Spontaneous chiral symmetry breaking is accepted to occur in low energy hadronic physics, resulting in the several successful theorems of PCAC. On the other hand scalar confinement is suggested both by the spectroscopy of hadrons and by the string picture of confinement. However these two evidences are apparently conflicting, because chiral symmetry breaking requires a chiral invariant coupling to the quarks, say a vector coupling like in QCD. Here we reformulate the coupling of the quarks to the string, and we are able to comply with chiral symmetry breaking, using scalar confinement. The results are quite encouraging.

1 Open Problem

Recently Bjorken asked, ” how are the many disparate methods of describing hadrons which are now in use related to each other and to the first principles of QCD?” Chiral symmetry breaking and scalar confinement are apparently conflicting, because chiral symmetry breaking requires a chiral invariant coupling to the quarks, say a vector coupling like in QCD. Here we try to solve this old conflict of hadronic physics, which remained open for many years.

The QCD Lagrangian is chiral invariant in the limit of vanishing quark masses. This is crucial because Spontaneous chiral symmetry breaking is accepted to occur in low energy hadronic physics, for the light flavors u, d and s, where, \( m_u, m_d \ll m_s < \Lambda_{QCD} < M_N/3 \). The techniques of current algebra led to beautifully correct theorems, the PCAC (Partially Conserved Axial Current) theorems. The QM (Quark Models) are widely used as a simplification of QCD, convenient to study quark bound states and hadron scattering. Recently 1 we have shown that these beautiful PCAC theorems, like the Weinberg theorem for \( \pi^- \pi^- \) scattering, are reproduced by quark models with spontaneous chiral symmetry breaking.

On the other hand the confining potential for constituent quarks is probably scalar. Scalar confinement is suggested both by the spectroscopy of hadrons, by lattice simulations and by the string picture of confinement. In a perturbative QCD scenario, the hadron spectroscopy would be qualitatively similar to electronic spectra of the lighter atoms. It is remarkable that the Spin-Orbit potential (also called fine interaction in atomic physics) turns out to be suppressed in hadronic spectra because it is smaller than the Spin-Spin potential (also called hyperfine interaction). This constitutes an evidence of non-perturbative QCD. Another evidence of non-perturbative QCD is present in the angular and radial excitations of hadrons, which fit linear trajectories in Regge plots, and suggest a long range, probably linear, confining potential for the quarks. This led Henriques, Kellet and Moorhouse, Isgur and Karl, and others 2,3 to develop a Quark Model where a short-range vector potential plus a long-range scalar potential partly cancel the Spin-Orbit interaction. The short range potential is inspired in the one gluon exchange, and
the quark vertex is a Coulomb-like potential, with a vector coupling $\bar{\psi}\gamma^\mu\psi$. The long range potential has scalar coupling $\bar{\psi}\psi$, and is a linear potential.

In this talk we define a quark model that matches the apparently conflicting vector coupling of QCD with a scalar confinement. We also solve the mass gap equation for the dynamical generation of the quark mass with the spontaneous breaking of chiral symmetry, and we indeed generate both the constituent quark mass and the scalar confinement.

2 The double vertex non-pertubative confining interaction

We aim to couple a quark line in a Feynman diagram with a scalar string, using the vector gluon-quark coupling of QCD. We remark that the a simple vertex does not solve this problem, therefore we reformulate the standard coupling of the quark to the confining potential. The coupling needs at least a double vertex, similar to the vertices that couple a quark to a gluon ladder in models of the pomeron. Our double vertex is depicted in Fig. 1.

To get the Dirac coupling of each gluon to a light quark, we follow the coupling obtained in the heavy-light quark system, computed in the local coordinate gauge. This results in a Dirac coupling a pair of $\gamma^0$ matrices, which is also compatible with the Coulomb gauge.

In the color sector, each sub-vertex couples with the Gell-Mann matrix $\lambda^a/2$. Moreover the string is also a colored object because it contains the flux of color electric field. In quark models the string usually couples with a $\lambda^a/2$ to the quark line, here it couples with to the two $\lambda^a/2$ of the sub-vertices. For a scalar coupling, which is symmetric, we use the symmetric structure function $d^{abc}$ defined with,

$$\{\lambda^a, \lambda^b\} = d^{abc} \lambda^c .$$

(1)

The dependence in the relative momentum must comply with the linear confinement which is derived from the string picture,

$$-i V(p - q) = -i \frac{\sigma}{C} \frac{-8\pi}{|p - q|^4}$$

(2)
\[ \langle \bar{\psi} \psi \rangle \left( \frac{2}{\pi} \sigma \right)^{\frac{3}{2}} \]

Figure 3: Testing the convergence of the numerical method with the quark condensate \( \langle \bar{\psi} \psi \rangle \).

where \( \sigma \approx \) is the string constant, and \( C \) is an algebraic color factor.

