Abstract

Consistent measurements of event-shape variables and $\alpha_s$ using the complete set of data taken with the ALEPH detector at LEP are presented. The strong coupling constant is determined from a fit of $O(\alpha_s^2)$+NLLA QCD calculations to distributions of 6 event-shape variables at 8 centre-of-mass energies. Combined results are compared to the expected evolution of $\alpha_s$.  

*Contributed Paper for LP01 and EPS HEP 2001*
1 Introduction

Studies of the production of hadronic events collected by the ALEPH detector at LEP between 91.2 and 206 GeV centre-of-mass energy are presented. Data presented in [1–5] up to 202 GeV have been re-processed using an improved selection as described in [6]. The measurements at 130-136 GeV, at 189-192 GeV, at 196-202 GeV and at 203-209 are combined, and a re-analysis of data at the Z boson resonance is included. The scope of this paper is a consistent measurement of event-shape variables and determination of $\alpha_s$ at LEPI and LEPII energies. In performing these determinations, the primary objective is to observe the running of the coupling with centre-of-mass energy; for this reason, the analyses at each energy point are designed, as far as possible, to be coherent with each other and to have correlated systematic errors. Final results of QCD analyses will be published in an upcoming paper.

This paper is organised as follows: In Section 2 a brief description of the ALEPH detector and of the overall event-selection and correction procedure is given, followed by measurements of event-shape distributions. These are used in Section 3 to determine $\alpha_s$. Conclusions are summarised in Section 4.

2 Experimental Procedure

A detailed description of the ALEPH detector is given in [7]. The event selection begins by selecting hadronic events as described in reference [3]. Hadronic events in which a significant fraction of the available energy was given-up as initial state photon radiation (ISR) are then removed in a procedure that has several steps. First, ISR photons observed in the detector are identified and removed as follows. The particles in the event are clustered into jets using the Durham algorithm [8] with a resolution parameter of $y_{\text{cut}} = 0.002$. Jets are identified as dominantly electromagnetic if the fraction of the jet’s energy carried by charged hadrons is less than 10% and there are either no charged hadrons or else less than half of the neutral energy is hadronic. From these ‘electromagnetic jets’, the photon and electron (or positron) candidates are taken as originating from an ISR photon and are removed; the latter are often the result of photon conversion in the material before the tracking chambers. Next, the invariant mass $M_{\text{vis}}$ of the system of remaining particles in the event is then computed; the reduced centre-of-mass energy $s'$ is also estimated by reclustering the remaining particles into two jets and calculating $s'$ based on the angles between them. Finally, the events with a large ISR energy component are rejected by requiring that $M_{\text{vis}} > 0.7$ and $s'/s > 0.81$. According to Monte Carlo studies based on the PYTHIA generator version 5.7 [9], the fraction of radiative events (defined by $\sqrt{s'/s} \leq 0.9$) in the selected sample is $\sim 4\%$ at 206 GeV. It should be noted that in the calculation of the event shape and other quantities in the final analysis, all reconstructed particles in the selected event are used.

The events passing the anti-ISR cuts still contain some background from four-fermion processes (WW, ZZ, $Z\gamma^*$). These are rejected by first clustering the particles to exactly four jets with the Durham algorithm. The energies of the jets are then rescaled, keeping their directions constant, such that the total energy of the event is equal to $E_{\text{cm}}$ and the total momentum is zero. The quantities

$$d^2 = \min \left[ \frac{(m_{ij} - M_W)^2 + (m_{kl} - M_W)^2}{M_W^2} \right],$$

with $M_W = 80.25$ GeV, and

$$c_{WW} = \cos(\text{smallest interjet angle})$$

are then computed, where for $d^2$ the minimum value is taken among all possible choices of jet pairings $ij$ and $kl$. Events are accepted if $d^2 \geq 0.1$ or $c_{WW} \geq 0.9$. The overall efficiency of the
event selection is about 20%. The integrated luminosities and numbers of events accepted and expected are shown in Table 1. The expected number of signal events has been obtained from the program KORALZ \([10]\), those for WW background from KORALW \([11]\) and for the ZZ and \(Z\gamma^*\) backgrounds from PYTHIA. The data taken at 130 and 136 GeV are averaged into a single

<table>
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<th>(\int L dt) (pb(^{-1}))</th>
<th>events found</th>
<th>events expected</th>
<th>expected signal</th>
<th>expected background</th>
</tr>
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<td>3590</td>
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<td>518</td>
</tr>
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</table>

Table 1: Integrated luminosities and numbers of accepted and expected events. There is an uncertainty of 2% in the predicted numbers of events.

data set at a nominal energy of 133 GeV. Weights proportional to the luminosity are applied and distributions are corrected to 133 GeV. The same procedure has been applied to the data sets at 189 and 192 GeV (averaged into a single set at nominal \(\sqrt{s} = 189\) GeV), at 196, 200 and 202 (averaged to \(\sqrt{s} = 200\) GeV) and to the data taken in the range from 203 to 209 GeV (averaged to 206 GeV).

