Toward
Observationally Determining
the Nature of the Missing Energy
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Why Go Beyond the CDM Model & its Variants

• **Precision** CMB, LSS, High–z (e.g. SNIa) tests may show we do not live in the 13D CDM space

• **Parameter estimation** and likelihood analysis is only as good as the **model space** considered

• Even if we **do live in CDM space** one would like to observationally prove the **dark matter is CDM** and the **missing energy is \( \Lambda \) or scalar field quintessence**

• Need to **parameterize the possibilities** with variables that go continuously from CDM space to more exotic possibilities → **generalized dark matter (GDM)**

• Determine the **clustering properties** of dark matter → **weighing neutrinos**, sound speed of dark matter

• Detect **anisotropies** in the **neutrino background radiation**
The Taxonomy of Structure Formation

S = Scalar  T = Tensor  V = Vector

Hu & Eisenstein (1998)
Einstein Equations
\[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \]

Homogen. & Isotropic Background
+ Linear Metric Perturbations

Scalar
Vector
Tensor
Scalar Perturbations
Stress + Smooth → Curvature

Stress Free
$S, S_A \ll \zeta_C$

Anisotropic Stress
$S_A \gtrsim \zeta_C, S \ll \zeta_C$

General Stress
$S_A \gtrsim \zeta_C, S_A \gtrsim \zeta_C$

Clustered
$\zeta_C = \text{const.}$
$\Phi = \text{backgrd. integral}$

General
$\zeta_C = \Phi = \text{stressed integral}$

Clustered
$\zeta_C = \Phi = \text{backgrd. ODE}$

General
$\zeta_C = \Phi = \text{stressed ODE}$

Clustered
$\zeta_C = \Phi = \text{stress integral}$

General
$\zeta_C = \Phi = \text{implicit integral}$

2 Component
$w_1, w_2$

CDM
ΛCDM
MDM
QCDM
ϕCDM
GDM

2–3 Component
$w_C = w_1$
$w_S = w_1, w_2$

CDM
string-DM
MDM
QCDM
ΛCDM
QMDM
ΛMDM
ϕCDM
OCDM
QϕMDM
OMDM
GDM

Superhorizon
$k\eta \ll 1$

Constant Entropy
PIB
Axion

All Type I (Adiabatic) Models

Non-Adiabatic Stress
External Source
Defects

Adiabatic Stress

WKB
$k_s \gg 1$

GDM

Two Component
$w_1 - w_2, k/k_{eq} \gg 1$

All Adiabatic Models

General Stress

WKB Integral
$k\eta \gg 1$

GDM

Two Component
$w_1 - w_2, k/k_{eq} \gg 1$

PIB

Hu & Eisenstein (1998)
Vector Perturbations

- **Stress Free**
  \[ \pi^{(\pm 1)} \ll \nu^{(\pm 1)} - B^{(\pm 1)} \]
  - Pure Decay
    - Decay of vorticity from arbitrary initial conditions

- **Anisotropic Stress**
  \[ \pi^{(\pm 1)} \gtrsim \nu^{(\pm 1)} - B^{(\pm 1)} \]
  - Stress Integral
    - Defects

Tensor Perturbations

- **Stress Free**
  \[ \pi^{(\pm 2)} \ll H_T^{(\pm 2)} \]
  - Free Gravity Waves
    - tCDM

- **Anisotropic Stress**
  \[ \pi^{(\pm 1)} \gtrsim H_T^{(\pm 2)} \]
  - Stress Integral
    - Defects
Generalized Dark Matter

• Arbitrary Stress–Energy Tensor $T_{\mu\nu}$
  16 Components

• Local Lorentz Invariance $\rightarrow$ Symmetric $T_{\mu\nu}$
  10 Components

• Energy–Momentum Conservation
  4 Constraints
  6 Components (Pressure, 5 Anisotropic Stresses)

