1. Introduction: The Language of Quantum Chromodynamics

The language of QCD is expressed in terms of Feynman diagrams and field theory. The fundamental particles of QCD are quarks and gluons, and the interactions are described by the strong force. The theory is based on the standard model of particle physics and is used to explain the behavior of hadrons. The language of QCD is crucial for understanding the strong interactions in the universe.
ground and paradigm for these now-mature concepts. It serves as a model for nonperturbative analysis beyond Standard Model physics. Quantum chromodynamics is, through its spontaneous chiral symmetry breaking, the origin of most of the mass of “bright” matter in the universe. Finally, it is a reservoir of benchmark problems, including confinement, and the description of its nonperturbative structure.

Quantum chromodynamics is a vast field, with subdisciplines, each characterized by its own degrees of freedom, including perturbative QCD, heavy quark effective theory, nonrelativistic QCD, chiral perturbation theory, lattice QCD, discrete light cone QCD, not to mention nuclear physics. In a sense, the great, nearly inexhaustible challenge of strong interaction physics is to delineate the applicability of these effective pictures, to connect them to each other, and to the underlying dynamics. At present, we have no universal approach adequate to treat all of these aspects of the theory. We have instead a family of languages, appropriate to different length scales.

Perturbative QCD

The use of perturbation theory in QCD is based in asymptotic freedom, in which the coupling of the theory vanishes as the logarithm of the momentum scale at which it is probed, \(\alpha_s(Q) \sim 2\pi/\beta_0 \ln(Q/\Lambda_{QCD})\). To use this property, however, we must identify observables that depend upon the theory at some specified scale, \(Q \gg \Lambda_{QCD}\). Such quantities, which are termed “infrared safe”, take the general form

\[
Q^2 \sigma_{SD}(Q, m) = \sum_n a_n \left( Q^2 / \mu^2 \right) \alpha_s^n(\mu) + \sum_p \frac{1}{Q^p} A_p \left( Q / \mu, \alpha_s(\mu) \right).
\]

Examples of infrared safe observables are jet, event shape and total cross sections in \(e^+e^-\) annihilation and weak vector boson decay, for which \(Q\) is the total center-of-mass energy scale and the vector boson mass, respectively. The parameter \(\mu\) is the renormalization scale, which may be chosen in these cases equal to \(Q\). We have exhibited as well power corrections in \(Q\), which are primarily nonperturbative in origin. We will encounter the important phenomenological role of power corrections below. The coefficients \(a_n\) may be calculated exactly. They provide, however, at best an asymptotic series for the observable in question.

Despite their variety, cross sections that are infrared safe are the exception rather than the rule at high energy, and are limited to those with purely leptonic initial states. The range of perturbative methods is essentially extended by identifying cross sections with large momentum transfer which, although not directly infrared safe, exhibit the factorization of perturbative short-distance from nonperturbative long-distance dynamics. Usually, such a factorization is manifested in the form of a convolution, for example, an integral over parton momentum fraction, “\(x\)”. Factorizations, in turn, imply evolutions, or resummations of logarithmic corrections.

In a simplified example, suppose that we can write a “physical” cross section describing momentum transfer \(Q\), as a product of short-distance and long-distance factors, which we may think of as coefficient functions and parton distribution functions (PDFs), respectively:

\[
Q^2 \sigma_{phys}(Q, m) = C_{SD} \left( \frac{Q}{\mu} \alpha_s(\mu) \right) f_{LD} \left( \frac{\mu}{m}, \alpha_s(\mu) \right) + \sum_n (1/Q^n) C_n,
\]

where the “factorization scale” \(\mu\) separates long- and short-distance dynamics. The nonleading, \(C_n\) terms, suppressed by powers of the momentum transfer, generally possess related, but more complex factorization properties. Since the choice of \(\mu\) is arbitrary, the \(\mu\)-dependence is determined to be power-like,

\[
\mu d\sigma_{phys}/d\mu = 0 \Rightarrow C \sim Q^{1/2}(\alpha_s(\mu)) \quad f \sim \mu^{1/(\alpha_s(\mu))}.
\]

The power, an “anomalous dimension”, can depend only upon \(\alpha_s(\mu)\), because this is the only scale held in common by the two functions. Then, because the coefficient function can depend upon \(Q\) only through the ratio \(Q/\mu\), the anomalous dimension controls the dependence of the physical cross section on the momentum transfer. Essentially all the evolutions and resummations of perturbative QCD may be understood in this language. Perturbative QCD is not all of QCD, but the two become equivalent for infrared safe quantities as \(Q \to \infty\).

The formal basis of factorization is in the quantum-mechanical incoherence of the long-distance dynamics that binds hadrons from the the short-distance dynamics that governs large momentum transfers, or the creation of extremely short-lived virtual states (sensitive to new physics). The product (or convolution) form can characterize factorized cross sections express this incoherence. The probabilities for incoherent processes may be multiplied, as if they were classical, and may be calculated using different methods. As long as \(\mu\) is large enough, however, perturbation theory may be employed to control logarithmic behavior in the factorization scale.
The techniques of infrared safety, factorization and evolution, involve a variety of expansions, in terms of the coupling $\alpha_s$, of course, but also, as we have seen in momenta $1/Q$. In the presence of additional scales, expansions in heavy quark masses, or nonrelativistic velocities in units of $c$, are appropriate in the same basic pattern.

Lattice QCD

Sometimes thought of as the opposite extreme from perturbative QCD, lattice QCD works best for static properties of the theory, dependent on its long-distance structure. Rather than cross sections at large momentum transfer, its stock-in-trade is operator expectation values, from which may be determined bound-state masses and, of increasing interest, matrix elements of local operators such as electroweak currents.

A typical lattice expansion, analogous to Eq. (1), for the expectation of operator $J$, relates the physical, or continuum expectation, to the corresponding matrix element in the lattice-regularized theory. This relation is of the general form,

$$\langle J^{\text{cont}} \rangle = Z (\mu a, \alpha_s(\mu)) \langle J^{\text{lat}}(\mu a) \rangle + \sum_n q^n \langle K_n (\mu, a, \alpha_s(\mu)) \rangle,$$

with $a$ the lattice spacing. The matrix elements here should be understood to include the nonlocal products of a few extra fields, through which expectations in hadronic states can be generated. In contrast to perturbative methods, the expectation on the right-hand side, $\langle J^{\text{lat}}(\mu a) \rangle$ may be computed numerically with high accuracy on the lattice. Numerical and finite-size issues aside, the challenge in this case is that the lattice theory is not the continuum theory, but is related to it by the limit of vanishing lattice spacing, $a \to 0$.

The relation between the continuum and lattice matrix elements bears a strong resemblance to the factorized cross section in Eq. (2), with the $Z$-factor playing the role of the short-distance coefficient function, now a function of $a$ in place of $Q$, and the expectation value playing the role of the long-distance matrix element. As in the previous case, and indeed even more so in the lattice relation, contributions that are power-suppressed in $a \approx 1/Q$ play a crucial role. Unlike a perturbative calculation, however, the output of the lattice calculation is the long-distance matrix element itself. The perturbative expansion of the $Z$-factor remains of importance to make the transition, and control over the expansion in $a$ requires special care. In realistic cases, quark masses also matter, and we may variously encounter expansions in inverse powers of heavy quark masses, and in (chiral) logarithms of light quark masses.

Field-theoretic consistency

Perturbative and lattice QCD are “first-principle” methods, exact up to corrections that are relatively easy to characterize, but are generally quite difficult to estimate. Not all valuable information is found in this fashion, however. The self-consistency of quantum chromodynamics itself, and of QCD with the remainder of the Standard Model, imply strong constraints on experiment, and provide powerful frameworks with which to study the theory.

From the consistency of perturbative QCD, supplemented by the operator product expansion, its nonperturbative spectrum and transition amplitudes, come QCD Sum Rules. Variants of this approach involve such concepts of very current interest as the local duality between partonic and hadronic degrees of freedom. From the necessary harmony of QCD with the electroweak content of the Standard Model, including the Higgs phenomenon, may be derived chiral perturbation theory, and its very successful description of the lowest-energy strong interactions.

Our understanding of the spectroscopy of QCD is still far from complete, and efforts to move the frontier forward rely upon all of these methods, from classic quark model concepts to the most mathematical sleights of hand of duality. Light-cone methods offer another synthesis of perturbative and lattice ideas [7].

QCD at Snowmass 2001

The QCD Working Group (P5) was convened by Brenna Flaugher, Ed Kinney, Paul Mackenzie and George Sterman. The results of the five topical subgroups are summarized in this report. With their conveners, they were: A. Parton distributions, spin and resummation (Walter Giele and Fred Olness), B. Fixed-order perturbation theory and top (Bill Kilgore), C. Jets and jet algorithms (Steve Ellis and Brenna Flaugher), D.
Diffractive and nuclear QCD (Ina Sarcevic) and E. Lattice QCD, light and heavy hadrons (Paul Mackenzie). George Sterman prepared the general sections of this report.

It is not possible below to summarize all the ideas that were discussed nor all the progress that was made at Snowmass, some of which is surely still latent in the minds of participants, and QCD is far too large a field to address fully even in an extended workshop. At the same time, QCD issues were so integral to the considerations of other Physics, Experimental and Accelerator working groups, that demands regularly outpaced supply, even for the deep intellectual resources assembled on the mountainside. For example, stimulating discussions were held on very high energy neutrino cross sections [8], on the CESR and JLab programs [10] in spectroscopy and on the interplay of chiral perturbation theory with lattice QCD, which we will not touch on in detail below. Another exciting but neglected area, closely related to those discussed at Snowmass and below, regards elastic scattering and rare B decays. Despite these omissions, we hope that this Summary may be successful in communicating at least some of the sense of breadth and excitement of the meeting and of the field.

II. PRECISION AND A NEW ERA OF QCD

In a sense, we may think of quantum chromodynamics as a vast continent, separating two oceans that represent its asymptotic limits: one to "high-\(Q\)" partonic degrees of freedom, the other to "low-\(Q\)" hadronic degrees of freedom. Lattice methods address the latter most directly; perturbative methods the former. Pursuing this metaphor, the aim of resummations, as illustrated by (3), is to extend these analyses into a region where the different pictures, in terms of different degrees of freedom overlap. The increased precision for which we strive in computing higher-order perturbative and power corrections in Eqs. (1) and (4) is the route inland to this continent.

![Image of QCD as a continent](image)

**FIG. 1: QCD as a continent.**

Increased precision leads to qualitative advances in understanding when it requires the simultaneous treatment of alternative degrees of freedom, the translation of one language of QCD to another. One example, now close at hand, is the role of hadronization and confinement effects in jet cross sections, a problem that is "pure QCD". The same principle of alternative degrees of freedom applies to the role of QCD in the search of new physics. The measurement of the strong coupling with an accuracy necessary to extrapolate it beyond the electroweak symmetry breaking scale will require precisely this control over nonperturbative power corrections in jet cross...
sections [3, 4]. Similarly, to detect physics at the TeV scale that starts and mixes with quantum chromodynamics will require an unprecedented ability to interpret the strong-interaction structure of final states [5, 6].

Both QCD dynamics and the search for new physics require reliable extrapolations from long to short distances. We may summarize this relationship with a variant of Eq. (2),

$$\sigma = C_{SD, \text{new}} \times Q^{\text{CD}}(Q, M_{\text{new}}, \mu) \otimes f_{\text{LD, QCD}}(\mu, m).$$

New physics at the scale $M \sim 1$ TeV, say, appears in the coefficient function, $C$, while the long distance function $f$ is controlled by QCD. In high-energy experiments that involve direct production of new states, the momentum transfer $Q \sim M$, and we will want to extrapolate $f$ over a long lever arm in $Q/\mu$, from precision experiments at current energies to the TeV range. In experiments, such as those at B factories, where we rely on rare events at fixed $Q (\sim 5$ GeV, say), we need an equally precise understanding of the long-distance function $f$, along with a careful calculation of $C$ in a variety of new-physics scenarios.

The nineteen eighties saw the confirmation of the basic ideas of quantum chromodynamics, which solidified its place in the Standard Model. The past decade has seen a transformation in QCD, and the focus has moved from “tests” of the theory, toward its exploration. In a large sense, this step forward has been made possible by a revolution in QCD data. The multi-GeV accelerator runs of the Tevatron, LEPI, SLC, HERA and LEPII have provided data that exposed the partonic degrees of freedom of QCD in an unprecedented fashion. To see the difference, one need only compare the illustrations from the Review of Particle Properties from 1990 and 2000.

For this reason, as well as because of advances in theory along many fronts, QCD is at the beginning of a new era. This sense informed our discussions at Snowmass, in a list of discussions which could hardly have been imagined at the end of the 1980’s, and which may serve as an outline for the remainder of this report.

- From Next- to next-to-next . . . (Section III.) Two-loop calculations for selected single-scale cross sections have existed for some time. The past year has seen the first calculations of true two-loop scattering amplitudes. These extraordinary new calculations are still some way from implementation as factorized cross sections, but the ice has been broken. Two-loop cross sections at large momentum transfer will make possible percent accuracy in a wide range of applications.

On a related front, from the design of detectors to the analysis of data, Monte Carlo event generators play a central role in modern high-energy physics. These computer programs generally dress lowest-order partonic cross sections with parton evolution based upon fragmentation, either independent or with QCD coherence effects taken into account to some approximation. The past few years has seen an increasing realization that the full potential of forthcoming experiments cannot be reached without the use of the information that comes from a more complete treatment of the hard scattering [11]. The challenge is to do so without double-counting, as a result of the definitions of the hard scattering and fragmentation parts. In a sense, this constitutes the problem of how to generalize factorization forms like Eq. (2) to arbitrary numbers, and quantum numbers, of outgoing particles. There is no generally accepted solution to this problem, but its great importance to QCD phenomenology makes it a central part of the long-term program in the field.

