If it is assumed that only luminous matter gravitates, then \( v \sim \frac{1}{\sqrt{r}} \) which is just Kepler’s law. In reality, the rotational curves have plateaux, as shown on Fig. 17.

This type of dependence can be derived with the assumption that the density of the halo of a galaxy changes with radius as \( \rho_{\text{dark}}(r) \sim 1/r^2 \), giving \( M_{\text{dark}} \sim r \). Quite a good overall fit is given by

\[ \rho_{\text{dark}}(r) = \frac{\rho_{\infty}^2}{4\pi G (r_c^2 + r^2)} , \]  

(79)

where \( \rho_{\infty} \) is the limiting velocity and \( r_c \) is the typical size of density distribution.

Thus, the total amount of matter in galaxies is considerably larger than their visible part, so cosmological data indicates that the Universe contains non-baryonic dark matter which clusters but does not shine with \( \Omega_M \simeq 0.3 \) and \( \frac{\Omega_{\text{dark}}}{\Omega_M} \simeq \frac{1}{r} \).
6.1.2 The accelerating universe

Further information about cosmological parameters comes from the observation of supernovae at cosmological distances. Two teams of observers have recently produced similar results: the Supernova Cosmology Project (SCP) and the High-Z Supernova Search (HZS). The idea is to take a number of “standard candles”, i.e. supernovae of type Ia, and to find their distance from the earth by comparing the known luminosity with the observed luminosity. In addition, one can also define their red shift. At small distances there is a linear dependence between the red shift and the distance, given by the Hubble law, whereas at large distances cosmological evolution has some effect and information on the acceleration or deceleration of the universe can be derived. The main result of this study is that the Ia supernovae with a high red-shift are observed to be dimmer than would be expected in an empty Universe, $\Omega_M = 0$ with no cosmological constant, see Fig. 18.

The result of fitting cosmological parameters, obtained by the SCP [58] (similar numbers from the HZS [59]), is

$$0.8\Omega_M - 0.6\Omega_\Lambda \simeq -0.2 \pm 0.1,$$

see Fig. 19.

Assuming that the Universe is flat ($\Omega_M + \Omega_\Lambda = 1$), this gives [58]

$$\Omega_M = 0.28^{+0.09+0.05}_{-0.08-0.04},$$

which is in reasonable agreement with the determination of $\Omega_M$ by other means.

6.1.3 Cosmic microwave background

Another important source of constraints on the cosmological parameters is related to cosmic microwave background radiation (CMB) and to the mass power spectrum. As we discussed, CMB anisotropy gives a snapshot of the Universe at redshifts of about 1000, corresponding to an age of $\sim 10^{12}$ s.
In the first approximation CMB is isotropic: the temperature does not depend on the direction. However, exact isotropy must not be expected, as the Earth, together with the solar system, moves through CMB with some velocity \( v \) and the temperature in the forward direction must be larger than in backward direction. The form of these deviations is obtained from the exact Planck curve by replacing the photon energy in the Planck distribution (7) with its Lorentz transformed value. For non-relativistic velocities we have \( \varepsilon \rightarrow \varepsilon(1 + v \cos \theta) \), where \( \theta \) is the angle between velocity \( v \) and the line of sight. Thus, the spectrum of deviations from the Planck curve is just the derivative of the equilibrium distribution with respect to frequency, see Fig. 20. Experimentally, this is indeed the case, \( (T_{\text{forward}} - T_{\text{backward}})/T \approx 1.23 \cdot 10^{-3} \) which corresponds to \( v = 371 \pm 0.5 \) km/s.

If dipole anisotropy is subtracted from the data on CMB, the fluctuations of the temperature on the level \( 10^{-5} \) are seen, as was first observed by COBE [60]. Nice colored pictures of COBE measurements can be found on the home page of this experiment, http://space.gsfc.nasa.gov/astros/cobe/. A review of different CMB experiments can be found in [61].