 Corrections for imperfections of the detector and for the residual effects of ISR are made by means of multiplicative factors, as done in \([5]\) \([6]\). These factors, which are derived from the Monte Carlo model KORALZ, are by construction approximately independent of the model used. For the simulation of hadronic final states in \(e^+e^-\) annihilation, JETSET version 7.4 \([9]\) and KORALZ are essentially equivalent. KORALZ is used for the detector corrections because of its more accurate description of initial state photon radiation.

The detector systematics were, when appropriate, estimated using the Z data collected in the same year as the high-energy data. The selection cuts on track parameters were changed in the Monte Carlo until the number of events selected per unit luminosity were the same in Monte Carlo and data. These changes were then applied for Monte Carlo only to the analysis of the high-energy events, and the change in the extracted values for each event-shape variable is taken as a systematic uncertainty.

 Event-shape distributions were measured using charged particle tracks and neutral energy clusters in calorimeters. To account for imperfections in the description of neutral objects, classes of objects in the range from 1 to 2 GeV were excluded from the analysis, and the change in the resulting distribution was taken as systematic error. For the systematic tests of the ISR and WW rejection and the event selection cuts, performed via cut variations, many Monte Carlo samples had been generated to estimate the statistical precision of the test. The dispersion of the results for a given test applied to the Monte Carlo samples was then compared with the result of the same test applied to data. If the change in data was greater than the expected precision then it was taken as a systematic error after the statistical precision had been subtracted in quadrature. For the cases where the data test was not significant, the largest upwards and downwards fluctuations were noted. Assuming a uniform probability distribution for this difference, an additional uncorrelated systematic uncertainty was derived.

 The systematic uncertainty due to a residual model dependence has been estimated by comparing with the results based on correction factors derived from HERWIG version 5.9 \([12]\)
and from ARIADNE 4.10 [13].

Variations in the WW cross sections used for background subtraction by ±2% led to negligible uncertainties in the corrected distributions.

In the event-shape distributions, the systematic uncertainty estimates in each bin are dominated by the small changes in the selected events and tracks as cuts are varied, and hence are very much limited in statistical precision. For this reason, the estimates for neighbouring bins have been averaged in groups of three.

For the measurement of event shapes at the Z boson resonance, about 1.1 million events were selected from the running period in 1994 and 1995. Where appropriate, the experimental systematic uncertainties was obtained in a similar way than at LEPII, but without statistical reduction for event selection cut variations.

The dominant experimental systematic uncertainty stems from the residual model dependence.

A definition of the event-shape variables studied here is given in [3]; these are thrust $T$ [14], heavy jet mass squared $M_H^2/s$ [15], wide and total jet broadening $B_W$ and $B_T$ [16], C-parameter $C$ [17] and $-\ln y_3$ [8].

### 3 Measurements of $\alpha_s$

Distributions of event-shape variables, which have been measured at energies between 91.2 and 206 GeV, are used in this section to determine the strength of strong interactions. The coupling constant $\alpha_s$ is determined from a fit of the perturbative QCD prediction to the measured event-shape distributions. The experimental situation at energies above $M_Z$ is different from measurements at $M_Z$. Statistical uncertainties are larger and background conditions are more difficult. In general theoretical uncertainties are limiting the precision of measurements, except for very small data sets at 161 and 172 GeV, where statistical errors dominate. It is in particular at these energies essential to combine measurements from different variables.

Event-shape distributions are fitted in the central region of 3-jet production, where a good perturbative description is available. The fit range is placed inside a region where hadronisation corrections are well under control, where detector corrections are stable and background corrections are small. In order to reduce the statistical error, the fit range was extended at high energies into the 2-jet region as much as possible. This generates larger perturbative uncertainties, which are avoided at $Q = M_Z$ by restricting the fit range. The data are corrected for detector effects, background from 4-fermion processes and for residual ISR contribution, as outlined in Section 2. Background from WW events is increasing with energy, and after subtraction some bins of the distribution become negative. This affects the choice of the fit range, which was restricted to regions with a good signal-to-background ratio.