- Parton distribution uncertainties. (Section IV.) One of the grand projects of QCD over the past thirty years has been the determination of parton distribution functions. Over the past few years, we have realized increasingly the importance of estimating uncertainties for these distributions, a complex enterprise involving estimates of theoretical as well as experimental, and systematic as well as statistical errors. They will be an essential ingredient in our analysis of new physics signals at hadronic colliders.

- Precision jet physics: measurements of $\alpha_s$. (Section V.) Some of the most striking results of the highest energy data, especially those from LEP, demonstrate the potential for a jet physics in which theory and experiment move from tens of percent toward the single percent level. Such an improvement of precision will make possible determinations of the strong coupling at a comparable level, making in turn much more precise extrapolations of the strong coupling, and hence of extensions of the Standard Model. It will stimulate an examination of both perturbative and nonperturbative jet physics at a similar level, illustrating again the interdependence of QCD physics and new physics searches.

- Resummation phenomenology. (Section VI.) The reorganization of perturbative corrections, to fold selected logarithmic contributions at all orders into exact low order calculations, is beginning to provide practical applications. Extensive studies have been carried out for event shape cross sections in $e^+e^-$ annihilation, and in the transverse momentum distributions of electroweak bosons produced through Drell-Yan processes in hadronic collisions. These may, in fact, be only the beginning.
Closely related to resummation is the analysis of power corrections, already begun for jet cross sections at LEP I and II, and at HERA. The applications of resummation noted above require nonperturbative corrections, both theoretically and phenomenologically.

- The dawn of unquenched lattice calculations. (Section VII.) Light quark loops are especially time-consuming in lattice calculations, and the “quenched approximation”, in which they are suppressed, has been hitherto a necessary evil for the majority of lattice results. These results can then reach an impressive precision, but they are not QCD, even on a lattice. Recent advances in lattice theory, at least as much as improvements in computing power, have made it possible to extrapolate a series of unquenched lattice calculations at the few percent level, and a transition from single-hadron expectations to multihadron amplitudes. Such calculations can play a unique role in the analysis of B decays, and the determination of parameters of the CKM matrix.

- Low-\(z\), diffraction and gaps, high-density QCD. (Section VIII.) The advent of high-energy nuclear scattering is beginning to make it possible to study high-density QCD in a partonic regime. The complementary program at HERA maps out the long-distance partonic content of the proton, at the edge of perturbative and nonperturbative dynamics.

- Polarized beams. (Section IX.) International programs are finding new windows into hadronic structure through polarized beams. Polarization capability can also provide sensitive tests of new physics, as shown at SLC.

III. FIXED ORDER CALCULATIONS AND TOP QUARK PHYSICS

This section was prepared by William B. Kilgore with input from Robert V. Harlander, Nikolaos Kidonakis, Steve Magill, Laura Reina and Zack Sullivan.

In the last several years, there have been a number of significant developments in the field of computing higher order QCD corrections: there have been technical developments in the computation of one-loop and two-loop matrix elements, a number of general-purpose formalisms for performing next-to-leading order Monte Carlo calculations have been developed, and resummation techniques have been employed to capture the most important components of higher order corrections to processes for which it is currently unfeasible to perform a full calculation beyond next-to-leading order. Top quark physics continues to be an active area of research with many important developments.

A. NLO

The process of performing next-to-leading order calculations is now quite mature. While the calculation of one-loop matrix elements is still quite challenging, the loop integrals involved are well-known and simplifying techniques (like the helicity method) for constructing the amplitudes are available. Once the matrix elements are known, there are a variety of well-established methods for constructing next-to-leading order Monte Carlo programs, including phase space slicing [12, 13], the subtraction method [14] and the dipole method [15, 16]. A thorough review of these methods and the various extensions to them has been made in reference [17].

Studies of QCD observables have revealed that leading order calculations are often little more than qualitative descriptions of the processes in question. While next-to-leading order corrections generally change the prediction for the total rate of a process one commonly finds that the corrections to the shapes of distributions of observables are far more important than the change to the total production rate. When the signal is prominent relative to the background, as in the case of massive vector boson production (decaying to leptons) in association with jets, or even dominant, as in the case of inclusive jet production, comparing measured distributions with NLO predictions allows one to extract more precise information about the process.

When backgrounds are large, however, the precise shapes of distributions of observables may be needed just to extract the signals. This observation points to a critical need for a much wider variety of next-to-leading order calculations. To date, the focus of such calculations has understandably been on interesting signal processes. However, it is expected that the distributions of background processes also change when computed at higher order. To some degree, this problem is avoided by measuring the background in side-bands and extrapolating into the signal region. When the signal lies in a steeply falling spectrum or at the tail of a background distribution, however, it may be difficult to make an accurate extrapolation. Thus, it is increasingly clear that it is not enough to merely compute the most important signal processes at NLO. The backgrounds to those
processes must also be computed at NLO if one is to extract the interesting signal on the basis of shapes of distributions.

In some cases, like Higgs production and heavy flavor production, leading order calculations severely underestimate the cross section of the processes in question. In such cases NLO calculations are necessary but are probably no more reliable than one would ordinarily expect leading order calculations to be and there is a dear need to extend calculations to still higher order.

B. NNLO

The developing state of the art in higher order corrections is next-to-next-to-leading order (NNLO). The first applications of NNLO calculations will be to the signal processes with the most important phenomenological applications. Indeed, the first NNLO calculations were to Drell-Yan production and the crossed process, deep inelastic scattering. In the last couple of years, the pieces have been assembled to permit NNLO calculations of inclusive Higgs production, dijet production in hadron-hadron collisions, $e^+e^-$ annihilation into three jets and the crossed process, dijet production in deep inelastic scattering.

To move beyond next-to-leading order QCD to next-to-next-to-leading order one needs two loop matrix elements. Two rather different techniques have been deployed recently. For some time now, the “integration by parts” method has been used to compute multi-loop vacuum polarization diagrams. Baikov and Smirnov [18] developed a technique to “open-up” the vacuum polarization diagrams, permitting the application of the integration by parts method to higher point amplitudes. This technique was exploited by Hahn and [19] to compute the two-loop virtual corrections to Higgs production by gluon fusion in the heavy top limit. With the virtual corrections available, it is now possible to compute inclusive Higgs production at NNLO [20, 21, 22].

Another extremely important development has been the computation of two-loop four point scattering amplitudes. The breakthrough came with the solutions to the scalar double box master integrals for all massless legs [23, 24] and the reduction of tensor double boxes to scalar master integrals [25, 26]. With the integrals in hand, it became possible to compute the two-loop amplitudes for scattering two massless partons into two massless partons (the amplitudes that contribute to single-jet-inclusive production) and these have all been computed [27, 28, 29].

More recently, the master double box integrals with a single massive external leg have been solved [30, 31, 32] and applied to the calculation of the scattering amplitude which describes such processes as $e^+e^- \rightarrow 3$ jets [33], dijet production in deep inelastic scattering and high $p_T$ Drell-Yan production. The phenomenological application of these processes will be profound. An NNLO calculation of $e^+e^- \rightarrow 3$ jets will permit a reanalysis of LEP data and provide a much improved measurement of $\alpha_s$.

In order to turn these two-loop amplitudes into NNLO calculations (for hadronic collisions), one also needs NNLO parton distribution functions (PDFs). The most important source for extracting this information is deep inelastic scattering (DIS). While the DIS coefficient functions have been known at NNLO [33, 34, 36, 37], only partial results are currently available for the three-loop anomalous dimensions describing parton evolution, in the form of a finite number of fixed Mellin moments [38, 39, 40, 41, 42, 43, 44, 45, 46]. These partial results have been used to construct approximations of the three-loop splitting functions [47, 48, 49, 50] which, in turn, have been used to extract approximate PDFs [51, 52].

To summarize the status of NNLO calculations: The Drell-Yan process [53] (inclusive production of massive lepton pairs) and the inclusive deep inelastic cross section [36] have been known for approximately ten years. The inclusive production of scalar Higgs bosons has recently been computed in the soft approximation and will soon be available for the full kinematic region. The matrix elements for the most important $2 \rightarrow 2$ (or $1 \rightarrow 3$) scattering processes are now known. The splitting functions for computing three-loop parton evolution are not yet fully known, but approximations based on a finite set of moments have allowed approximate extractions of NNLO parton distribution functions. The algorithms for performing NNLO Monte Carlo calculations have not yet been worked out and currently stand as the largest barrier to the application of perturbative QCD at next-to-next-to-leading order.

C. Applications of Resummation to Fixed Order Calculations

For many interesting processes, it is not yet possible to perform fixed order calculations beyond next-to-leading order. Still one would like to make use of the all-orders information available from resummation techniques. This is particularly true when one would like to make measurements near a production threshold, where phase space restrictions can lead to large logarithmic corrections. These logs occur at all orders in perturbation theory...
beyond leading order) and can lead to large corrections in the threshold region. Thus, it is important to capture these terms to obtain more accurate predictions than NLO calculations can provide.

Threshold resummation \cite{54, 55, 56, 57, 58, 59, 60} is performed in moment space and uses the factorization properties of QCD to compute the large logarithmic corrections to all order in $\alpha_s$. To take this all-orders information back to momentum space, one needs to invert the Mellin transform. This inversion involves crossing the Landau pole of the QCD coupling and requires a prescription. Unfortunately, different prescriptions lead to different results for the sub-leading logarithms. Without a direct calculation beyond NLO, it is impossible to accurately determine the sub-leading log terms.

A different technique is to expand the moment-space result in powers of $\alpha_s$ and then transform the fixed order result to momentum space, matching to the known NLO result. This avoids prescription dependence and unphysical terms and allows the accurate calculation of leading and sub-leading logarithmic terms \cite{61, 62}. The current state of the art in such calculations obtains results at NNLO-NLL (next-to-next-to-leading order and next-to-next-to-leading logarithm) \cite{63}. Such analyses have recently been applied to the inclusive jet production at NNLO-NLL and top quark production at NNLO-NLL \cite{63}.

D. Top Quark Physics

Top quark physics promises to be an active field of study in the coming years, offering a variety of important measurements at Run II of the Tevatron, at the LHC and at an $e^+e^-$ linear collider. There have been numerous surveys of top quark physics at various colliders \cite{3, 64, 65} so here we limit ourselves to developments presented at Snowmass 2001.

One of the most important measurements at the Tevatron and LHC will be the top production cross section. At the Tevatron, the cross section is strongly affected by threshold corrections. Threshold resummation captures the most important logarithmic corrections, but is plagued by prescription ambiguities. By expanding the (moment space) resummed result in $\alpha_s$ before inversion to momentum space, ambiguities can be avoided and one can obtain reliable predictions for the most important logarithmic terms (up to next-to-next-to-leading log) at NNLO \cite{58, 59, 63}.

The associated production of a Standard Model Higgs boson, $h_{SM}$, with a pair of top-antitop quarks can play a very important role at the Tevatron and at the LHC, both for discovery and for precision measurements of the top Yukawa coupling \cite{66}. Recently the total cross sections for $p\bar{p}\to t\bar{t}h_{SM}$ \cite{67, 68, 69} and $pp\to t\bar{t}h_{SM}$ \cite{68} have been calculated at $O(\alpha_s^3)$, i.e. at next-to-leading order (NLO) in QCD. The calculation of both virtual and real $O(\alpha_s)$ corrections required the generalization of existing methods to the case in which several external particles are massive.

The NLO QCD corrections slightly decrease the total cross section for $p\bar{p}\to t\bar{t}h_{SM}$ at $\sqrt{s} = 2$ TeV, when the renormalization and factorization scales are varied between $m_t$ and $2m_t$, while it increases it mildly for scales roughly above $2m_t$. On the other hand, the impact of NLO QCD corrections on the total cross section for $pp\to t\bar{t}h_{SM}$ at $\sqrt{s} = 14$ TeV is slightly positive over a broad range of scales. More importantly, the NLO results show a drastically reduced renormalization and factorization scale dependence as compared to the Born result and leads to increased confidence in predictions based on these results.

IV. PARTON DISTRIBUTION FUNCTIONS (PDFS)

This section was prepared with input from Csaba Balazs, Richard Ball, Edmond Berger, Walter Giele, Nikolaos Kidonakis, Pavel Nadolsky, Carlo Oleari, Fred Olness, Wu-Ki Tung and Zack Sullivan.

Reliable knowledge of parton distribution functions (PDFs) is required for detailed QCD studies and crucial for many searches for new physics signals in the next generation of experiments. There has been significant recent progress in quantifying the uncertainties of the PDFs. The task is formidable and requires continuing study. We briefly discuss these issues, and examine prospects for improvement on both the theoretical and experimental fronts \cite{70}.

A. PDF Uncertainties: the Challenge

Quantitative estimates of PDF uncertainties are of great importance for current and future hadron collider experiments. These uncertainties affect determinations of important quantities such as the $W$-boson mass, and they have an impact on the new physics potential of current and proposed hadron colliders. Compositeness searches in the Run Ia data on the one-jet inclusive transverse energy distribution \cite{71} are a case in point.
Prior to this measurement, a series of global fits to PDFs, combining data from a variety of experiments with next-to-leading order (NLO) perturbative QCD, had worked well, and were a showcase for the success of the QCD framework. Confronted with the relatively precise CDF one-jet inclusive measurement, they produced an apparent signal of quark compositeness. However, these PDFs could be adjusted to accommodate the high transverse energy excess [72], while remaining consistent with previous data. To many, this demonstrated dramatically the need to quantify uncertainties in PDF determination at this level of measurement accuracy.

Quantifying PDF uncertainties is difficult for many reasons: 1) From the practical side there are problems in combining many different experiments in a consistent single global fit. 2) From the more theoretical side, one has to address many issues to ensure correct error estimates, such as the choice of PDFs, validity of Gaussian approximations, likelihood estimators, and other, equally technical considerations.