Why are there fluctuations in CMB?

(i) “Experimental” reason. We do know that the Universe is clumpy at “small” scales of the order of supercluster size or below. There must therefore have been some fluctuations in matter distribution at decoupling. As matter and radiation were interacting at that time, there must be fluctuations in CMB as well. The theory of structure formation indicates that one should expect to have \( \delta T/T \sim 10^{-5} \).

(ii) Theoretical reason. Quantum fluctuations of the scalar field-inflaton during inflation lead to adiabatic density perturbations that result in fluctuations in CMB.

Using theoretical machinery to study anisotropies of the microwave background, a first step consists of expanding the CMB temperature with the help of spherical harmonics \( Y^m_l(\theta, \phi) \),

\[
\frac{\Delta T(\theta, \phi)}{T} = \sum_{l,m} a_{lm} Y^m_l(\theta, \phi) \tag{82}
\]
and defining $C_l$ as

$$C_l = \frac{1}{2l+1} \sum_m |a_{lm}|^2.$$  \hspace{1cm} (83)

For a particular cosmological model, values of $C_l$ can be predicted. Schematically, one should take an initial spectrum of density perturbations (given, e.g., by inflation) and write kinetic equations for photons, taking into account various factors such as the interaction of photons with gravity potential induced by matter fluctuations (Sachs-Wolfe effect), the peculiar velocities of plasma (Doppler effect), damping of fluctuations, etc. Calculation then gives $C_l$ as a function of cosmological parameters ($\Omega_\Lambda$, $\Omega_M$, baryon-to-photon ratio) and other parameters essential for the model (for example, neutrino masses).

Recent experimental results, shown in Fig. 21, come from two different balloon experiments: Boomerang, [62] and Maxima [63]. In Fig. 22 the regions of cosmological parameters coming from different types of data are shown.

6.1.4 Global fit of cosmological data.

The results from different sources can be summarized as follows. A simple homogeneous cosmological model with two parameters $\Omega_M \simeq 1/3$ and $\Omega_\Lambda \simeq 2/3$ fits all the data well and the cosmological constant is nonzero at a confidence level of 99.7% ($3\sigma$) [58, 59]. With this set of parameters the Universe accelerates and will expand forever, provided the simplest cosmological model is correct. The total energy density in the Universe is close to the critical one [64]. This could be considered as an indication of the validity of the inflationary-universe scenario.

The age of the universe, according to [64], for $\Omega_{\text{tot}} = 1$ is $11.8 \pm 0.8$ Gyr, and the value of the Hubble constant is $H = 75 \pm 10$ km s$^{-1}$Mps$^{-1}$.
Thus, modern cosmological observations suggest that the Universe consists of baryonic matter, $\Omega_B \simeq 0.045$, non-baryonic dark matter with $\Omega_{DM} \simeq 0.3$, 

$$\frac{\Omega_{DM}}{\Omega_B} \simeq 7,$$  \hspace{1cm} (84)

and a cosmological constant with $\Omega_A \simeq 0.7$, 

$$\frac{\Omega_A}{\Omega_M} \simeq 2.$$ \hspace{1cm} (85)

In fact, all the components that we see in the Universe represent a challenge for the standard model of elementary particle interactions. We have already considered the problem of the origin of matter – baryon asymmetry of the Universe – which points towards physics beyond the standard model. The problems of dark matter and the cosmological constant remain to be discussed.

### 6.2 Origin of dark matter

Observations of cluster dynamics suggest that the amount of matter that can cluster in the Universe is about $\Omega_M \simeq 0.3$, while analysis of nucleosynthesis indicates that the amount of baryonic matter is much smaller, $\Omega_b \simeq 0.045$. The macho abundance found by the Eros collaboration [65] reveals that machos cannot contribute more than 20% of the galactic halo and thus cannot be used to explain the rotational curves of the galaxy. Thus, there should exist some non-baryonic dark matter.