Distributions of infrared- and collinear-safe observables at partonic level can be computed in perturbative QCD to second order in $\alpha_s$ using the ERT matrix elements [18]. In addition, the variables used in this analysis exhibit the property of exponentiation so that leading and next-to-leading logarithms can be resummed to all orders in $\alpha_s$ into analytic functions [19–22]. These resummed calculations, valid in the semi-inclusive region, have to be matched to the fixed order part in order to obtain an improved prediction over the entire phase space. It is worth noting that for this analysis modified Log(R) and R [19] matching schemes are used. In the modified matching schemes a kinematic constraint is imposed such that the predicted distributions vanish at a given boundary value $y_{max}$. The formulae are given for convenience in Appendix A.

All these calculations above neglect quark masses. Quark mass effects are relevant for the b-quark at $M_Z$, where the fraction of b-quarks is large in an inclusive sample, while $Q$ is still moderate. Calculations including a quark mass indicate that the expected change in $\alpha_s$ is of the order of 1% at $M_Z$. The effect is scaling with $m_b^2/Q^2$ and decreases to 0.2-0.3 % at 200
GeV. Mass corrections were computed using the matrix elements of [23], to second order. No corrections are yet available for the resummed calculations, the full theoretical prediction can consequently account for the quark mass effect only in the perturbative region. The perturbative QCD prediction is corrected for hadronisation and resonance decays by means of a transition matrix, which is computed with Monte Carlo generators. Corrected measurements of event-shape distributions are compared to the theoretical calculation at particle level.

The value of $\alpha_s$ is determined at each energy from a binned least-squares fit. The resulting measurements of $\alpha_s(Q)$ are given in Table 2 for 91.2 to 172 GeV and in Table 3 for 183 to 206 GeV. Systematic theoretical and experimental uncertainties are discussed below.

<table>
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<th>$Q = 91.2$ GeV</th>
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<th>$M_H$</th>
<th>$C$</th>
<th>$B_W$</th>
<th>$B_T$</th>
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<td>$\alpha_s$</td>
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<td>0.1201</td>
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<td>0.1225</td>
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<td>0.0002</td>
<td>0.0002</td>
<td>0.0001</td>
<td>0.0002</td>
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<tr>
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<tr>
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<td>0.0012</td>
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<th>$B_T$</th>
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<tr>
<td>theo. error</td>
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<table>
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<tr>
<td>theo. error</td>
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Table 2: Results on $\alpha_s(Q)$ as obtained from fits to distributions of event-shape variables at 91.2, 133, 161 and 172 GeV.

The value of the renormalisation scale is set to $\mu = Q$ in the parton level prediction, which is then folded to the hadron level with the use of a transition matrix computed with PYTHIA. The modified Log(R) matching scheme is used to merge fixed order and resummed calculations. Mass corrections are computed for a pole b-quark mass $M_b = 5$ GeV and Standard Model values are taken for the fraction of b-quarks. Nominal fit results using the modified Log(R) matching scheme are shown in Figs. 1 and 2 together with the measured distributions.

Experimental systematic uncertainties of $\alpha_s$ are estimated in a similar way than for the event-shapes themselves, as described in Section 2. Changes in $\alpha_s$ under variations of cuts are
Figure 1: The measured distributions, after correction for backgrounds and detector effects, of thrust, -ln y₃, heavy jet mass and wide jet broadening at energies between 91.2 and 206 GeV together with the fitted QCD predictions. The fit ranges cover the central regions indicated by the solid line, the theoretical predictions extrapolate well outside the fitted ranges, as shown by the dotted lines. The plotted distributions are scaled by arbitrary factors.
Table 3: Results on $\alpha_s(Q)$ as obtained from fits to distributions of event-shape variables at 183, 189, 200 and 206 GeV.

in general dominated by statistical fluctuations. To reduce the statistical component, the expected precisions of tests were included as additional statistical errors in the fits. Only changes in $\alpha_s$ larger than the precisions are taken as systematic effect, others are accumulated, and the largest upward and downward fluctuations are taken as uncorrelated systematic error. This procedure is still under evaluation. It is evident that the experimental systematic error is still subject to statistical fluctuations. A new method to assess experimental uncertainties is being developed. The sources of uncertainties are identical from 161 to 206 GeV, but size and composition of backgrounds are energy dependent. Dominant systematic uncertainties are related to the dependence of the detector corrections on the Monte Carlo model and at LEPII to background rejection cuts. These uncertainties result in errors of the order of 1-2%.