In the last few years, much progress has been made to address these issues and obtain reliable PDF sets that include uncertainties. For example, if a Gaussian probability distribution is assumed for the PDFs, systematic and statistical errors can be computed following general statistical procedures based on the Bayesian treatment of uncertainties. In this approach, a covariance matrix of errors can be built by computing a second order Taylor expansion in PDF parameter-space around the maximum likelihood solution. This method is guaranteed to work (in the sense of a local minimum) if there are no directions in the parameter space where the second derivative, computed with respect to the maximum likelihood solution, is zero. In order to avoid this possibility, one has to find and fix the parameterization of the PDFs, both to fit the data and to avoid zero modes. This method was first used in Ref. [73] to fit H1, ZEUS, BCDMS and NMC data. Other experiments were included in the fit in Refs. [74, 75], and in Refs. [76, 77] a procedure was proposed to maintain the “global fitting” philosophy, including as many experiments as possible.

An alternative method developed in Ref. [76] uses Lagrange multipliers to estimate PDF uncertainties for a specific observable. Changing the observable necessitates re-fitting the PDFs. However, using this method one does not need to assume a Gaussian distribution for the PDFs.

A more general method that does not require a Gaussian approximation is the Monte Carlo approach in which the space of PDF functionals is sampled and probability weights are assigned to each PDF set [78, 79]. While the required computing resources are larger, one can in principle incorporate any error analysis. Moreover, the method can accommodate complicated topological regions of PDF parameter space that have a constant probability measure. This approach opens the way to describe the PDF functionals with a complete set of functions, with as many parameters as are needed numerically, and it removes the issue of parameterization dependence. This method maximizes the PDF uncertainty and enhances the ability to consistently accommodate a large number of functional forms.

This strategy can be used to directly determine the luminosity at hadron colliders from W-boson and Z-boson counts, as in Fig. 2. In Ref. [79], the luminosity was derived from the D0 Run I vector-boson events, and the result was compared with the traditional method. Additionally, the corresponding implications for the top quark cross section uncertainties were investigated using this extracted luminosity. This study underscores the importance of this source of uncertainties, the understanding of which is crucial for a reliable luminosity determination and accurate theoretical predictions.

B. Projects at Snowmass

Having posed the problem of extracting PDFs and their uncertainties, we briefly mention some the ongoing work that was performed during the Snowmass workshop. Details can be found in the corresponding individual contributions contained in these proceedings.

PDF Standard User Interface

In a continuation of the work initiated at the Les Houches Workshop [80], a standard user interface was presented and discussed. The purpose of this user interface is to provide a simple function call which gives access to all PDFs with or without uncertainties. The standard proposed has a number of improvements over previous standards such as the CERN PDFLIB. Some features of the Les Houches standard are: 1) The ability to easily access complete sets of PDFs, to facilitate calculations of the PDF uncertainties in various observables. 2) The definition of PDFs through external files. This makes the actual interface code independent of the specific PDF set used, and makes the use of different sets within one analysis trivial.

PDF Uncertainties in W-Higgs Production

In Ref. [81], the authors estimate the uncertainty in associated W-Higgs boson production at Run II of the Tevatron due to imprecise knowledge of PDFs. They use a method proposed by the CTEQ Collaboration [77] to estimate the uncertainties using a set of orthogonal PDF parameters, obtained by the diagonalization of the
matrix of second derivatives of \( \chi^2 \) (Hessian matrix) near the minimum of \( \chi^2 \). The PDF uncertainty for the signal and background rates was found to be of the order of 3\%. The uncertainties on important statistical quantities (significance of Higgs boson discovery, accuracy of the measurement of the Higgs boson cross section) are significantly smaller (\( \sim 1.5\% \)) due to the strong correlation of the signal and background. To summarize, the PDF error for the W-Higgs production at Tevatron is well under control.

**Heavy-quark parton distribution functions and their uncertainties**

The uncertainties of the heavy-quark (c, b) PDFs in the zero-mass variable flavor number scheme were investigated in Ref. [82]. Because the charm- and bottom-quark PDFs are constructed predominantly from the gluon PDF, their uncertainties are commonly assumed to be the same as the gluon uncertainty. While this approximation is a reasonable first guess, it was found to work better for bottom quarks than for charm quarks. The heavy-quark uncertainties have a weak logarithmic dependence on \( Q^2 \) and approach the uncertainty of the gluon only for small \( x \). As an application, the PDF uncertainty for \( t \)-channel single-top production is calculated, and a cross section of \( 2.12 \pm 0.32 - 0.29 \text{ pb} \) at Run II of the Tevatron is predicted.

**Differential distributions for NLO neutrino-production of charm**

In Ref. [83], the charged current DIS charm production cross section is computed at NLO in QCD. While the inclusive calculation for this process has been published previously, the fully differential distribution is necessary to properly model the experimental detector acceptance. This full distribution, in turn, is crucial for correctly
extracting the strange-quark PDF from the data. This work is a collaborative effort between theorists and experimentalists from the NuTeV collaboration, and this method will be implemented in the analysis of the NuTeV di-muon data set.

C. Summary

In recent years, new information has become available concerning parton distributions and their uncertainties. The issues have become more important with the realization that these uncertainties could be hampering searches for physics beyond the Standard Model. To achieve the goal of quantitative PDF uncertainties will require a comprehensive and collaborative investigation involving both theorists and experimentalists. If we are to make the best use of data and discoveries from hadron experiments, the area of PDF uncertainties must receive high priority.

V. JET PHYSICS

This section was written by Steve Ellis.

As already noted, one of the most instructive arenas for the exploration of QCD has been the study of jets in hadronic final states. The underlying picture is that a parton, which is isolated in momentum space due to some short-distance hard scattering process, evolves via higher order corrections, showering and hadronization to its long-distance asymptotic state consisting of a "spray" of jets of hadrons, which is approximately aligned with the direction of the initial parton. By assigning these associated hadrons to a single jet, i.e., by summing over their kinematic properties, one constructs a measure of the hadronic final state that is infrared finite and that tracks the kinematic properties of the underlying short-distance partons. Thus jet cross sections in $e^+e^-$ collisions can be directly addressed in QCD perturbation theory. In the case of lepton-hadron or hadron-hadron collisions, jet ideas can be used to control potential infrared singularities arising due to the final state interactions, while factorization techniques are employed to control the singularities in the initial state. Thus, much like the various shape parameters employed so successfully to characterize $e^+ e^-$ final states, e.g., Thrust, Energy-Energy Correlations, etc., jet cross sections were first used as tests of our understanding of QCD. As the physics focus has shifted, as described elsewhere in this Summary, to the question of precision strong interactions and new physics searches, the role of jet physics has also shifted. The study of jets is not only central to precision QCD measurements, but also essential as a tool for mapping the observed long-distance hadronic final states onto the underlying short-distance partonic states. While we get to measure only the former, it is the latter that we can must recognize the W and Z bosons and heavy quarks, which we expect to arise from Higgs and Supersymmetric physics. A central element of jet physics is the "jet algorithm", which is the set of rules that specify how to identify and combine the hadrons (on the experimental side) or the partons (on the theoretical side) into jets. As our control of higher order corrections, resummations and nonperturbative contributions and parton distributions improves, as outlined elsewhere in this Summary, we are faced also with the task of improving the way in which we define jet physics [84].

Much of the discussion of jet physics at this Workshop focused on improving the definitions of jets. Traditionally, two types of jet algorithms have been employed at colliders. Both are based on the assumption that hadrons associated with a jet will in some sense be near each other. At a previous Snowmass Workshop it was argued [85] that the correct approach at hadron colliders should define jets based on nearness in angle, or more specifically nearness in rapidity and azimuth. This idea is the basis of the cone jet algorithm, which has been successfully employed to test QCD at the Tevatron [86], matching data to theory at least at the 10% level. The second alternative is based on nearness in momentum space, and is typically called the $k_T$ algorithm. It has been employed particularly at $e^+e^-$ colliders and more recently $e^p$ machines. Suggestions [87] that the $k_T$ algorithm be used also at the Tevatron are also now yielding results [88]. A characteristic feature of both of these algorithms is that they identify and assign final state hadrons to unique jets event-by-event. While this may seem like a perfectly natural procedure, it is not without ambiguity, certainly if our goal is precision at the 1% level. The short-distance partons are, after all, QCD color nonsinglets while the hadrons in the jets are color singlets. Thus the color singlet jets constructed from the observed final states must arise from the correlated evolution of color singlet combinations of partons in the short-distance scattered state, not from single partons. These effects are power-suppressed in $p_T$, but not necessarily negligible for that reason [89]; see section V.C below. Thus it is essential that the jet analysis serve to suppress any sensitivity to this inherent ambiguity, especially as it arises in the stochastic processes of showering and hadronization. Of course, at lepton-hadron or hadron-hadron colliders the color of the hard scattered parton leading to the jet could also be balanced by color charge carried by remnants of the initial hadrons - the spectator partons. For hadron-hadron
colliders these spectators lead to the very important and interesting question of the role of the underlying event, which also factors from the hard-scattering process only to leading power. Since this contribution to the event does not decouple completely from the hard scattering, and also contributes hadrons to the jets by any measure of nearness, it must be understood if we are to reach our goal of precision jet physics. Note that, while a perturbative analysis can say something about the initial state and final state radiation contributions to the underlying event, the contributions of the beam remnants are totally outside of the scope of QCD perturbation theory at leading power.

A. Underlying Event

This important subject is treated in some detail in a separate contribution to these Proceedings [90]. The underlying event may be defined as everything except the two most energetic jets in the final state, i.e., the underlying event includes both initial and final state radiation and the beam-beam remnants. The new results presented at the Workshop include a detailed comparison of CDF data with results from three Monte Carlo event simulators, ISAJET, HERWIG and PYTHIA. The analysis makes insightful use of the concept of the “transverse” activity in the events, where transverse here means transverse to the direction of the approximately back-to-back energetic jets. If the leading jet defines \( \phi = 0 \), the transverse region corresponds to \( 60^\circ < |\phi| < 120^\circ \) in the same region of rapidity as the jets. On an event-by-event basis, the activity in the 2 transverse regions (\( \phi > 0 \) and \( \phi < 0 \)) is ordered as the “transMAX” and “transMIN” regions depending on the level of activity, which are studied separately and as the sum and difference. The transMAX region preferentially correlates with the “hard” part of the underlying event (initial and final state radiation plus extra jets), while the transMIN region is correlated with the beam remnants, although the separation is, of course, not precise. The ISAJET results, which assume independent fragmentation, exhibit too many soft particles in the underlying events and the wrong correlation with the \( P_T \) of the leading jet. In contrast, both HERWIG and PYTHIA include some level of color coherence effects (angular ordering) in the parton showering. However, HERWIG, like ISAJET, exhibits a \( P_T \) spectrum for the beam remnant particles that is too steep and thus does not generate enough remnant particles above \( P_T = 0.5 \) GeV. PYTHIA, which includes the effects of multiple parton scattering, seems to come closest to agreement with the data, suggesting that such extra underlying scattering processes are an important part of the correlation with the hard scattering. That is, it appears that hard scattering events are more central collisions, and hence more likely to yield secondary parton scattering processes. This conclusion, however, implies sensitivity to multiparton correlations. Fully understanding the underlying event is clearly a demanding challenge.

B. \( k_T \) Algorithm

Discussions of the hadron collider application of the \( k_t \) algorithm included a presentation of the recent results from D0 [88]. While these first results indicate reasonable agreement with NLO theoretical expectations, the experimental results lie systematically above the theory, especially for jet \( E_T < 150 \) GeV. At least naively, this can be ascribed to the expectation that the \( k_T \) algorithm will tend to “vacuum-up” unassociated but nearby energy, which is not present in the perturbative analysis. With our goal set at 1% jet physics, it will be important to test whether the \( k_T \) algorithm can reach this precision.

C. Cone Algorithm

Detailed results concerning the efficacy of the cone algorithm were reported at the Workshop, and are presented in a separate contribution to these Proceedings [93]. In the last year several previously unappreciated features of the cone algorithm have come to light. Recall that the cone algorithm identifies discrete jets event-by-event by requiring that “found” jets correspond to “stable” cones. The algorithm requires that the set of hadrons (or calorimeter towers) that make up a jet not only lie within radius \( R \) (in \( \eta, \phi \) space) of the center of the cone \((\eta, \phi)\) but also that the \( E_T \) weighted centroid of the set coincide with the center of the cone. In equations this looks like

\[
k \in J : (\phi_k - \phi_J)^2 + (\eta_k - \eta_J)^2 \leq R^2,
\]

\[
\eta_J = \sum_{k \in J} \frac{E_{\tau, k + \eta_J}}{E_{\tau, k}}, \quad \phi_J = \sum_{k \in J} \frac{E_{\tau, k + \phi_J}}{E_{\tau, k}},
\]

\[
E_{\tau, J} = \sum_{k \in J} E_{\tau, k}.
\]

(6)
It is this latter “stability” constraint that results in the new issues. When smearing is included, i.e., when the energy of an underlying short-distance parton is spread out in \( \eta, \phi \), as presumably necessarily happens due to showering and hadronization, stable solutions present at the perturbative level disappear in the smeared final state. This occurs both in a simple theoretical calculation of the smearing effect and in studies of more realistic simulated final states. In the latter studies of Monte Carlo events, sizeable amounts of nearby energy in the LEDO plot are simply not identified with what is obviously an associated (i.e., nearby) jet. While the overall impact on jet rates is at only the 5 to 10% level, this effect is clearly relevant to 1% jet physics. The study also confirms that a simple “fix” using a smaller cone size during the jet discovery phase, but not during the jet construction phase, removes much of the problem when applied to the Midpoint Algorithm recommended by the Run II Workshop [84]. On the other hand this fix is unlikely to help the Seedless Algorithm [84] address this issue. An interesting historical note is that the traditional CDF Cone algorithm, JetChu [92], has a (largely undocumented) feature that seems to help with the difficulty discussed here. JetChu (like the D\( \Omega \)) cone algorithm only looks for stable cones in the vicinity of energetic calorimeter towers (or clusters of such towers), the seed towers. Any towers inside an initial seeded cone remain associated with that cone even as it migrates in \( \eta, \phi \) towards a stable location. (The cone algorithm is applied in an iterative process using the \( E_T \) weighted centroid for a given cone center to define the next cone center until the two coincide.) The sticking of the towers to the initial cone is labeled as ratcheting. Ratcheting is not simulated in any way in the perturbative analysis.