The minimal standard model does not provide any candidate for the non-baryonic dark matter and, therefore, cosmological observations again point in the direction of physics beyond the standard model. A minimal extension of the standard model that gives neutrino mass through higher-order operators does not help for a number of reasons. Firstly, to explain the rotational curves of some dwarf galaxies, neutrino mass should be larger than 100 eV, which is in conflict with the cosmological upper bound on neutrino mass. Secondly, neutrinos form a so-called “hot” dark matter, and the theory of structure formation
Fig. 21: The power spectrum seen by Boomerang, Maxima and COBE-DMR experiments. Best fit corresponds to $(\Omega_{\text{tot}}, \Omega_\Lambda, \Omega_b h^2, \Omega_{CDM} h^2) = (1.2, 0.5, 0.03, 0.12)$ and to $(1.0, 0.03, 0.17)$ if $\Omega_{\text{tot}}$ is taken to be 1. The initial spectrum of perturbations is assumed to be scale-invariant. From ref. [64].

says that, with hot dark matter, small structures, such as galaxies, are not formed. Finally, if there is no extreme degeneracy between different neutrino flavors, the SuperK value of the neutrino mass splitting $\Delta m_\nu \sim 0.1 \text{ eV}$ is of the order of the neutrino mass, but then this value is too small to make a significant contribution to the dark matter. Thus, it looks as though more drastic modifications of the standard model are necessary in order to accomodate the cosmological data.

Particle physics provides a general answer to the question of dark matter by stating that there exist new stable objects (perhaps particles) which were produced in the course of the expansion of the universe. The non-observability of dark-matter particles is explained by their very weakly interacting character (if there are plenty of them) or by the fact that they are very rare, in which case they could be strongly interacting.

There are quite a number of particle-physics candidates for dark matter, for example axion, related to a solution of the strong CP problem (for a review of cosmological constraints on axion see, e.g., [66]) or the lightest supersymmetric particles, which may be stable due to R-parity conservation (neutralino, axino, gravitino). Neutralino is a Majorana fermion, which is a mixture of photino, zino and higgsino. It annihilates into lighter particles and its relic concentration can be computed in a particular supersymmetric extension of the standard model.

In a more general way, one can assume that dark matter consists of weakly interacting massive particles, characterized by their mass, concentration (mass and concentration are related to each other if the dark matter density is fixed from observation) and a cross-section of interaction with ordinary matter. One expects that their mass may be in the region of, say, $10 - 100 \text{ GeV}$ and that their average velocity is of the order of the gravitational galaxy escape speed $300 \text{ km/s}$. These particles may be searched for via their elastic scattering on ordinary mass particles, with a typical event rate of 1 event/kg/day, for existing detectors. There are several experiments looking for this type of event. I will just mention
strong constraints on the SUSY relic coming from the Heidelberg-Moscow experiment [67] and unusual events seen by the DAMA collaboration [68].

There are also indirect methods for WIMP searches. Dark matter particles may be trapped by the Earth or by the Sun and concentrate in their centers. Annihilation of WIMPs will result in $\nu \bar{\nu}$ production in the centers of the Sun and Earth. These neutrinos can be detected by large neutrino telescopes, such as Amanda [69] or Antares [70]. To be compatible with the direct searches, the volume of neutrino telescope should be of the order of $1 \text{ km}^3$.