Various sources of theoretical uncertainties have been investigated. Here the main uncertainty is stemming from missing higher orders in the perturbation series. These have been estimated in the following way. First an error for the matching scheme uncertainty is assigned, which is taken to be the difference of fit results obtained with the modified R and Log(R) matching schemes at $\mu = Q$. Then the renormalisation scale is varied in the range $0.5 \leq x = \mu/Q \leq 2$ for the modified Log(R) matching scheme. The maximum spread of the fit results at the scale variation endpoints are taken as scale uncertainty error. In addition an error for the mass
The measurements obtained with different variables are combined into a single measurement per energy using weighted averages. A weight is assigned to each observable dependent measurement \( \alpha_s \) proportional to the inverse square of its total error \( w_i \propto 1/\sigma_i^2 \). The weighted average \( \bar{\alpha}_s \) is then given by:

\[
\bar{\alpha}_s = \frac{1}{N} \sum_{l=1}^{N} w(l)\alpha_s(l) ,
\]

and the combined statistical error is

\[
\sigma_{\alpha_s}^{\text{stat}} = \sqrt{\sum_{i \neq j}^{N} (\sigma(i)w(i))^2 + 2\rho_{ij}\sigma(i)w(i)\sigma(j)w(j)} .
\]

In order to obtain the statistical error of the weighted average, the correlation coefficient \( \rho_{ij} \) is needed. This is the correlation between fits of \( \alpha_s \) to different variables, which has been obtained by fitting a large number of Monte Carlo data samples. The linear correlation coefficient is typically 60-80 %. The correlation of systematic errors is taken into account in such a way, that the weighted average is recomputed for all variations of the analysis, and the change in \( \alpha_s \) with
respect to the nominal value is taken as error. Combined results are given in Table 4. The combined experimental systematic uncertainty at LEPII energies is obtained from a luminosity-weighted average of the uncertainties between 133 GeV and 206 GeV. They are also shown in

\[
\begin{array}{|c|cccccccc|}
\hline
Q \text{ [GeV]} & 91.2 & 133 & 161 & 172 & 183 & 189 & 200 & 206 \\
\hline
\alpha_s(Q) & 0.1203 & 0.1182 & 0.1170 & 0.1050 & 0.1085 & 0.1107 & 0.1081 & 0.1049 \\
\text{stat. error} & 0.0001 & 0.0030 & 0.0048 & 0.0054 & 0.0028 & 0.0018 & 0.0021 & 0.0018 \\
\text{exp. error} & 0.0013 & 0.0012 & 0.0012 & 0.0012 & 0.0012 & 0.0012 & 0.0012 & 0.0012 \\
\text{theo. error} & 0.0046 & 0.0051 & 0.0058 & 0.0041 & 0.0043 & 0.0043 & 0.0039 & 0.0036 \\
\text{total error} & 0.0048 & 0.0060 & 0.0076 & 0.0069 & 0.0053 & 0.0048 & 0.0046 & 0.0042 \\
\hline
\end{array}
\]

Table 4: Combined results of \(\alpha_s(Q)\) as obtained with weighted averages.

Fig. 3, together with a fit of the QCD expectation. The curve is seen to be in excellent agreement with the measurements. Note that in the definition of the \(\chi^2\) of the fit only the uncorrelated component of the errors is taken into account, which excludes the theoretical error.

In a second step combined measurements between 133 and 206 GeV are evaluated at the scale of the Z boson mass. This is done by using the predicted energy evolution of the coupling constant at 3-loop level [24]. Once all measurements are evolved to the same scale, they can again be combined into a weighted average of \(\alpha_s(M_Z)\). In contrast to the combination from different variables, the measurements here are statistically uncorrelated. Systematic error correlations are taken into account, all variations of the determination of \(\alpha_s\) have been performed for the weighted average. The result is given in Table 5. This measurement is in good agreement with previously published measurements of \(\alpha_s\) [25–28]. A combined measurement of \(\alpha_s\) at LEPI and

\[
\begin{array}{|c|c|}
\hline
\text{data set} & \text{LEPII} \\
\hline
\alpha_s(M_Z) & 0.1211 \\
\text{stat. error} & \pm 0.0013 \\
\text{exp. error} & \pm 0.0015 \\
\text{theo. error} & \pm 0.0052 \\
\text{total error} & \pm 0.0056 \\
\hline
\end{array}
\]

Table 5: Weighted average of combined measurements at LEPII of \(\alpha_s(M_Z)\) obtained at energies from 133 GeV to 206 GeV.