\section{Jet Energy Flow}

In an effort to study a related but quite different approach to jet physics, the Jet Working Group actively considered the jet energy flow (JEF) approach and has made a separate contribution to these Proceedings [93]. The JEF approach is an effort to explore, apply and extend a formalism introduced and extensively developed by Tkachov, in terms of observables based on calorimetric measurements, which he termed “C-continuous” [94]. In Ref. [94], the algorithmic counting of jets of definite energies and directions is replaced by a class of continuous measurements of the flow of energy [95].

As suggested above, a characteristic (and troubling) feature of traditional jet algorithms is the event-by-event identification of individual hadrons with discrete jets, and thus with a single (or color nonsinglet pair of) underlying short-distance parton(s) in the perturbative calculation. We know this connection cannot be made precise in detail, and we expect this mismatch to enhance the sensitivity to the effects of showering and hadronization (where the extra color correlations must appear). The issues raised above concerning the cone algorithm are presumably of just this nature. Thus it is appealing to consider other procedures for mapping the hadronic final state onto the underlying short-distance partonic state. The JEF approach is just such a procedure. It accepts the reality that the hadronic final state represents the collective radiation from several out-flowing color charges, i.e., the underlying short-distance partons. No attempt is made to associate individual hadrons with unique jets, i.e., with unique underlying partons, on an event-by-event basis. Yet the energy flow pattern of an event still provides a footprint of the underlying partons, from which much of the same information provided by the standard jet algorithms can be extracted. The basic idea is to generate a jet energy flow distribution to describe each event instead of a discrete jet of jets. This is accomplished by convoluting the observed energy distribution with a simple smearing function as suggested in the figure, where the smearing function in this case is just a constant inside the circle (cone) and zero outside. In equations, we start with the underlying 4-vector distribution for the event (the color coded pixels in the figure) defined by

\[ P_{\mu} (\vec{p}) = \sum_{i=1}^{N} p_{\mu}^i \delta (\vec{p} - \vec{p}_i), \]

where the directional unit vector is defined by \( \vec{p} (m) = \vec{p} / |\vec{p}| \) with the 2-dimensional angular variable defined as \( m = (\eta, \phi) \) (typical of hadron collisions, where \( \eta \) is the pseudorapidity, \( \eta = \ln (\cot \theta / 2) \)) or \( m = (\theta, \phi) \) (typical of lepton colliders). Eq. (7) is the building block from which all weights and correlators of energy flow may be constructed [94]. For example, it defines the underlying energy flow via \( E_T (m) = P_\parallel (m) \) or (for the case of hadronic collisions) the more familiar underlying transverse energy flow is defined by the composite quantity \( E_T (m) = \sqrt{P_\parallel^2 (m) + P_\perp^2 (m)} \). Clearly many quantities can be constructed from the 4-momentum distribution of Eq. 7, including the usual cone jet algorithm. We define the jet energy flow (JEF) via a smearing or averaging function \( A \) as

\[ J_\mu (m) \equiv \int dm' P_\mu (m') \times A (m' - m), \]
where $A$ is normalized as

$$\int dm \, A(m) = 1. \quad (9)$$

A simple (but not unique) form for the averaging function in terms of the general 2-tuple of angular variables $m = (\alpha, \beta)$, which provides a direct comparison with the jet cone algorithm, is

$$A(m) = A(\alpha, \beta) = \frac{\Theta (R - r(\alpha, \beta))}{\pi R^2} = \frac{\Theta \left( R - \sqrt{\alpha^2 + \beta^2} \right)}{\pi R^2}. \quad (10)$$

where $R$ is the cone size and $r(\alpha, \beta)$ is the distance measure in the space defined by $(\alpha, \beta)$. The specific example of the jet transverse energy flow (transverse JEF)

$$J_T(m) = \int dm' \, E_T(m') \times A(m' - m) = \int dm' \, \sqrt{P_T^2(m') + P_\phi^2(m')} \times A(m' - m) \quad (11)$$

for the case of $R = 0.7$ is suggested in the figure. Note that the transverse JEF is smeared on a scale $R$ compared to the underlying $E_T$ distribution. For comparison, the Snowmass cone jet algorithm [85, 86] identifies jets at a discrete set of values of locations $m_j$ defined by the $E_T$ weighted cone “stability” constraint. These stable cone locations $m_j$ are the solutions of the equation

$$\int dm' \, E_T(m') \times (m' - m_j) \times A(m' - m_j) = 0. \quad (12)$$

The corresponding cone jet $E_T$ values are found by evaluating Eq. (11) (times $\pi R^2$) at the jet positions, $E_{T,j} = \pi R^2 \times J_T(m_j)$. 

FIG. 3: Plot of energy flow in $\eta, \phi$ indicating the region averaged over to determine the JEF value at the center of the circle. In this figure, red (dark) pixels indicate high energy. The event was created with PYTHIA.
A primary strength of the JEF approach is that, in contrast with the usual algorithmic approach to jet identification, the JEF formalism generates a smooth distribution, \( J_P(m) \), event-by-event, and, in that sense, the JEF formalism is more analytic. For example, in the application of the cone algorithm the goal of identifying unique jets leads to the "stability" constraint as noted above. The non-analytic character of the jet cone algorithm arises from the need to solve Eq. (12) and then work with only the discrete set of solutions, i.e., the jets in an event. Only limited (and typically complicated) regions of the multiparticle phase space contribute to a jet. No such constraint arises in the JEF analysis. This distinction has several important consequences, including the expectation that the more inclusive and analytic calculations characteristic of a JEF analysis are more amenable to resummation techniques and power corrections analysis \cite{96} in perturbative calculations; that, since the required multiparticle phase integrations are largely unconstrained, i.e., more analytic, they are easier (and faster) to implement; that since the analysis does not identify jets event-by-event, the analysis of the experimental data from an individual event should proceed more quickly; and that the signal to background optimization can now include the JEF parameters (and distributions). As emphasized in \cite{94}, the representation of the final state provided by the JEF, or more generally in terms of \( C \)-continuous observables, is expected to be a more reliable footprint in the sense of exhibiting a reduced sensitivity to the showering and hadronization processes. The challenge in the JEF type analysis is to define quantifiable observables. Several simple examples are provided in the separate contribution to these Proceedings.

E. Conclusions

To make the best possible application of the increased precision of our understanding of QCD outlined elsewhere in this Summary, it is essential the our understanding of jets and how to identify them also improve. Several relevant issues were addressed at the Workshop and are discussed briefly in this summary and elsewhere in the Proceedings. The overall conclusion is that 1% jet physics is an important and possible goal but there is much work still to be done.

VI. RESUMPTION

This section was prepared with the participation of Csaba Balazs, Anna Kulesza, Nikolaos Kidonakis, Pavel Nadolsky and George Sterman.

A. Threshold Resumtion

The goal of threshold resummation is to resum potentially large logarithms that appear in the cross section for production of a given high-mass system, \( F \), near threshold, to all orders in the strong coupling \( \alpha_s \). Choices for \( F \) include Higgs and weak vector bosons, heavy quarks and high-\( p_T \) jet pairs, with fixed or integrated rapidities. The logarithms in question arise from the incomplete cancellation of infrared divergences between virtual and real graphs near threshold in the partonic system \( a + b \to F + X \), where there is restricted phase space for real photon emission.

Threshold resummation is derived from the factorization properties of the hadronic cross section \cite{54, 55, 56, 57, 58, 50, 60}. For example, the heavy quark cross section can be written as

\[
\sigma_{h_a h_b \to Q X} = \int d x_a d x_b f_{q_a / h_a} (x_a, \mu^2_F) f_{q_b / h_b} (x_b, \mu^2_F) \sigma_{q_a q_b \to Q X} (s_q, t_i, m^2_q, \mu^2_F, \alpha_s (\mu^2_R)),
\]

(13)

For the production of heavy quark \( Q \), we define \( t_i = (x_i p_i - p_Q)^2 - m^2_Q \), with \( i = a, b \). As usual, the non-perturbative parton distributions \( f_{q / h} \), describe the long-distance physics that is factorized from the hard scattering, and \( \sigma_{q_a q_b \to Q X} \) is the perturbative partonic cross section, which still displays sensitivity to gluon dynamics that is perturbative, but soft compared to \( m_Q \). This sensitivity manifests itself in plus distributions of the type \( \left[ \ln^{2n} (s_q / m^2_q) / s_q \right]_+ \), \( m \leq n - 1 \) at \( n \)th order in \( \alpha_s \). The variable \( s_q \equiv s + t_a + t_b \), and \( s_q \) vanishes at partonic threshold.

A further re-factorization separates the non-collinear soft gluons, described by a soft gluon function \( S \), which is a matrix in color space, from the hard scattering \( H \). From the renormalization properties of these functions one may obtain evolution equations that lead to the resummed cross section \cite{55, 57, 58, 59, 60}. Generally, the resummed cross sections are specified in a Mellin or Laplace moment space, with the latter defined by \( \sigma (N) = \int (d s_q / s) e^{-N s_q / \mu^2} \sigma (s_q), \) with \( N \) the moment variable. Note that under moments, \( \left[ \ln^{2n-1} (s_q / m^2_q) / s_q \right]_+ \to \ln^{2n} N \), and our aim becomes to resum logarithms of \( N \).
The resummed cross section at next-to-leading logarithmic (NLL) and higher accuracy can be written as [63]:
\[
\sigma_{ab \to QX(N)} = \exp \left\{ \frac{\mathcal{L}^2}{N^2} \frac{m^2}{\mu^2} \left[ A_2(\alpha_s(m)) + A_b(\alpha_s(m)) \right] \ln \left( \frac{N m}{Q} \right) \right\} \\
\times \exp \left\{ H \left[ \int_0^{m/N} \frac{d\mu}{\mu} \Gamma_s \left( \frac{m^2}{N^2} \right) \exp \left[ \int_0^{m/N} \frac{d\mu}{\mu} \Gamma_s \left( \frac{m^2}{N^2} \right) \right] \right] \right\},
\]
where the first factor gives the double-logarithmic $N$-dependence from the incoming partons, in terms of anomalous dimensions $A_2(\alpha_s) = (\alpha_s/\pi) C_2 + \ldots$ with $C_2 = C_F$ and $C_2 = C_A$, while $\Gamma_s$ is an anomalous dimension matrix of soft gluons, which controls single logarithms associated with coherent jet radiation [67, 71, 85].

Threshold resummation was first derived for the Drell-Yan cross section [97, 98] and has been applied to a large number of processes in the last five years. Resummed cross sections and their finite-order (NNLO and higher) expansions have now been derived for heavy quark hadroproduction [57, 61, 62, 99, 100, 101] and electroproduction [102, 103], direct photon production [60, 104, 105, 106], $W$ + jet production [107], and dijet [58, 59] and single-jet [108] production. For a review see Ref. [109]. It has been found that the threshold corrections are more or less sizable, depending on the specific process and kinematics, and that the factorization scale dependence of the cross sections for these processes is significantly reduced relative to NLO. However, ambiguities remain in the cross section, due to contributions from subleading logarithms [62, 99] and due to uncertainties from the kinematics (single-particle-inclusive versus pair-inclusive) [61].

**B. $k_T$ Resummation**

The soft-gluon resummation formalism for the inclusive production of colorless massive states [110] may be the most mature, best understood, and best tested formulation among the resummation techniques. Its phenomenological advantage over threshold resummation is that the effects of resummation are readily observable in the distribution of the transverse momenta. Indeed, it is not possible, as in the case of threshold resummation, to define a finite differential cross section without resummation.

Recently both theoretical refinement and phenomenological applications of the original formalism have received considerable attention. On the theory side, the improvement of its non-perturbative sector [111], the introduction of resummation schemes [112], and the development of combined threshold and transverse momentum ($Q_T$) resummation [113] are important milestones. The formalism is successfully applied to $W^\pm$, $Z^0$ boson, diphoton [114], heavy quark pair [115], and direct photon production [116] at the Tevatron, to DIS [117], and to Higgs production at the LHC [118]. The latter application plays a critical role in the search for a light Higgs boson.

In recent papers, the results of the full NLL calculation of the Higgs $Q_T$ distribution were presented [119]. In recent papers [120], Catani, de Florian and Grazzini observed that NNLO soft terms in resummed small-$p_T$ cross sections can be obtained by combining coefficients of lower orders and two-loop splitting functions. Their result substantially simplifies calculation of higher-order terms for resummed cross sections in a variety of processes, including Higgs boson production at the LHC. When included in the Higgs boson cross section, the soft NNLO terms significantly alter the shape of the small-$p_T$ distribution [121], which suggests the importance of even higher-order corrections (and of more work to achieve adequate understanding of this observable).