In addition to WIMPs, there are some “new” particle-physics candidates that have been suggested during the last three years. The first one is a superheavy particle relic. Usually it is assumed that dark-matter particles are relatively light. The reason is that if stable particles are in thermal equilibrium, their freezing concentration can be readily computed if the annihilation cross-section is known. Using the unitarity bound on the annihilation cross-section and requiring that these particles do not overclose the universe one can derive an upper bound on their mass, $M < 500 \text{ TeV}$ [71]. A possible loop-hole in this consideration is that the particles could never be in thermal equilibrium, so that their abundance may always be smaller than the freezing concentration in the equilibrium case. A hypothesis that superheavy particles play the role of dark matter requires that these particles be stable on the cosmological scale and that they be produced in sufficient amounts during the evolution of the Universe. In principle, these particles could be thermally created after inflation, provided their masses were not very much larger than the reheating temperature [72, 73]. Heavy particles might also be produced because of inflaton oscillations in parametric resonance [15]. Finally, they might be produced gravitationally [73, 74, 75].

Another example of a superheavy relic is provided by supersymmetric non-topological solitons. The existence of Q-balls is a generic feature of SUSY models [76]. In supersymmetric theories with low-energy SUSY breaking and flat directions in the effective potential ($V(\phi) \propto V_0$ as $\phi \rightarrow \infty$) there are stable states, Q-balls, that carry baryon number [77, 78]. The mass of a soliton with baryon charge $B$ varies as [77] $M \sim V_0^{\frac{1}{4}} B^{\frac{3}{2}}$. Thus, non-topological solitons cannot decay into baryons for $B > V_0/m_p^4$. 

Fig. 22: The phase space of the Universe as determined from supernovae observations and from CMB. From ref. [64].
where $m_p$ is the proton mass. In a number of gauge-mediated models, $V_0$ can be as small as $(1 \text{ TeV})^4$ and Q-balls are stable with respect to decay into protons if their charge is greater than $10^{12}$. Supersymmetric Q-balls might be produced in the early Universe through the decay of Affleck-Dine condensate together with ordinary baryons [78]. The search for Q-balls is possible [79, 80] in a number of working and projected experimental installations, such as Baikal, Macro, SuperK, Antares, etc., and some constraints are already available [81, 82].

### 6.3 The cosmological constant problem

The set of observations discussed above suggests that the cosmological constant is nonzero and positive, $\Omega_\Lambda \simeq 0.7$. This value of the cosmological constant corresponds to the energy density

$$V_0 = \left( \frac{\Lambda}{8\pi G} \right)^{1/2} \sim 10^{-3} \text{ eV} \sim 0.01 \text{ cm}^{-1}.$$  \hspace{1cm} (86)

Does this mean that a new scale in physics has been discovered?

The nonzero cosmological constant introduces several fine-tuning problems. The first comes from comparison between the magnitude of the scale (86) and other known scales, for example $\Lambda_{QCD}$, $M_W$, $M_{GUT}$ and $M_{Pl}$. Why is the scale associated with $\Lambda$ so small compared with the other scales?

Another problem arises from comparison of $\Omega_M$ and $\Omega_\Lambda$. At the present stage of expansion of the Universe they are of the same order of magnitude. However, matter energy varies as $\rho_M \sim 1/a^3$, radiation energy varies as $\rho_\gamma \sim 1/a^4$, whereas vacuum energy does not change during the expansion of the universe. In other words, $\Omega_\Lambda \sim 10^{-120}$ on the Planck scale, and the question is whether this huge hierarchy could have any physical explanation.

Using a $\Lambda$ term to fit the cosmological data is the simplest (but certainly not the only) possibility. It would also be reasonable to assume that extra matter in the Universe has a more general equation of state [83, 84],

$$p = \omega \epsilon,$$  \hspace{1cm} (87)

where $p$ is pressure, $\epsilon$ is an energy density and $\omega$ is a constant to be determined from observations. Equation (87) can be used to fit the cosmological parameters globally. The use of a set of cosmological observations (such as the Hubble constant, fraction of baryon mass, cluster abundance, age of the Universe, mass power spectrum, supernovae data, gravitational lensing and large scale structure) implies that $\omega$ is negative, $\omega < -0.5$ [85], see Fig. 23.