LEPII energies does not improve the result using LEPI data alone, if total errors are used as weights. Another way to combine measurements is to perform simultaneous fits to data sets at all energies. Effectively, this implies using statistical significances as weights, so this result is dominated by the precise data at \(M_Z\), and the result is almost the same as without high energy data. The combined result using simultaneous fits is given in Table 6.

\[
\begin{array}{|c|c|}
\hline
\text{data set} & \text{LEPI+LEPII} \\
\hline
\alpha_s(M_Z) & 0.1203 \\
\text{stat. error} & \pm 0.0001 \\
\text{exp. error} & \pm 0.0013 \\
\text{theo. error} & \pm 0.0046 \\
\text{total error} & \pm 0.0048 \\
\hline
\end{array}
\]

Table 6: Combined result of \(\alpha_s(M_Z)\) obtained at all energies using simultaneous fits.
Figure 3: The strong coupling constant $\alpha_s$ measured between 91.2 and 206 GeV. The measurements using different event-shape variables are combined, correlations are taken into account. The outer error bars indicate the total error. The inner error bars exclude the theoretical error, which is expected to be highly correlated between the measurements. A fit of the 3-loop running formula is shown, where the hatched area corresponds to the statistical uncertainty of $\Lambda_{\overline{MS}} = 244 \pm 12$ MeV.

4 Conclusions

New results are presented for a consistent measurement of event-shape variables recorded by ALEPH at a centre-of-mass energies between 91.2 GeV and 206 GeV. The energy evolution of the strong coupling constant $\alpha_s$ has been investigated.

For $\alpha_s$, the distributions of thrust, C-parameter, heavy jet mass, $-\ln y_3$, wide and total jet broadening have been compared to calculations of perturbative QCD, and the strong coupling constant has been measured at all energies. The new combined result is $\alpha_s(M_Z) = 0.1203 \pm 0.0048$, combining only LEPII data yields $\alpha_s(M_Z) = 0.1212 \pm 0.0054$. The results are found to be in good agreement with the expected energy evolution of the running coupling constant.

Acknowledgements

We wish to thank our colleagues from the accelerator divisions for the successful operation of
LEP. It is also a pleasure to thank the technical personnel of the collaborating institutions for their support in constructing and maintaining the ALEPH experiment. Those of the collaboration not from member states thank CERN for its hospitality.

References

A Theoretical predictions

To second order in $\alpha_s$, the distribution of a generic event-shape variable $y$ is given by:

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma(y)}{dy} = \alpha_s(\mu^2) A(y) + \left(\alpha_s(\mu^2)\right)^2 \left[A(y) 2\pi b_0 \ln \left(\frac{\mu^2}{s}\right) + B(y)\right],$$

where $\alpha_s = \alpha_s^2 / 2\pi$, $b_0 = 33 - 2n_f / 12\pi$, $\mu$ = renormalisation scale.

The coefficient functions $A$ and $B$ are obtained from ERT matrix element integration. Consider the cumulative cross section (Radiator):

$$R(y, \alpha_s) \equiv \frac{1}{\sigma_{\text{tot}}} \int_0^y \frac{d\sigma(x, \alpha_s)}{dx} dx,$$

which may be cast into the second-order form

$$R^{(\alpha^2)}(y, \alpha_s) = 1 + A(y)\alpha_s + B(y)\alpha_s^2,$$

where $A$ and $B$ are integrated forms of $A$ and $B$, and the explicit scale dependence has been dropped for clarity.

The prediction of the Log(R) matching scheme is given by:

$$\ln R(y, \alpha_s) = L g_1(\alpha_s L) + g_2(\alpha_s L) - (G_{11} L + G_{12} L^2)\alpha_s$$

$$- (G_{22} L^2 + G_{23} L^3) \alpha_s^2 + A(y)\alpha_s + B(y) - \frac{1}{2} A^2(y)\alpha_s^2,$$

with $L = \ln(1/y)$ for $y = 1 - T, M^2_{\text{R}}/s, y_3, B_T, B_W$ and $L = \ln(6/C)$ for C-Parameter. Expressions for the functions $g_1$ and $g_2$, which resum leading and next-to-leading logarithms to all orders in $\alpha_s$, can be found in the literature [19] [20] [21] [22].