In the formalism of Ref. [110], resummed cross sections receive relatively unknown non-perturbative “power” contributions in the impact parameter $b$ from the region of large $b$. Qin and Zhang [111, 122] suggested that the need for such contributions can be reduced by extrapolation of leading logarithmic terms from the perturbative region, combined with the lowest-order power corrections. They found good agreement of their approach with data on vector boson production. As an alternative to the numerical Fourier transform from the impact parameter space, one may try to estimate sums of leading logarithmic series directly in transverse momentum space. Kulesza and Stirling [123] proposed a method for the systematic summation of leading logarithmic towers in $p_T$-space while preserving some advantages (e.g., transverse momentum conservation) of the $b$-space formalism. They find that the Tevatron $W$ and $Z$ data can be described reasonably well by the lowest four towers of logarithms, except at $p_T \lesssim 2-3$ GeV, where non-perturbative effects become important. The method of joint resummation [113] provides some of the advantages of both these approaches, with a cross section that can be defined consistently in perturbation theory, and a consistent picture of nonperturbative corrections for both $k_T$ and threshold resummations.
C. Resummation for Final States: Power Corrections

Jet properties and event shapes are another successful application for resummation techniques. Technically similar to threshold resummation, they share with $k_T$ resummation a direct relation to observables.

Consider the resummed cross section for $e^+ e^- \rightarrow Z^*(Q) \rightarrow F$, in the elastic limit, where the final state $F$ consists of two jets with invariant masses much less than $Q$. In terms of the familiar “thrust” variable, we take the limit $1 - T \sim (m_1^2 + m_2^2)/Q^2 \to 0$. In this case, the light-like relative velocities between the jets imply factorization, evolution and, eventually a cross section resummed for the singularities in $t \equiv 1 - T$,

$$\frac{1}{\sigma_{tot}} \frac{d\sigma_{\text{PT}}(1 - T)}{dT} \sim \frac{d}{dT} \exp \left[- \int_t^1 \frac{\alpha_s(Q)}{y} \int_{Q^2}^{y Q^2} \frac{m^2}{\alpha_s(m)} A(\alpha_s(m)) \right],$$

shown here to leading logarithm. The function $A$ is exactly the same function of the coupling encountered in threshold resummation.

In the resummed cross section of (15), we encounter the “Landau” pole of the perturbative running coupling in the limit $y \to 0$. In the analysis of power corrections, we make a virtue of the necessity of dealing with the Landau pole. Roughly speaking, we discover that, unaided, perturbation theory is not self-consistent, and we search for a minimal class of nonperturbative corrections to restore self-consistency [124]. This approach has had considerable phenomenological successes, and is beginning to shed light on the relationship between perturbative and nonperturbative dynamics in QCD.

More insight can be gained by expressing the exponent in more detail. The following form is equivalent to (15) at leading logarithm, but actually incorporates all logarithmic behavior in the eikonal approximation,

$$\int_0^1 dy \frac{\alpha_s(y)}{y} \int_{Q^2}^{y Q^2} \frac{m^2}{\alpha_s(m)} A_0(\alpha_s(m)) \sim \frac{2}{T Q} \int_0^{\beta_0} dm \, A_0(\alpha_s(m)) + \ldots,$$

where on the right we have exhibited the leading behavior in $Q$ of the integral for low values of the momentum scale $m$, with $\beta_0$ a fixed cutoff $> A_0 Q$. In these terms, it is natural to think of the $1/Q$ term as the dominant power correction implied by the ambiguity of the perturbative series. In addition, because the function $A$ appears often (recall threshold resummation), it is tempting to think of the correction as in some sense universal.

These ideas have been tested most extensively in electron-positron annihilation, making use of the energy lever arm of the LEPI and II programs. Resummations similar to that for thrust can be carried out for a variety of event shape functions measured at LEPI and elsewhere, which measure various kinematic properties of jets, and which are labelled by a small alphabet soup of letters, which we denote by $\epsilon \equiv 1 - T, B_T, C, \ldots$. All of the $\epsilon$'s are defined to vanish in the elastic limit.

As a minimal approach, we assume that we only have to worry about the first term in the expansion (16). In general, the overall perturbative coefficients of the integral over the running coupling differ from event shape to event shape. Taking this into account, we define [125],

$$\lambda_\epsilon \sim \frac{d(\epsilon)}{d\epsilon} \cdot \int_0^{\beta_0} dk \, \alpha_s(k),$$

where, adopting one popular notation, we replace the function $A(\alpha_s)$ by the combination $M_s \alpha_s$. The constant $M_s$ incorporates certain higher-order effects that depend, in part, on the event shape in question. In this formalism we identify a “universal” nonperturbative parameter as the integral of the running coupling from zero to some fixed scale $\mu_0$, which again has the interpretation of a factorization scale.

Adding such a term to a suitably-defined perturbative exponent produces a simple shift of the cross section,

$$\frac{d\sigma(\epsilon)}{d\epsilon} = \frac{d\sigma_{\text{PT}}(\epsilon - \lambda_\epsilon/Q)}{d\epsilon} + \mathcal{O} \left( \frac{1}{k^2 Q^2} \right).$$

This result gives a number of consequences, for instance, that the true first moments of event shape $\epsilon$ are related to the perturbative predictions for the first moments by $\langle \epsilon \rangle = \langle \epsilon \rangle_{\text{PT}} + \lambda_\epsilon/Q$, with similar predictions for second moments.

The phenomenology of various event shapes [126] shows quite good agreement with this picture for the first moments, as shown in Fig. 4. At the same time, there are unexpectedly large $1/Q^2$ corrections in second moments, such as $\langle (1 - T)^2 \rangle$. First moments, however, are relatively blunt instruments to study the perturbative-nonperturbative interface, and there are interesting alternative explanations of the data [127].
A natural extension of the shift (18), to take into account soft radiation from dijets beyond the pure 1/Q correction, is to generalize the shift to a convolution [96],

$$\frac{d\sigma}{d\epsilon} = \int_0^{1/Q} d\epsilon f_\epsilon(\epsilon) \frac{d\sigma_{\text{PT}}(\epsilon - \epsilon/Q)}{d\epsilon} + O\left(\frac{1}{\epsilon^2}\right),$$

where the nonperturbative “shape function” $f_\epsilon$ is independent of $Q$. It is therefore sufficient to fit $f$ at $Q = 1/Q$ to derive predictions for all $Q$. A justification for (19) may be found in an effective theory for soft gluon radiation by 3 $\otimes$ 3$^*$ sources, which represent a quark pair, and are given by products of ordered exponentials in the directions of the jets’ momenta. The phenomenology of shape functions has been discussed in [128] and in [129], while in [130], it was shown that the double distribution determined phenomenologically in [129], follows from rather general considerations. In considerations such as these, resummation opens the path from perturbative to nonperturbative QCD.

The extension of these methods to hadronic scattering has begun relatively recently [131, 132]. This effort should enable us to use energy flow analysis in jet and other hard-scattering events to make hadronic final states more accessible to both perturbative and nonperturbative QCD. In studying differential hadronic final states, we may also hope to promote techniques introduced for threshold resummation to a greater phenomenological relevance. To make use of these ideas will require an approach to hard-scattering events closer to the philosophy of “jet energy flow”, described in Section V above. The textures of events contain a wealth of information that may be lost if they are analyzed for single purposes and then forever discarded. This suggests the daunting challenge of somehow “archiving” data for a future in which new ideas will arise, and be answered by asking new questions.

VII. LATTICE QCD

This Section was written by Paul Mackenzie, who thanks Peter Lepage and Andreas Kronfeld for their input.

QCD is the theory of all of strong interaction physics, but the traditional perturbative methods of field theory are only suitable for investigating strong interactions at short distances, where asymptotic freedom
causes the effective coupling constant to be small. Lattice gauge theory provides a way of understanding the effects of strong interactions at all distance scales, including the long distance effects that are the hallmark of strong interactions and for which traditional methods fail. Accurate lattice calculations are necessary for showing that we understand QCD at long as well as at short distances. Lattice calculations are required for extracting non-QCD standard model effects such as Cabibbo-Kobayashi-Maskawa (CKM) matrix elements from experiment. They also provide prototypes for investigations of nonperturbative effects in strongly coupled beyond-the-standard-model theories, which have been little studied so far because of the incompleteness of field theoretic methods.

Lattice QCD has gone through a revolution in the last five to ten years. Ten years ago, there were very few phenomenological lattice results that were solid enough to be of much interest to particle physicists outside the lattice community. Today, there are many such results in the quenched approximation (omitting the effects of light quark loops), and decent unquenched calculations are becoming more and more common. Solid results are appearing for some of the nonperturbative quantities of greatest importance to standard model phenomenology, such as the amplitudes for determining CKM matrix elements. In comparison with perturbative QCD calculations, solid lattice calculations were slow to develop. After the invention of QCD, the methods of perturbation theory were applied to short distance strong interactions physics and produced quantitative results immediately, in the parton distribution functions of deep inelastic scattering for example. The application of QCD to nonperturbative long distance phenomena, by contrast, required the development of new methods and took much longer. Wilson proposed formulating QCD on a lattice soon after the discovery of QCD in 1973, with the aim of making a wider array of calculational methods applicable. Almost ten years passed before the establishment of the current calculational paradigm of large-scale Monte Carlo simulations of the QCD path integral, and more than another ten years after that before reasonably accurate phenomenological results became common.

A. The Path to High Precision

The accuracies so far achieved for important phenomenological calculations are far below what will be required by experiment. For example, B factories and collider experiments will determine the experimental amplitudes for $B\bar{B}$ oscillations in $B_d$ and $B_s$ mesons to an accuracy of 1% or better. Determining the controlling CKM matrix element requires lattice calculations of the associated hadronic amplitudes to the same accuracy, far beyond current lattice standards of perhaps 10%. For all the progress that has been made in making lattice calculations more precise, much more is required.

The ever-increasing power of computers, from around one floating point operation/second/dollar (flops/$\$) on the Vax 11/780s on which the first Monte Carlo spectrum calculations were done, to around 1,000,000 flops/$\$ on one of today's Pentium 4 servers, has been an essential element of the improving accuracy of lattice calculations. However, the accuracy $\epsilon$ improves slowly with CPU power, typically as $\epsilon \sim \text{CPU power}^{-1/p}$ where $p$ is somewhere between six and nine. Investigation of algorithms that minimize this scaling power is an important and ongoing avenue of lattice research [133]. Even an order of magnitude increase in computing power improves accuracy by only tens of per cent, so much more than brute force computing improvement alone is necessary to bring us to the required accuracy. Increasingly accurate approximations to the normalization factors $Z(\mu, \sigma_{\Lambda}(\mu))$ and the higher dimension correction operators $K_{\Lambda}(\mu, a, \sigma_{\Lambda}(\mu))$ in Eqn. (4) have also been required for higher precision lattice calculations, and continue to be required for further progress.

There are three main families of lattice actions in current use for light fermions. Wilson fermions can be formulated for any number of quark flavors, at the expense of adding an $O(a)$ discretization error to the quark action. A nonperturbative renormalization program for Wilson fermions has been carried out through $O(a)$ corrections [134]. Staggered and naive fermions have only even dimensional correction operators and are relatively quick to compute in unquenched simulations, but are naturally formulated with four or sixteen degenerate flavors in the fermion determinant [135]. Calculations with physical numbers of flavors must be done by taking a root of the determinant, eliminating the possibility of formulating the theory in terms of a local action and thus requiring more careful attention to the locality of the resulting theory. The use of domain wall/overlap fermions has made it possible to formulate lattice theories with exact chiral symmetry, and without fermion doubling, which was the most significant technical challenge in lattice QCD left unsolved by Wilson [136]. This allows a clean definition of the bare quark mass, avoidance of chirality forbidden mixings in lattice operators, and other advances. They have only even dimension correction operators, but are more expensive to simulate since they introduce and extra space-time dimension or a large number of additional fermion fields. All three families of actions are currently under active investigation.

As with all applications of QCD, high precision lattice calculations will require next-to-next leading order perturbation theory (or its nonperturbative lattice equivalent) for the normalizations $Z(\mu, a, \sigma_{\Lambda}(\mu))$ in Eqn. (4).
Although lattice perturbation theory expressed in terms of the bare coupling constant often converges poorly, when formulated properly it converges at roughly the same rate as dimensionally regulated perturbation theory [137]. Automated perturbation theory techniques make possible the efficient handling of the complicated actions of lattice gauge theory [138]. Furthermore, more short distance tools are available on the lattice than in dimensional regularization. Nonperturbative short distance calculations may be used to test and extend conventional lattice perturbation theory calculations. Short distance lattice calculations may be formulated entirely nonperturbatively, so that only the dimensionally regularized part of a lattice-MS matching is done perturbatively [134].

B. Applications of Lattice Gauge Theory

Lattice QCD calculations serve to test lattice methods (and QCD, too, if you prefer), to extract standard model information (such as quark masses and CKM matrix elements) from data, and to serve as prototypes for investigations of strongly coupled beyond-the-standard-model theories. For recent reviews of applications of lattice gauge theory, see [139]. Some of the most interesting and important applications of lattice QCD are also among the most solid. Determinations of the strong coupling constant, the quark masses, and the CKM matrix elements can all be made using lattice calculations having a single, hadronically stable meson on the lattice at a time. For example, most constraints on the $\rho - \eta$ plane will ultimately be governed by such lattice QCD calculations. These can be considered golden calculations of lattice QCD. Stable particles are easier to analyze than unstable particles. Stable mesons are small (thus, having smaller finite volume errors) and light (thus, having smaller statistical errors). The heavy quark masses and strong coupling constant are particularly easy to obtain since they can be obtained from heavy quarkonia, where nonrelativistic methods are also available to help analyze corrections. Good prototype calculations exist for all the quantities in this category, and no roadblocks are known to ever increasing accuracy. Many stable meson masses are known experimentally with high precision that can test lattice mass predictions, as well as provide determinations of quark masses and the strong coupling constant. Lattice calculations can use data from BaBar, Belle, and the Tevatron to provide accurate determinations of CKM parameters. These data do not provide any high precision tests of lattice amplitudes, however. Cleo-c will provide new lattice-based determinations of CKM matrix elements in the charm sector, and will also provide some much more accurate tests of lattice amplitudes, for example using the ratio of the $D$ meson decay constant and the semileptonic amplitude $D \rightarrow \pi \nu$.