A model for a substance with a general equation of state can be provided by a uniform scalar field $\phi$ [86]-[95], for which

$$\omega = \frac{\dot{\phi}^2 / 2 - U(\phi)}{\dot{\phi}^2 / 2 + U(\phi)}.$$  \hspace{1cm} (88)

Depending on the field evolution, one can set $-1 \leq \omega \leq 1$, where $\omega = -1$ and $\omega = +1$ correspond to the dominance of potential and kinetic energies, respectively.

At the present time an explanation of the first problem (why the cosmological constant is so small) is absent, whereas some solutions of the second problem (why energy density is roughly the same as the matter energy density) have been suggested [86]-[95]. The main idea is that the extra substance in the Universe is not a $\Lambda$-term but a time-dependent scalar field with an unusual type of potential (ground state at $\phi \rightarrow \infty$). The evolution of the scalar field is such that it adapts its energy to the energy contained in matter, so that its late time evolution practically does not depend on the initial conditions and is established dynamically (the so-called attractor solutions).

A simple example is provided by the exponential potential $V(\phi) = V_0 \exp(-\lambda \phi / M_{Pl})$ [86, 88, 91]. This potential does not have a ground state and $\dot{\phi}$ is nonzero at all times, see Fig. 24. The equations
Fig. 23: Constraints on the equation of state from supernova data. From ref. [85].

Fig. 24: The run-away potential used in quintessential models.

of motion (in conformal time, $a d\tau = dt$) are:

$$
\frac{1}{a^2} \frac{d}{d\tau} (a^2 \dot{\phi}) + a^2 V'(\phi) = 0 ,
$$

$$
H^2 = \frac{1}{3M_{Pl}^2} (\frac{1}{2} \dot{\phi}^2 + a^2 V(\phi) + a^2 \rho_n) .
$$

Here $\rho_n$ is a matter density that obeys the equation of state:

$$
\dot{\rho}_n + nH \rho_n , \quad \rho_n \sim 1/a^n ,
$$

with $n = 3$ for matter and $n = 4$ for radiation. This system of equations has the following attractor solutions: if $\rho_\phi \gg \rho_n$ then $\rho_\phi$ scales faster than $1/a^n$, but if $\rho_\phi \ll \rho_n$ then $\rho_\phi$ varies more slowly than $1/a^n$ ($\rho_\phi$ is an energy density of field $\phi$). This means that after some time $\rho_\phi$ varies as a dominant component and

$$
\Omega_\phi = \frac{\rho_\phi}{\rho_\phi + \rho_n} = \frac{n}{\lambda^2} .
$$
In other words, the scalar field is self-tuned to the energy density of the dominant component, see Fig. 25. Thus, the value of $\Omega_0$ is now not related to initial conditions (as was the case for the cosmological constant problem) but to parameters of potential. For this particular model the parameter $\omega$ in the equation of state is that of the dominant matter component, i.e. $\omega = 0$ at the present time, which is in disagreement with the analysis of [84]. Other types of potentials may give different types of behavior: in particular it is possible to obtain a negative value of $\omega$ at the present stage of expansion of the Universe, which is consistent with observations [93]. The main problem of this approach is that it does not give any solution to the cosmological constant problem, while the particle-physics origin of a scalar field with the necessary dynamics (existence of a correct attractor solution) remains to be found.

Fig. 25: Attractor solutions.

7 CONCLUSIONS

According to cosmological observations, most of the energy density of the Universe ($\sim 95\%$) cannot be described by the physics of the Standard Model. Roughly a third of this energy density can cluster and represents cold dark matter with unknown particle content, while almost two thirds of the energy appears to be uniform. There are quite a few theoretical proposals for cold dark matter particles and for the uniform component, but the existing cosmological data is not sufficient for a choice to be made. Future experiments, such as Planck and MAP in the area of cosmic microwave background, are extremely important as they will bring cosmology into a new era of precision and define constraints on drastically different cosmological and particle-physics models and even, perhaps, single one out.

References


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