The gluon propagators and the different sub-vertices result in a distribution in the loop momentum \( k \). Here different choices would be possible. For simplicity we assume that the relative momentum \( p - q \) flows equally in the two effective gluon lines. We also remark that the distribution is \( k \) normalized to unity once the correct string tension is included in the relative potential \( V(p - q) \). This amounts to consider that the momentum \( k \) distribution is a Dirac delta,

\[
(2\pi)^3 \delta^3 \left( k - \frac{p + q}{2} \right).
\]

We finally compute the vertex, decomposing the Dirac propagator in the convenient particle and anti-particle propagators, computing the energy loop integral, and summing in color indices,

\[
\mathcal{V}_{\text{eff}} = \lambda \left( S_k + C_k \hat{k} \cdot \gamma \right) \Bigg|_{k = \frac{p + q}{2}},
\]

where \( S_k = m_k / \sqrt{k^2 + m_k^2} \), \( C_k = k / \sqrt{k^2 + m_k^2} \), and where \( m_k \) is the constituent quark mass, to be determined in the next section.

3 Mass gap equation

We derive the mass gap equation projecting the Schwinger-Dyson equation with Dirac spinors,

\[
\overline{\pi}_s(p) S_{0}^{-1}(p) v_{s'}(p) - \pi_s(p) \Sigma(p) v_{s'}(p) = 0
\]

where \( S_0 \) is the free Dirac propagators, and where \( \Sigma(p) \) is the self-energy, depicted in Fig. 2. The self energy consists in a three loop Feynman diagram, including one loop in each double vertex and a third rainbow-like loop for the string exchange interaction. Nevertheless each double vertex is simple, see eq. (4), and we get for the self-energy term,

\[
\pi_s(p) \Sigma(p) v_{s'}(p) = \int \frac{d^3 q}{(2\pi)^3} \left[ \left( C_k^2 - S_k^2 \right) S_q C_p + 2 S_k C_k S_q S_p \hat{k} \cdot \hat{p} - C_0 S_p \hat{q} \cdot \hat{p} - 2 S_k C_q C_p \hat{k} \cdot \hat{q} + +2 C_k^2 C_q S_p \hat{k} \cdot \hat{q} \hat{k} \cdot \hat{p} \right] V(|p - q|).
\]
Figure 4: The $m_k$ solutions of the mass gap equation in units of $\sqrt{2\pi\sigma}$.

The mass gap equation is a difficult non-linear integral equation, that does not converge with the usual methods. Here we develop a method to solve it with a differential equation, using a convergence parameter $\lambda \rightarrow 0$. Our technique consists in starting with a large infrared cutoff $\lambda$, where the integral term in eq. (6) is small. Then eq. (5) for the chiral angle $\varphi_p$ is essentially a differential equation which can be solved with the standard shooting method. Next we decrease step by step the $\lambda$ parameter, using as an initial guess for the evaluation of the integral the $\varphi_p$ determined for the previous value of $\lambda$. We test the convergence of the method computing the quark condensate, $\langle \bar{\psi}\psi \rangle$, see Fig. 3.

4 Results and conclusion

In this talk we build a model for the coupling of quark to a scalar string. Double vector vertices are used, and the quark confining interaction has a single parameter, the string tension $\sigma \simeq 200 \text{MeV}/Fm$. We solve the mass gap equation for the spontaneous breaking of chiral symmetry. In Fig. 4 we compare the constituent quark mass $m_k$, computed with our double vertex defined in eq. (vertex result), with the mass computed with a simple Coulomb gauge vertex $\lambda^c \gamma^0$. It turns out that the dynamical quark mass $m_k$ is larger when the double vertex is used, and this is a good point for the present work.

In the chiral limit of a vanishing quark mass, the effective vertex (4) $V_{\text{eff}} \rightarrow \lambda^c \hat{k} \cdot \gamma$ is proportional to the $\gamma^\mu$ and is therefore chiral invariant as it should be, whereas in the heavy quark limit, $V_{\text{eff}} \rightarrow \lambda^c$ is simply a scalar vertex. The dynamical generation of a quark mass $m_k$ also generates a scalar coupling for light quarks.

The results are encouraging, and we will now try to reproduce the whole hadron spectrum.