All of nonperturbative QCD is ultimately approachable through lattice calculations, and not just these simplest quantities. Recent progress has been made in formulating the lattice amplitudes for kaons decaying into multiple pions, required for lattice calculations of the $CP$ violation parameter $\epsilon'/\epsilon$ [141]. More work is required to formulate the equally important amplitudes when the decaying particle is heavy, such as a $B$ meson. QCD at finite temperature and density is an ongoing area of research [142].

Lattice QCD calculations are exciting not only because of their direct usefulness, but also as prototypes of general nonperturbative field theory calculations. QCD exhibits many of the properties expected in other strongly coupled theories: confinement, chiral symmetry breaking, instantons, etc. Since many solutions of the gauge hierarchy problem, both supersymmetric and nonsupersymmetric, rely on a strongly coupled sector of a new gauge interaction, this is an area of phenomenological as well as theoretical importance. As noted above, the formulation of theories with chiral fermions was an outstanding unresolved issue in lattice field theory. In the last few years, the problem of formulating a vector-like theory with an exact chiral symmetry has been solved. It has also been shown how to formulate $N=1$ supersymmetric Yang-Mills theories (in which the fermions are real) [143]. Enormous progress has been made in the last few years in the development of nonvector-like chiral gauge theories [136, 144]. Active research is underway investigating the remaining technical and algorithmic challenges to performing Monte Carlo simulations for theories with complex actions.

C. Computing Facilities for Lattice QCD

Large scale computing facilities are a crucial component of accurate lattice calculations. The scale of the required facilities, although small on the scale of high energy physics experiments, is larger than usual for theoretical physics research. Multiteraflop projects have been established in Europe and Japan for this purpose. To meet these challenges in the US, the DoE has established a coordinated planning process for US lattice computing needs. It is led by an executive committee consisting of Bob Sugar (chair), Norman Christ, Mike Creutz, Paul Mackenzie, John Negele, Claudio Rebbi, Steve Sharpe, and Chip Watson [145]. The two most important avenues for hardware are being investigated, large clusters of fast, cheap commodity hardware such as Pentium servers, and more tightly integrated purpose built equipment. The DoE's Scientific Discovery through
Advanced Computation (SciDAC) program has funded a software development program and medium-sized prototype hardware, which is now being constructed. Planning is underway for terascale hardware in '03/'04.

D. Summary

Lattice calculations have become an essential quantitative tool for strong interaction physics. For some of the most important lattice quantities in standard model phenomenology, increasingly precise results are immediately foreseeable. These calculations require computational resources that are larger than has been customary for theoretical physics, but are still tiny compared to the the experiments that require them.

VIII. FROM SMALL $x$ TO NUCLEI

This section written by Ina Sarcevic, Jamal Jadidian-Marian and George Sterman.

Unlike electrons or quarks, hadrons have structure, an inside and an outside, stabilized by ongoing interactions. Their factorized, partonic description, however, is in terms of noninteracting states. The interactions that bind the hadron are expressed only in the distributions of these states. A doorway from the partonic to hadronic descriptions is the study of electromagnetic structure functions, and consequently of PDFs, in the small-$x$ limit. Experiments at HERA [146], and also at CERN [147] and the Tevatron [148], have already begun to build a bridge between hard and soft reactions, between the partonic and hadronic pictures of QCD. Much of the intellectual excitement engendered by these developments comes from the realization that they afford a window into a new regime of quantum field theory: high density, nonlinear interactions and yet weak coupling. And beyond that, they link the short distance picture of QCD even further, to its phase structure; from the $Q^2 - x$ plane of momentum transfer and parton fraction [149], to the $T - B$ plane of temperature and baryon number density [150]. A few of the important questions involved in this expansion of interest are given below.

- Saturation: Do parton densities at small $x$ access the high-density/weak-coupling regime?
- The QCD pomeron: What is the true high-energy behavior of hadron-hadron scattering?
- Diffraction & rapidity gaps: How can we understand the persistence of diffractive events in the presence of hard scattering?
- Colored glass condensate: Is it possible to compute bulk properties of the initial state in hadronic (nuclear) collisions?
- Jet quenching: What is the behavior of high-energy partons traversing dense matter?
- Quark-gluon plasma; color superconductivity: What is the phase structure of QCD?

Even to a casual observer, these terms and questions relate to areas in which particle and nuclear physics have influenced one another deeply over the past decade. The first three are perhaps more “particle”, the latter three, more “nuclear”. Given the forum and constraints of space, we will concentrate on the former. Nevertheless, an equally important remaining question is how to foster this interchange between the two fields [151, 152], given intellectual history and the realities of funding.

A. Small $x$ and Saturation

In deep-inelastic scattering, $x = Q^2 / 2 p \cdot q = Q^2 / (s + Q^2)$, with $p$ the momentum of the target hadron, $q^2 = -Q^2$ the momentum transfer through an electroweak current, and $s \equiv (p + q)^2$. The limit $x \to 0$ at fixed momentum transfer implies an unlimited growth of center-of-mass energy in the target-current system [153]. The limit $s \to \infty$ with momentum transfer fixed is the Regge limit, and the behavior of DIS structure functions as $x \to 0$ is a species of total cross section, related by the optical theorem to the forward scattering amplitude of an off-shell electroweak boson with the hadronic target.

In terms of the hadronic tensor written as a Fourier transform, 

$$W^{\mu \nu}(x, Q^2) = \frac{1}{8 \pi} \int d^4 z e^{-i q \cdot z} \langle p | \mathcal{F}^\mu(z) \mathcal{F}^\nu(0) | p \rangle,$$

(20)
$x \ll 1$ translates to $z^2 \sim 1/Q^2$, $p \cdot z \sim 1/x \gg 1$, and thus a correlation between electroweak currents $J$ separated by a distance of order $p \cdot z$ out along the light cone. As $x$ vanishes, this separation exceeds the size of the hadron, and the measurements from which we determine PDFs take place not in the hadron proper, but in a region where the hadron has dissolved into the surrounding vacuum. In this configuration, it may be better to think of the PDF not so much as a measure of pre-existing partons within the hadron, as the probability that a quark pair, a component of the nearby vacuum, polarized by the incident electroweak boson, scatters from the target as a whole. In this picture, at large $s$, the electroweak boson fluctuates into a virtual hadronic state that is typically off-shell by order $Q$, but whose (Lorentz dilated) lifetime is long compared to its transit time across the target.

Rephrased in partonic language, at small $x$ the dynamics that determines the distribution of the quarks to which an electroweak boson couples is to be found, not inside the target, but rather outside in the (target-independent) vacuum. At small impact parameter, the extended, virtual, hadronic states will interact with a nucleon with essentially unit probability, that is, with a cross section determined by the geometric size of the target. This picture is often realized in terms of the perturbative coupling of a photon to a dipole in the form of a quark pair, whose interaction with the target can then be modeled [154]. When translated back to a PDF, this implies saturation, a limit beyond which (in $x$) the distribution cannot grow. On the other hand, at peripheral impact parameters, the scattering can be diffractive, transforming an incoming photon, for example, into recognizable hadronic states, such as vector quarkonia. These can be thought of as messages from the QCD vacuum, carried by electroweak currents.

The qualitative pictures of partons within and without the hadron have quantitative analogs in complementary factorizations and evolutions, commonly referred to as DGLAP and BFKL. Both may be thought of as describing the population of regions of phase space that open up in high energy scattering.

The classic factorization for DIS, in terms of parton momentum fraction, leads, as we have described above, to an evolution in momentum transfer:

$$F(x, Q^2) = \int dz \ C(z, x, \mu) f(z, \mu) = \mu \frac{\partial f(x, \mu)}{\partial \mu} = \int dz \; P(x, z) f(z, \mu). \quad (21)$$

This is DGLAP evolution, and corrections to it are suppressed by powers of $\mu$. The evolution kernels, $P$, describe the formation of virtual states through radiation processes that decrease momentum fraction but increase transverse momentum. A similar relation may be written down for diffractive scattering at large momentum transfer, leading to the concept of diffractive parton distributions [155]; the likelihood of striking a parton with momentum fraction $x$ at a short-distance resolution $1/Q$ and, still scattering the target elastically.

At small $x$, in contrast, the relevant region of phase space is in rapidity ($y \sim \ln(1/x)$). The universality of the vacuum is expressed as a factorization in rapidities, with a reference rapidity $y$, playing the role of the factorization scale, $\mu$, above, and with a convolution in transverse momentum. Once again, the factorization leads to evolution [156]:

$$F(x, Q^2) = \int d^2 k_T d^2 k_T' C(x, k_T - k_T', y) G(k_T', y) = \mu \frac{\partial G(k_T, y)}{\partial y} = \int d^2 k_T K(k_T - k_T') G(k_T', y). \quad (22)$$

This result, BFKL evolution, has, in general, corrections that are power-suppressed in rapidity, rather than momentum transfer. Such corrections, for purely practical reasons, tend to be rather more important, and harder to control. At the same time, the two evolutions complement each other, and a great deal is to be learned by exploring which each means for the other [157]. The story of HERA small-$x$ data is in large part the recognition that structure functions increase rapidly toward $x = 0$, with a slope that increases with the momentum transfer. The pendulum has swung from early enthusiasm for a BFKL explanation, to a realization that the data, even at moderate $Q^2$, are describable within the standard PDF-DGLAP formalism, back to a realization that at $Q$'s below a few GeV, structure functions may be showing signs of power behavior, and even saturation [149].

**B. The Pomeron in 2001**

Through $x \rightarrow 0$, DIS, the quintessentially partonic process, leads back to the questions of hadronic scattering that were, some decades ago, displaced at the center of strong interaction physics: how can we understand the behavior of the total cross section at high energy? Is the Froissart bound saturated by the strong interactions, and if so, what is the dynamical source of that saturation? What is the makeup of the total cross section in terms of diffractive and highly inelastic scattering, and what does this tell us about hadronic structure? What is the singularity structure of hadronic scattering amplitudes in the complex plane of angular momentum?
Beyond their undeniable historic interest, these questions highlight the challenges in constructing a theory that makes the transition from partonic to hadronic degrees of freedom. They are a set of questions that were, for a time, edged off center stage as we learned that rare, high-momentum events were a window to QCD in terms of the most elemental degrees of freedom. The questions remained, however, and we can recognize them now as a variant of one of the essential intellectual challenges: how complex phenomena arise out of simple, underlying laws. The spirit of these questions may be summarized by the technical query: what is the QCD pomeron [158]?

The pomeron is, to many of us, an elusive concept, with origins in a Regge technology that is no longer widely accessible. It can also be addressed in the very modern language of duality [159]. In some sense, however, the concept is not a difficult one. It is an empirical fact that most inelastic collisions result in energy loss through the production of particles with relatively low transverse momenta, and without strong correlations to the initial-state hadrons. As initial-state energy increases, the phase space increases, and the lack of correlations with the initial-state particles implies a universality for the bulk of the event. The optical theorem – that is, the conservation of probability – identifies the total cross section with the elastic scattering amplitude at zero momentum transfer. The pomeron can be thought of as the sum total of what is left behind when two hadrons collide, insofar as it is encoded into the elastic scattering amplitude. The universality of central particle production is essential here, and this universality ensures that the dominant behavior of the DIS cross section is related to the very same pomeron that governs the total pp and pp cross sections at high energy. To the extent that elastic scattering is thought of as resulting from the exchange of an object, that object may be thought of as the pomeron. To identify it with a virtual particle, however, is certainly an oversimplification.

The universality of the pomeron appears not to be absolute, but to depend at least on the reality of the initial states to which it couples, or alternately whose evolution into virtual states it describes. Thus, as $Q^2$ increases, the $x$, or equivalently $s$, dependence of a PDF is expected to deepen, corresponding to the short-distance size of the initial state at the current end. Depending on the kinematics, the actual increase may be determined by the DGLAP or BFKL equation, or either, or something else [160]. For a pomeron "pinned down" to short distances at both ends, it is possible that a really reliable perturbative approach might be possible. An example will be afforded by the total cross section for two virtual photons, with $S_{\gamma\gamma} \gg Q_1^2$, $Q_2^2 \gg \Lambda_{QCD}$. Such measurements, already made at LEP, would be an exciting program for a future linear collider [3, 4].

C. Diffraction and Gaps

One of the striking developments of Run I at the Tevatron was the observation of the diffractive production of heavy states, involving jets, electroweak bosons and heavy quark pairs [161]. At the percent level, a few high-$p_T$ processes stand like isolated islands above a background of little or no radiation. At low overall momentum transfer, it appears natural to describe this as the result of pomeron exchange between the hadrons. As in DIS, we may formulate, and to some extent measure, diffractive parton distributions in hadron-hadron scattering, although they need not be, and indeed are not, the same as in DIS. Other outstanding results from Run I involved dijet rapidity gaps [162, 163], with little radiation between jet pairs resulting from large momentum transfers, also seen at HERA [164], and the observation of intra- and interjet coherence effects in final states [165].

The continued study of diffractive and rapidity gap final states at Run II of the Tevatron and at HERA is likely to be very fruitful, independently and in their comparison. Diffractive and gap events should be thought of as windows into QCD evolution over long times [166]. A gold mine of information on the transition of QCD dynamics from short distances to long is encoded in correlations of radiation in diffractive final states, and their analogs in minimum-bias [167] and hard-scattering events. The huge lever arm of the LHC, and of a TeV-scale lepton beam, either in conjunction with a fixed hadronic target or a hadron beam, could allow, if the concepts are in place, a precision study of QCD time evolution. This is an opportunity that will only be realized through a collaborative effort of theory and experiment, beyond what we have accomplished in the recent past.