A kinematic constraint is imposed to the modified Log(R) matching scheme to guarantee that the prediction of the distribution vanishes at a given value $y_{\text{max}}$.

$$\ln R(y_{\text{max}}) = 0,$$

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma(y)}{dy} \bigg|_{y=y_{\text{max}}} = \frac{dR}{dy} \bigg|_{y=y_{\text{max}}} = 0.$$

To fulfill this constraint $L$ is replaced by $\tilde{L} = \ln(1/y + 1/y_{\text{max}} + 1)$, respectively $\tilde{L} = \ln(6/C + 6/C_{\text{max}} + 1)$ for C-parameter. The values of $y_{\text{max}}$ and $C_{\text{max}}$ are given in Table 7. Hence the
Table 7: Values of $y_{max}$ at which distributions are forced to vanish.

<table>
<thead>
<tr>
<th>variable</th>
<th>$1 - T$</th>
<th>$M_{H}/s$</th>
<th>$- \ln y_3$</th>
<th>$B_T$</th>
<th>$B_W$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{max}$</td>
<td>0.5</td>
<td>0.47</td>
<td>$\ln 3$</td>
<td>0.41</td>
<td>0.35</td>
<td>1</td>
</tr>
</tbody>
</table>

The prediction of the **modified Log(R)** matching scheme is simply obtained by replacing $L$ by $\tilde{L}$ in equation (5).

The expression for the $R$ matching scheme reads as

$$R(y, \alpha_s) = (1 + c_1 \pi_s + c_2 \pi_s^2) \exp \left[ Lg_1(\alpha_s L) + g_2(\alpha_s L) + G_{21} L \pi_s^2 \right]$$

$$- G_{21} L \pi_s^2 - \left[ C_1 + G_{11} L + G_{12} L^2 \right] \pi_s$$

$$- \left[ C_2 + C_1 (G_{11} L + G_{12} L^2) + \frac{1}{2} (G_{11} L + G_{12} L^2)^2 + (G_{22} L^2 + G_{23} L^3) \right] \pi_s^2$$

$$+ A(y) \pi_s + B(y) \pi_s^2.$$  

The constraints for the **modified R** matching are

$$R(y_{max}) = 1,$$  

$$\frac{1}{\sigma_{tot}} \frac{d\sigma (y)}{dy} \bigg|_{y=y_{max}} = \frac{dR}{dy} \bigg|_{y=y_{max}} = 0.$$  

Here a simple modification of $L$ does not satisfy the second constraint. Therefore, $L$ is modified and the matching coefficients $G_{11}$ and $G_{21}$ become functions of $y$ according to the condition:

$$\tilde{L}(y_{max}) = 0, \quad \tilde{G}_{11}(y_{max}) = 0, \quad \tilde{G}_{21}(y_{max}) = 0.$$  

This is achieved with the following modification:

$$\tilde{L}(y) = \ln \left[ \frac{1}{y} - \frac{1}{y_{max}} + 1 \right]$$

$$\tilde{G}_{11}(y) = G_{11} \left[ 1 - \frac{y}{y_{max}} \right]$$

$$\tilde{G}_{21}(y) = G_{21} \left[ 1 - \frac{y}{y_{max}} \right],$$

with the special case of $C$-parameter $\tilde{L}(C) = \ln \left[ 6/C - 6/C_{max} + 1 \right]$.

Finally the expression for the **modified R** matching scheme can be written as

$$\tilde{R}(y, \alpha_s) = (1 + c_1 \pi_s + c_2 \pi_s^2) \exp \left[ \tilde{L}g_1(\alpha_s \tilde{L}) + g_2(\alpha_s \tilde{L}) - \frac{y}{y_{max}} G_{11} \pi_s \tilde{L} + \tilde{G}_{21} \pi_s \tilde{L}^2 \right]$$

$$- \tilde{G}_{21} \pi_s \tilde{L}^2 - \left[ C_1 + \tilde{G}_{11} \tilde{L} + G_{12} \tilde{L}^2 \right] \pi_s$$

$$- \left[ C_2 + C_1 (\tilde{G}_{11} \tilde{L} + G_{12} \tilde{L}^2) + \frac{1}{2} (\tilde{G}_{11} \tilde{L} + G_{12} \tilde{L}^2)^2 + (G_{22} \tilde{L}^2 + G_{23} \tilde{L}^3) \right] \pi_s^2$$

$$+ A(y) \pi_s + B(y) \pi_s^2.$$