• Linear Perturbations
  1 Pressure (Isotropic Stress)
  1 Scalar Anisotropic Stress
  2 Vector Anisotropic Stresses (2 Vorticity)
  2 Tensor Anisotropic Stresses (2 GW Polarizations)

• Homogeneity & Isotropy
  1 Background Pressure
  1 Pressure Fluctuation
  1 Scalar Anisotropic Stress Fluctuation

• Gauge Invariance $\rightarrow$ Parameterized Stresses
  1 Equation of State $w=p/\rho$
  1 Sound Speed $c_{\text{eff}}^2=\delta p/\delta \rho=$Adiab.+Non-Adiab.
  1 Anisotropic Stress $c_{\text{vis}}^2$ (viscosity)

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Clustering Regime

- Energy–Momentum Conservation + Causality
  \(\rightarrow\) Potential \(\Phi(w)\) \([w=\text{background eqn. of state}]\)

- Weak Decay as \(w\) decreases
  \(\rightarrow\) Minimal CMB anisotropies

\[
\frac{\Phi}{\Phi_0} = \frac{1}{1 + y/2w_2 + y^2/12w_2^2}
\]

Radiation Domination

Hu & Eisenstein (1998)

\(w_2 = -1\)

\(w_2 = -1/6\)

Matter Domination

GDM Domination
“Smooth” Regime

• Stress stabilization for \( k^{-1} < s = \int c_{\text{eff}} \, d\eta \) 
  \([c_{\text{eff}}= \text{effective sound speed}]\)

• Maximize CMB anisotropies \( c_{\text{eff}} \to 1 \) [scalar fields]

• Maximize LSS Features \( c_{\text{eff}} \to 0 \) [HDM]

Radiation Domination

\[ w_2 = -1 \]

\[ w_2 = -1/6 \]

\( \Phi/\Phi_0 \)

\( y/(1+y) \)

Boltzmann

Analytic

Hu & Eisenstein (1998)

Matter Domination

GDM Domination
Is the Missing Energy a Scalar Field?


- **Scalar Fields** have maximal sound speed
  \[ c_{\text{eff}} = 1, \text{ speed of light} \]

- **CMB+LSS** → Lower limit on \( c_{\text{eff}} > 0.6 \) at \( w_g = -1/6 \)
  
  [2.7\( \sigma \): MAP+SDSS; 7.7\( \sigma \): Planck+SDSS]
  [in 10d parameter space, including bias, tensors]

- Constraints weaken as \( w_g \) decreases

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**Large Scale Structure**

\[
P(k) = \begin{cases} 
10^4 & \text{if } c_{\text{eff}}^2 = 0 \\
10^3 & \text{if } c_{\text{eff}}^2 = 1/6 \\
10^2 & \text{if 1 scalar fields} 
\end{cases}
\]

\[
k (h \text{ Mpc}^{-1})
\]

**CMB Anisotropies**

\[
\left( \frac{\Delta T}{T} \right)^2 \times 10^{-10}
\]

\[
l
\]

\[
c_{\text{eff}}^2 = 0 \quad w_g = -1/6
\]

\[
1/6
\]

1 scalar fields
Hot Dark Matter as GDM

- Hot Dark Matter is a component of matter going nonrelativistic near last scattering (eV range)
- Possesses a time–dependent equation of state, sound speed, and viscosity
- Well–modeled by GDM of $c_{\text{eff}}^2 = c_{\text{vis}}^2 = w_g$ at low computational cost

![CMB Anisotropies](image1)

![Power Spectrum](image2)
Weighing Neutrinos

Hu, Eisenstein, & Tegmark (1998)
Eisenstein, Hu, & Tegmark (1998)

• **Massive neutrinos** suppress power strongly on small scales \[\Delta P/P \approx -8\Omega_\nu/\Omega_m\]

• CMB signal small

• Degenerate with other effects [tilt \(n\), \(\Omega_m h^2\) ...]