D. Nuclei

High energy scattering on nuclei has led over the years to a series of surprises and challenges for QCD. The Cronin effect (enhancement of single-particle spectra at $p_T \sim$ few GeV), the EMC effect and shadowing at small $x$ (deviations from $F_{2N} / A = A F_{2N}$) were surprises at the time, which led to deeper understanding of the meaning of parton distributions, and of the role of multiple scattering at high energy. In factorized cross sections, nuclei differ from an incoherent collection of nucleons in that their nonperturbative distributions may – indeed must – differ, and also in that power corrections in momentum transfer are enhanced at least by a factor $A^{1/3}$, due
to the longer path length in the nucleus. This nuclear enhancement is characteristic of "rare" hard scatterings, in which the active projectile parton is unlikely to encounter more than a single parton of the target. In such cases, multiple scattering should be thought of as a correction, of the generic magnitude \( T_4 A^{1/3}/Q^2 \), where \( T_4 \) is a nuclear (higher twist) matrix element with mass dimension of momentum transfer squared [168]. In this case, nuclear effects give insight into long-range correlations in the nucleus.

At small \( x \), the difference between nucleon and nuclear scattering can become much more pronounced. For example, as we have observed above, the nucleus may interact not with a single parton, or electroweak boson, but with a long-lived hadronic virtual state, often modelled by a QCD dipole. For small impact parameter in this scattering, the interaction can be geometrical, and grow as the surface area \( \sim A^{1/3} \). In this case, a nuclear target is expected to accentuate the transition from evolution in transverse momentum to evolution in rapidity, and consequent saturation, and the achievement of high-density dynamics at weak coupling.

In the past decade, the discipline of nuclear physics has been transformed, by the importation and further creation of ideas and methods in QCD. The reason for these developments are complex, but there is no doubt that the construction of the Relativistic Heavy Ion Accelerator at Brookhaven, with the aim of observing a quark-gluon plasma, has had a tremendous impact. As this project has matured, the shared interests of high energy and nuclear physics have become more and more clear. A keen appreciation of theoretical efforts in the full range of QCD has characterized the RHIC project from the start, and has stimulated ideas which are equally relevant to the high energy program. Prominent among these are the polarization studies, which we will review below, but the full range of "pA" and "AB" collisions planned at RHIC will pick up where programs at the ISR and Tevatron left off. One of the opportunities and challenges facing the high energy community is how to interact constructively with this effort, how to benefit from and contribute to the study of QCD in a nuclear environment.

1. Nuclear Targets

It is widely held that semihard processes play a increasingly important role in ultrarelativistic heavy-ion collisions at \( \sqrt{s} \geq 200 \text{ GeV} \), in describing global collision features, such as particle multiplicities and transverse energy distributions. These processes can be calculated in the framework of perturbative QCD and they are crucial for determining the initial condition for the possible formation of a quark-gluon plasma. To test the applicability of the factorization theorems to these processes, it is essential to know the parton distributions in nuclei. For example, to calculate so-called "minijet" production for \( p_T \sim 2 \text{ GeV} \), or charm quark production in the central rapidity region \( (y = 0) \), one needs to know the nuclear gluon distribution \( x G_A(x, Q^2) \) at \( Q^2 \sim m_c^2 \) or \( m_c \) and \( x = x_m = 2m_T/\sqrt{s} \) or \( 2m_c/\sqrt{s} \). For \( \sqrt{s} \geq 200 \text{ GeV} \), the corresponding value of \( x \) is \( x < 10^{-2} \). Similarly, nuclear quark densities, \( x f_Q/A(x, Q^2) \), are needed for the Drell-Yan lepton-pair production process.

The attenuation of the quark density in a nucleus has been firmly established experimentally at CERN [147] and Fermilab [148] in the region of small values of \( x = Q^2/2mv^2 \) in DIS on nuclei, where \( Q^2 \) is the four-momentum transfer squared, \( v \) is the energy loss of the lepton and \( m \) is the nucleon mass. The data, taken over a wide (but correlated) kinematic range, \( 10^{-5} < x < 0.1 \) and \( 0.05 \text{ GeV}^2 < Q^2 < 100 \text{ GeV}^2 \), show a systematic reduction of the nuclear structure function \( F^A_2(x, Q^2)/A \) with respect to the free nucleon structure function \( F^A_2(x, Q^2) \). There are also some indications of gluon shadowing from the analysis of lepton suppression in hadron-nucleus experiments [169]. Unfortunately, the extraction of the nuclear gluon density is not unambiguous since it involves the fitting of initial parton energy loss and final state interactions [170]. These measurements present a tremendous challenge to the theoretical models of nuclear shadowing (for recent reviews, see [171]).

The partonic description of nuclear shadowing has been extensively investigated in the context of recombination models [172, 173, 174] as well as Glauber multiple scattering theory [175, 176, 177, 178, 179, 180]. These approaches predict different scale \( (Q^2) \) dependence of the nuclear structure function than DGLAP evolution in transverse momentum space. Qualitatively, the scale evolution of a nuclear structure function is slower than for a nucleon, leading to a perturbative mechanism for the depletion of the parton density at large momentum transfer. The quantitative prediction of nuclear shadowing at large \( Q^2 \) in general depends on the initial value of shadowing at a small momentum transfer, where a perturbative calculation breaks down and a non-perturbative model and/or experimental input is needed. The so-called vector-meson-dominance (VMD) model [181] has been successful in predicting the quark shadowing at small \( Q^2 \) where the experimental data is rich [147, 148].

2. Small-x Gluon Distribution in a Nucleus

Presently there is rather little experimental information on the initial gluon shadowing value at some small \( Q^2 \). This, in principle, could make the prediction on gluon shadowing very uncertain. There exists, however, a
typical scale $Q_{3H}^2 \simeq 3 \text{ GeV}^2$ in the perturbative evolution of the nuclear gluon density beyond which the nuclear gluon shadowing can be unambiguously predicted in the context of perturbative QCD [182]. This small hard scale $Q_{3H}^2$ is determined by the strength of the scaling violation of the nucleon gluon density in the small-$x$ region, at which the anomalous dimension $\gamma \equiv \partial \ln x g_3 / \partial \ln Q^2$ is of order unity. As $Q^2$ approaches $Q_{3H}^2$, the gluon shadowing ratio rapidly approaches a unique perturbative value, determined by Glauber multiple scattering theory. At $Q^2 > Q_{3H}^2$, the nuclear gluon density, in contrast to the quark case, is almost independent of the initial distribution at $Q_{0}^2$, and its evolution is mainly governed by DGLAP evolution.

3. Nuclear Collisions

The high parton density phase of QCD will play a central role in high energy nucleus-nucleus collisions such as those at the RHIC and LHC. The primary purpose of these experiments is to create a quark-gluon plasma, a stage through which our expanding universe must have gone, and to investigate its properties. In this process, the state of high parton density is thought of as an initial condition for the creation of a plasma, resulting immediately after the collisions of the QCD fields of the initial-state nuclei. Energy loss [183], resulting in so-called jet quenching, the softening of high-$p_T$ spectra in nuclear collisions, may be a sensitive tool to study the evolving partonic state [184]. Phenomena of this kind are typical of the current overlap of nuclear and high energy physics.

To define the total number, $N$, of gluons in a hadron requires a resolution scale. A natural intrinsic scale, which grows at small $x$, is

$$\Lambda = \frac{1}{\pi R^2} \int dy$$

HERA data [146, 153] already suggests that $\Lambda \gg \Lambda_{QCD}^2$, so that $\alpha_s(\Lambda) \ll 1$, i.e., we are in the regime of weakly coupled QCD. Furthermore, in case of a nucleus, the intrinsic scale,

$$\frac{1}{\pi R^2} \int dy \sim A^{1/3} x^e$$

and at small $x$ or large $A$, classical field methods may be applicable [185]. As $x$ decreases, $\Lambda$ increases, corresponding to larger $p_T$. A typical parton size is of the order $1/p_T$. Thus we increase the number of gluons by adding more gluons with smaller and smaller sizes, which cannot be seen by probes of size resolution $\geq 1/p_T$ (at fixed $Q^2$), and thus do not contribute to the cross section (and the cross section does not vary unitarity). At the saturation momentum, $Q \sim Q_s$, the quantity $(1/\pi R^2 Q^2) x G(x, Q)$ approaches unity, and all powers of $Q$ (twists) become relevant. The saturation momentum depends on the hadron in question and on the longitudinal momentum fraction of the gluon. At the RHIC and HERA, one encounters saturation scales of about 1 GeV, at the boundary of the perturbative region. At the LHC, saturation scales in the 2-3 GeV range are anticipated.

One of the interesting theoretical developments in connection with the RHIC project is an effective field theory approach for studying small $x$ physics in the regime of high density [186]. The small $x$ effective action was obtained by successively integrating out the modes at larger values of $x$, which leaves the form of the action unchanged, while the weight satisfies a Wilsonian nonlinear renormalization group equation. If the parton density is not too large, the RG equation can be linearized and it becomes BFKL equation. Furthermore, in the double logarithmic approximation (small $x$, large $Q^2$), the renormalization group equation can be expressed as a series in inverse powers of $Q^2$, where the leading term is the small $x$ DGLAP equation and the first subleading term corresponds to GLR equation. It is argued in Refs. [185, 186] that the high parton densities in a large and energetic nucleus at the RHIC and later the LHC, allow one to formulate the collision classically, and to determine the initial conditions of such collisions from QCD, and to predict multi-particle and entropy production at high energies from first principles. The configuration of gluon fields in such a picture has been picturesquely dubbed the “colored glass condensate” [187].

A further proposal is to build an Electron-Ion Collider (EIC) at Brookhaven National Laboratory. This would use the already existing heavy ion beam from the RHIC, at 100 GeV to collide with an electron beam of 10-15 GeV, which will achieve a center of mass energy of $\sqrt{s} \sim 65$ GeV. This machine would expand the covered $x$ and $Q^2$ kinematic region by an order of magnitude over previous or current experiments such as NMC at CERN and E776 at Fermilab. It would allow one to measure nuclear structure functions at very small $x$ and moderate $Q^2$, the ideal region for investigating high parton density effects using weak coupling methods. It would further shed light on the nature of nuclear shadowing and its $Q^2$ dependence at small $x$, which would be crucial for understanding nuclear shadowing from QCD. For the first time, one would also be able to measure the longitudinal structure function $F_L$ directly by tuning the beam energy. $F_L$ is known to be a sensitive probe
of high parton density dynamics. The knowledge of nuclear structure functions at small $x$ and moderate $Q^2$ would also be required in the upcoming experiments at the LHC in its nuclear mode of operation, where parton distribution functions at $x \sim 5 \times 10^{-4}$ at $Q^2 \sim 1$ GeV$^2$ will be needed.

IX. POLARIZATION

This section was prepared by George Sterman and Werner Vogelsang.

In polarization studies at high energy, we address the essence of what QCD has to offer: the experimental study of strong coupling physics through degrees of freedom that interact weakly. More and more, particle and nuclear physics facilities complement each other in this area, as at the HERA experiment, HERMES, and at the RHIC spin project at Brookhaven. The future development of polarization physics in the high energy program will depend in large part on a creative dialog between the disciplines. In the high energy community, however, proton structure is often thought of as relevant only through its impact on new physics searches at hadron colliders. Perhaps it is worth noting that the most ambitious programs for revealing large extra dimensions, or black holes, involve the same basic approach: the study of strong coupling (string theory) through the weakly interacting quanta of the Standard Model. Should there come a time when arguments from duality reach a sophistication that allows the construction of brane world scenarios from M theory, the same methods will most likely be testable by measuring the spin structure of the nucleon.

The past of polarization studies has been in deep-inelastic scattering. Its future will complement these studies with, for the first time, hadronic collisions at RHIC [188], starting at the time of this writing.

A. Deep-inelastic Scattering

The analysis of polarized deep-inelastic scattering begins with the hadronic tensor,

$$ W^{\mu\nu}(P, q, S) = \frac{1}{4\pi} \int d^4z \, e^{i q z} \langle P, S | \hat{J}^\mu(z), \hat{J}^\nu(0) | P, S \rangle $$

$$ = W^{\mu\nu}_{\text{unpol}}(P, q, S) + i M \varepsilon^{\mu\nu\rho\sigma} q_\rho \left[ \frac{S_\sigma}{P \cdot q} g_1(x, Q^2) + \frac{S_\sigma(P \cdot q) - P_\sigma(S \cdot q)}{(P \cdot q)^2} g_2(x, Q^2) \right], $$(25)

where of the two new structure functions, $g_1$ is the better-known, because it appears at leading power in $Q$. In addition, it has a convenient parton-model interpretation, in terms of polarized parton distributions, $g_1(x) = \frac{1}{2} \sum_q \epsilon_q^2 \left[ \Delta q(x) + \Delta \bar{q}(x) \right]$ where $\Delta f(x, \mu) \equiv f_+ (x, \mu) - f_- (x, \mu)$ denotes the difference between densities of partons with positive and negative helicity. $g_1$ is accessible in experiments with longitudinally-polarized targets, $g_2$ with transverse polarization.

The past decade has seen considerable progress in the measurement [189] of $g_1$, so that roughly speaking this polarized structure function is known about as well as $F_2$ was in the mid-eighties. Evolution effects are well-observed, and the difference between proton and neutron is well enough known that the Bjorken sum rule [100], relating $\int dx (g_1^p(x) - g_1^n(x))$ to the axial charge $g_A$ that governs the beta decay of the neutron, is tested at around the ten percent level [189]. As a result, there is now an abundance of fits to polarized parton distributions [191], which, however, are not always mutually consistent. The range of these models underscores the importance of a global approach to polarized parton distributions. The practical problem is that the inclusive DIS cross section only sees the gphon at next-to-leading order. This is its great strength as a determination of quark distributions, but leaves the gphon in the dark, as it were.