• CMB breaks degeneracies

• 2\(\sigma\) Detection: 0.3eV [Map (pol) + SDSS]

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Power Suppression

[Graph showing power suppression with different neutrino masses]

Complementarity

[Graph showing complementarity with SDSS and MAP data points]

\[\Omega_\nu/h^2 = m_\nu/94\text{eV}\]
Angular Diameter Distance Degeneracy

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- If $w_g < 0$, GDM has no effect on acoustic dynamics → $k_{\text{peaks}}$, heights independent of $w_g, \Omega_g, c_{\text{eff}}, c_{\text{vis}}$

- Angular diameter distance to last scattering → $d_A = f(w_g, \Omega_g ...)$

- One measurement two parameters

\[ (\Delta T/T)^2 \propto \frac{l \times d_A(-1) / d_A(w_g)}{c_{\text{eff}}^2} = \begin{cases} 1 & \text{for } w_g = -1/6 \text{ to } -1 \end{cases} \]
Equation of State and Density
(Complementarity and Consistency)

- **CMB** determines angular diameter distance
  \[d_A = f(w_g, \Omega_g, ...)]

- **Galaxy surveys** determines shape parameter
  \[\Gamma = (1 - \Omega_g)h\] and baryonic acoustic features

- **SNIa** determines luminosity distance
  \[d_L = f(w_g, \Omega_g)\]

![Diagram showing the relationship between \(\Omega_g\) and \(w_g\) for different surveys and combinations: MAP (no pol with pol), SNIa Only, SDSS Only, MAP + SNIa, MAP + SDSS.](image)
Approaching $\Lambda$

- **Smooth** and **Clustered** distinction disappears
  \[ c_{\text{eff}}^2 \text{ significantly constrained only if } w_g \gtrsim -2/3 \]

- **ISW** and **Clustering Features** disappear
  \[ d_A \text{ degeneracy must be broken by SNIa, } P(k), \text{ etc} \]

- **Complementarity** and **Consistency** more important
Measuring $H_0$ and $\Omega_m$
from CMB+Galaxy Surveys

Eisenstein, Hu, & Tegmark (1998)

- Neither CMB nor Galaxy Surveys have direct information on $H_0$
- Morphology of acoustic peaks calibrates the sound horizon and creates a standard ruler
- Measurement in redshift space determines $H_0$
- CMB measurement of $\Omega_m H_0^2$ determines $\Omega_m$
- Relies on high redshift physics near recombination
- Robust to uncertainties in the missing energy
- Requires only observable acoustic oscillations
Detecting the Neutrino Background Radiation


- Neutrino number $N_\nu$ or temperature $T_\nu$ alters the matter–radiation ratio
- Degenerate with matter density $\Omega_m h^2$
- Break degeneracy with SDSS baryon/CDM ratio and/or NBR anisotropies

![Graph showing $N_\nu$ vs. $\Omega_m h^2$ with and without anisotropies]
Detecting Anisotropies in the Neutrino Background Radiation


- Neutrino quadrupole anisotropies alter the gravitational potentials that drive acoustic oscillations
- Anisotropies well modeled by viscosity in the GDM $c_{\text{vis}}^2 = 1/3$ but largely degenerate
- Detectability: $1\sigma$, MAP (pol); $3.5\sigma$, MAP+SDSS; $7.2\sigma$, Planck (pol); $8.7\sigma$, Planck+SDSS

\[
\left( \frac{\Delta T}{T} \right)^2 \times 10^{-10}
\]

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Summary

• Upcoming high precision cosmological measurements may be inconsistent with our simple structure formation models and/or

• Provide the opportunity to observationally determine the nature of the dark matter but

• Requires the complementarity and consistency of multiple cosmological data sets

• If the equation of state of the missing energy differs significantly from $\Lambda$ then we can effectively probe its clustering properties with CMB+LSS

• If we live in a simple CDM space then we can effectively bound the mass of the neutrinos and detect anisotropies in the neutrino background radiation