The measurement of $g_2(x)$, on the other hand, is much more difficult than that of $g_1(x)$, because $g_2$ only appears in terms that are power-suppressed by factors of $m_q/Q$ in standard cross sections. Nevertheless, $g_2$ is a linear combination of leading-twist polarized quark distributions, which depend on matrix elements in the proton of the form $\langle p | \bar{q} q | p \rangle$, with $q$ a quark field, and higher-twist quark-gluon correlations, $\langle q | \bar{q} F(q) | p \rangle$ [192], with $F$ the gphon field strength. The recent precise measurements [193] thus shed light on the importance of these elusive windows into nucleon structure.
B. Hadronic Beams and $\Delta G$

The limitations of DIS for determining the polarized gluon distribution have been a large part of the impetus for a new generation of polarized beams of hadrons. The spin program at RHIC [188] will provide, for the first time, colliding beams of both longitudinal (and transverse) polarization, which will bring into reach processes such as high-$p_T$ direct photon production, for which polarized gluon distributions appear at leading order. If the momentum transfer is large, these cross sections have the standard factorized form,

$$d\Delta \sigma^\gamma_{AB} = \sum_{a,b} \int dx_a dx_b \, \Delta f_a(x_a, \mu) \, \Delta f_b(x_b, \mu) \, d\Delta \sigma_{ab}^\gamma(x_a P_A, x_b P_B, p_T^\gamma, \eta, \mu),$$

with $\Delta \sigma$ the helicity-dependence of the short-distance cross section, $d\Delta \sigma_{ab}^\gamma = \{1/2\} [d\sigma^\gamma_{a}(x a, \mu) - d\sigma^\gamma_{b}(x b, \mu)].$

As in the case of unpolarized distributions, the eventual determination of polarized PDFs is required more than a single input, and hadron + hadron $\to \gamma + X$ will be complemented by other channels in proton-proton scattering, such as jet or heavy flavor production. A promising example is also forward photoproduction [194, 195, 196], for which the gluon fusion processes $\gamma_g \to g\gamma$ is relevant at leading order in $a_s$. In measurements of semi-inclusive DIS, such as those already performed by SMC [197] and HERMES [198], one looks for specific hadronic final states and can tag individual polarized quark flavor distributions. Similarly, the RHIC experiments can achieve this through W boson production [188]. In the not-too distant future, we will have available the information necessary for a truly global fit for parton helicity distributions [199], hopefully with quantifiable uncertainties.

C. Beyond Helicity

The experiments discussed so far are all aimed at the helicity distributions $\Delta f$ of the nucleon, the difference between parton densities of flavor $f$ with helicities along and against the helicity of the parent hadron. There is much to learn, however, from transverse polarization as well. Historically, fixed target single-spin experiments studied transverse polarization in the initial state [200] and hyperon spins in the final state [201]. These experiments often showed striking spin dependence, which may be interpreted in partonic language in terms of higher-twist quark-gluon correlations [202], of roughly the same form as encountered in $g_2$. The limited kinematic range available to these older experiments make this interpretation far from conclusive. The capability of the Tevatron in mapping these higher-twist effects has, unfortunately, not been exploited.

Even at leading twist, however, there is another matrix element accessible in polarized hadronic scattering, the transversity [203]. Transversity measures the interference between states in which a parton carries opposite helicity, which can be nonzero because of the spontaneous breaking of chiral symmetry in QCD, and to a lesser extent because the quarks are massive. Defining projections for helicity and transverse spin of quarks by $P_{\perp}^\pm \equiv (1 \pm \gamma_5)/2$ and $P_{\perp}^{\perp} \equiv (1 \pm \gamma_1 \gamma_5)/2$, respectively, the helicity and transversity distributions can be put into similar forms,

$$\Delta q_\perp(x) \sim \frac{1}{2} \sum_X \delta(P_{\perp}^+ - (1 - x) P_{\perp}^-) \left| \langle X | \tilde{P}_{\perp}^{\perp} q(0) | P, \lambda, S_{\perp} \rangle \right|^2 - \left| \langle X | P_{\perp}^{\perp} q(0) | P, \lambda, S_{\perp} \rangle \right|^2.$$

where the helicity $\lambda$ and the transverse spin $s_{\perp}$ are $1/2$ in each matrix element. In inclusive DIS, the interference terms decouple from the hard scattering, except for the small helicity-mixing effects of quark masses. In principle, the measurement of transversity is a straightforward application of Eq. (26) for the hard scattering of two transversely-polarized colliding beams [188]. In practice, this is also difficult, primarily because the hard-scattering functions $\Delta \sigma$ that are sensitive to transversity turn out to be disappointingly small [204].

The most promising window into transversity appears to be by matching its helicity mixing with corresponding effects in fragmentation. Here, the most useful observable is the “Collins effect” [205], which depends on the observation that the nonperturbative joint distribution $D^b(z, k_{\perp})$, with $k_{\perp}$ the hadronic transverse momentum relative to the axis of a jet, factorizes from the hard scattering in the same fashion as do distributions in momentum fraction $z$ alone. It is then possible that we may observe nonzero values for such correlations as

$$D^b(z) = \int d^2 k_{\perp} \, D^b(z, k_{\perp}) \cos(\phi),$$

where $\cos(\phi) \equiv k_{\perp \cdot \bar{v}}/|k_{\perp}| |\bar{v}|$, with $\bar{v} \equiv S_T \times \bar{k}'$ in terms of the momentum $\bar{k}'$ of the outgoing electron. Within the past year, hints of this effect have been reported [206], although the energies and momentum transfers...
involved leave its interpretation in partonic language still problematic. A dedicated transverse-spin program at the RHIC can, in principle, provide the lever arm in energy to explore these, and related [207], signs of helicity mixing in hadronic structure.

Much of the current interest in polarized scattering resulted from the the so-called spin crisis, which boils down to the observation that the total helicity of the proton appears not to be carried primarily by quarks. While this came as something of a surprise, given the general success of partonic language in the description of DIS, the identification of nucleon with parton helicity is in no way a prediction of QCD, perturbative or otherwise. Nevertheless, if we must look elsewhere for the proton's spin, orbital effects are the natural choice. Formally, one has in mind operator expectations of the angular momentum tensor [208, 209],

$$1/2 = \langle P, 1/2 | J_3 | P, 1/2 \rangle = \langle P, 1/2 \rangle \int d^3x \, M_{012}(x) \left| P, 1/2 \right|.$$  \hfill (29)

The analysis of such matrix elements has led to the introduction of a wider class of distribution, the so-called off-diagonal or skewed parton distributions [209, 210], which take the general form $\langle p + \Delta \bar{\xi}|q(p)\rangle$, in terms of some momentum transfer $\Delta$. Matrix elements of this form interpolate between DIS structure functions and elastic form factors, as well as diffractive production amplitudes, through the unitarity properties of the theory. They are measurable in principle through "deeply virtual Compton scattering" [209]: $\gamma^* p \rightarrow \gamma p$, which has been observed both at HERA and Jlab quite recently [211]. We are still far from the quantitative experimental surveys of DVCS and related processes that would allow us to work backwards to new insights into off-diagonal matrix elements and angular momentum. Nevertheless, a direction has been set.

D. At the Boundary of Nuclear and Particle Physics

The proton is the simplest nucleus, and nucleon substructure has become more and more a subject of investigation for the nuclear community. When it comes to polarization studies in particular, it is not possible to make a clean distinction between the viewpoints of the two disciplines. This should be thought of as a positive development, and indeed a reprise of the heritage of high-energy physics. In some areas, particularly the study of off-diagonal matrix elements, the initiative is more on the side of nuclear physics. Others, such as the study of DIS distributions, seem more like particle physics. Machines now under considerations again know no boundary between specialties, including an electron-ion collider (EIC [212]) with, like the RHIC, a polarization capability, and a polarized fixed-target option for TESLA (TESLA-N [213]) or another TeV-scale linear collider. In our view, this overlap of specializations within shared interests constitutes an opportunity for future collaboration.

X. PROGRAMS FOR THE COMING DECADES

Whether QCD? The rules of quantum mechanics ensure that quantum chromodynamics is a nearly ubiquitous feature in high energy physics, a key player in nearly every experimental program, present and projected.

Among current colliders, the importance of determining parton distributions, and quantifying their uncertainties has been described above. We also emphasize the desirability of archiving data in a form that is amenable to further analysis, if possible allowing for the application of the novel ideas and new methods that are sure to come. Lattice QCD shares many methods and problems with perturbation theory, and we have seen how it may interact with it, as with experiment, in the coming years. Initiatives such as the SciDAC program will be needed to ensure that improved computing capabilities will be available to exploit theoretical advances in this direction.

Although neutrinos do not themselves interact strongly, their inelastic and elastic scattering on hadrons, which dominate their tiny cross sections, are both a source of information about hadronic structure, and a source of uncertainty in the interpretations of oscillation and related experiments. QCD will therefore play a crucial role in the analysis of both cosmic and accelerator-based neutrino experiments [214], whose full potential will be limited in some cases by strong interaction theory. The atmospheric, water and ice cross sections of ultrahigh energy neutrinos [215] are, in principle, sensitive to hitherto unexplored ranges in parton momentum fraction.

The continued planning for, and data analysis at, the Large Hadron Collider will depend on the very parton distributions being hammered out now. The LHC itself will provide a new frontier for QCD in a variety of ways, by probing nucleons at smaller scales [5], offering new tests of quark compositeness and of the theory of hadronization, extended ranges for diffractive and rapidity gap physics, windows into the evolution of color degrees of freedom from partonic to hadronic quantum numbers.
The Large Hadron Collider will also realize in a single facility the shared goals of particle and nuclear physics [216], on the investigation of new phases of QCD, and on their connection to the real of low parton-\(x\) whose study began in earnest at HERA.

Looking forward to a new linear collider, a vast field of QCD investigations opens up [3]. Top pair production allows studies of QCD at the electroweak symmetry breaking scale, and a realization of the exquisite physics of production thresholds. Photon-photon cross sections at a linear collider will offer the first entry into forward scattering in a perturbative region, and a direct test of the celebrated BFKL program. Precision top quark physics will become a reality, with implications for both Standard Model and post-Standard Model physics. A linear collider will offer jet fragmentation over unprecedented ranges, and a lever arm for event shape and energy flow analysis that will realize the promise of studies at LEP. With higher-loop calculations in hand, and an improved theory of power corrections, it should be possible to measure the strong coupling to the one percent level, which will offer a searchlight toward the physics of unification. Studies of QCD at a linear collider will both benefit from and strengthen QCD, and hence searches for new physics, at hadronic colliders, both the LHC and a VLHC or other successor.

Finally, should supersymmetry be discovered, from Fermilab on up, a new world of strongly interacting particles will emerge. New problems, and new rewards will flow from studies of QCD in this even larger context.

While it is notoriously difficult to predict new directions in theory, it seems safe to predict substantial progress in the following directions, driven by the capabilities of experiment:

- Next-to-next-to-leading order phenomenology of jet cross sections, with the aim of percent-level precision.
- Parton distribution uncertainties.
- Next-to-leading order event generators.
- Unquenched lattice calculations at the percent level.
- General analysis of power corrections in infrared-safe cross sections, heavy quark decays and hard-scattering functions.
- Theories of QCD energy flow in jet events and diffraction in hard scattering.
- Behavior of the total cross section at high energy.
- Elucidation of the polarization structure in the nucleon.
- High density and high temperature QCD in nuclear collisions.

Progress on some of these projects, particularly the first two, is well underway in the high energy community. In some others, progress will come increasingly from the interplay of high energy and nuclear experiment and theory. In yet others, only halting first steps have been taken. A theory of hadronization, and eventually of confinement is sure to come some day, perhaps, as has often been suggested, out of string theory. In any event, there will certainly be other important steps forward. The most important of these will help to bridge further the gaps between the languages of quantum chromodynamics.

XI. CONCLUDING SENTIMENTS

Quantum chromodynamics is an essential ingredient in the future of particle physics on the basis of its intrinsic interest. As a theory with weak and strong coupling phenomena, it is an inexhaustible testing-ground for advances in quantum field, and string theory, and as a component of the Standard Model its exploration is basic science at its best.

If there is a concern for the future of QCD in high energy physics, it is that it is sometimes taken for granted. It is common enough to hear, in particular, that perturbative QCD is well-understood, and that nonperturbative is synonymous with progress, which leads to the idea that the remaining frontiers of QCD are irrelevant to high energy physics, and vice versa. As we have stressed above, every experiment that involves hadrons (even when only virtual, \(\mu\)on \(g - 2\) [217]) is sensitive to QCD dynamics at all length scales. The beauty of QCD is to be found in its wealth of observables, its "infinite variety", that can in principle test every concept of quantum field theory. We would like to suggest that many of these concepts may not yet be developed, but will take form as we ask new questions of this inexhaustible theory. We suggest to both theorists and experimentalists to take QCD seriously as an open field of inquiry in its own right, to return to its basics, to realize that much
theoretical formalism was built up in a different environment, before the data revolution of the nineties. Old ideas should be reevaluated and reworked when necessary. Some aspects of theory must be reinvented for new experimental realities, as can be seen now at the B factories, or in polarization programs.

The boundary between QCD for its own sake, and QCD as a servant of new physics is an open one. Studies in QCD and searches for new physics will go hand-in-hand in the coming decade. Indeed, the complexity of new physics signals, documented again and again at this conference, challenges us to develop our control over QCD evolution at a level beyond what we have today. In particular, a TeV-scale linear collider program concurrent with the Large Hadron Collider is a natural match for QCD studies, and for the capability of QCD analysis to contribute in the search for new physics. To accomplish this development requires a broad, vigorous program in high energy physics in the intervening time, with dynamic interplay between theory and experiment at the Tevatron, HERA, the LHC along with the RHIC at Brookhaven and the electron accelerator at Jlab. It will need coherent long-range planning that incorporates an appreciation of current projects and facilities, as well as their support in theory.

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[214] C. Albright et al. (E1 World wide Web page), these proceedings, hep-ph/0111030.

