A Detailed Simulation of the CMS Pixel Sensor

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Abstract

This note describes a detailed simulation of pixel sensors called PIXELAV. It is not “fast” and is not intended as a replacement for the CMSIM/OSCAR pixel simulation but rather as a partial replacement for the test beam. It incorporates much of the currently known physics of charge deposition and transport in silicon. Some additional test beam data are still necessary to validate the simulation and to verify that the electronics works as designed, but hopefully, the simulation will reduce our reliance upon (expensive) test beam running. The simulation is intended to aid in: the fine-tuning of the pixel system design; the development of more realistic reconstruction algorithms; the tuning of the “fast” simulation to more accurately model the physical pixel system; and perhaps most importantly, the continuing calibration of the charge-sharing functions (needed for simulation and reconstruction) as the detector is radiation damaged during operation.

The simulation is already contributing to many of these goals. It gives a good description of charge sharing measurements made by the Atlas Collaboration and helps to resolve a mild controversy with the Atlas Collaboration regarding charge sharing after irradiation. The simulation indicates that the readout chip saturation does not limit but actually enhances the detector resolution by suppressing large fluctuations. The simulation demonstrates the importance of reversing the fan blade angles in the two endcaps (it gains a factor of 2 in resolution). It provides the charge sharing functions needed to reconstruct short cluster hits. It helps select the best long pixel reconstruction algorithm and provides bias corrections after radiation-induced trapping degrades the determination of one cluster end. It can be used to provide an independent calibration of the forward pixel charge sharing functions after irradiation and will probably provide similar information for the barrel reconstruction.
1 Introduction

This note describes a detailed simulation of the pixel sensors called PIXELAV. The pixel simulation contained within CMSIM/OSCAR uses idealized models of charge deposition and transport. These idealizations are a completely reasonable starting point to study the physics capabilities of the detector and to begin the development of the reconstruction codes. The idealized simulation is also “fast” in that it can function as part of the larger CMS simulation without significant CPU overhead. The simulation described in this note is not “fast” and is not intended as a replacement for the CMSIM/OSCAR code. PIXELAV is really intended as a partial replacement for the test beam. It incorporates much of the currently known physics of charge deposition and transport in silicon. Some test beam data are still necessary to validate the simulation and to verify that the electronics works as designed, but hopefully, the simulation will reduce our reliance upon (expensive) test beam running. The simulation is intended to aid in: the fine-tuning of the pixel system design; the development of more realistic reconstruction algorithms; the tuning of the “fast” simulation to more accurately model the physical pixel system; and perhaps most importantly, the continuing calibration of the charge-sharing functions (needed for simulation and reconstruction) as the detector is radiation damaged during operation.

This note is organized as follows: a brief pedagogical introduction to the pixel system is given in Section 2, the ingredients of the simulation are described in Section 3, some tuning and cross checking is discussed in Section 4, the reconstruction of the simulated data is described in Section 5, the results of simulations for the barrel and forward regions are summarized in Section 6, and some concluding remarks are given in Section 7.

2 CMS Pixel Tracking

A complete description of the CMS pixel tracking system is given in the Technical Design Report [1]. As is shown in Fig. 1, the system consists of 2(3) cylindrical barrels and 1(2) forward disks which provide 2(3) hit coverage over the region of pseudorapidity $|\eta| \leq 2.4(2.1)$. The barrels and disks are composed of planar detector elements which have the sandwich structure shown in Fig. 2. Plaquettes or rectangular arrays of (rectangular) silicon diodes are bump-bonded to readout chips. Each $150 \times 150 \mu m$ diode is connected to an individual readout circuit having exactly the same dimensions. The readout chip, which provides 6-8 bits of dynamic range, is described in detail in Ref. [1].

A simplified cross sectional view of a sensor array is shown in Fig. 3. The array is made from an n-doped bulk silicon wafer of approximately $300 \mu m$ thickness. The doping density of the wafer is typically $1-3 \times 10^{12} cm^{-3}$ (designated as n). One side of the wafer is implanted with acceptor impurities at much larger density (the doping density is typically $10^{14} cm^{-3}$ and is designated as p+). The other side is implanted with an array of donor implants of high density (the doping density is comparable to the p+ implant and is designated as n+). The n+ implants are metallized and bump bonded to the readout chip. They are held at ground potential and the p+ implant is maintained at negative high voltage which reverse biases the diodes. If the voltage is large enough, all of the free charge is swept out of the sensor leaving a non-zero electric field across detector. The profile of the electric field across the diode for this “fully depleted” case is shown at the bottom of Fig. 3. The field is largest at the pn...
junction and decreases away from the junction. If the applied voltage is too small, the field will vanish in part of the bulk and the detector is said to be “partly depleted”. When a charge particle traverses the detector, an average 25,000-30,000 electron hole pairs are created (for a normally incident particle, the number can be much larger for inclined tracks). The electrons drift under influence of the field to the n+ implants and the holes drift to the p+ implant. A well-known problem is caused when positive charges become trapped on the n+ side between the bulk and the surface oxide layer (not shown). Electrons are attracted to the surface and can move between the n+ implants causing cross talk. To suppress this, the n+ implants are surrounded by p+ implants which provide channel-to-channel isolation.

Fig. 3 shows the same detector after a radiation exposure of more than $10^{13}$ charged hadrons per cm$^2$. The radiation exposure produces lattice defects which, on balance, create acceptor states. The doping density of the bulk material changes from n- to p- (called type inversion). The pn junction moves from the interface between the p+ implant and the bulk to the interface between the n+ implants and the bulk. The bias voltage drops from it’s initial value (typically 200 V) to a small value near zero (at type inversion) and then increases again as the material becomes more and more p-like. The electric field profile across the diode is shown at the bottom of the Figure. Note that largest fields are now near the n+ implant and the smallest are near the p+ implant. This fact will have a number of consequences when Lorentz drift and trapping are discussed. Note that it is possible to operate the detector in a partly depleted mode (which would be impossible if holes were collected at a segmented p+ implant).

The resolution of the pixel system is better than that of a simple hodoscope of the pixel-sized elements because of charge sharing. Tracks that are angled with respect to the normal direction (see Fig. 5) deposit charge in several pixels in the plane of the magnetic field. This is precisely what happens in the pixel barrel for along the z-direction (except at small pseudorapidity where the tracks are normal to the sensors). In the transverse plane (the azimuthal direction in the barrel), a strong Lorentz force causes the deposited charge to drift transversely so that it is typically detected in two rows of pixels (see Fig. 6). In both planes, it is possible to localize the track to a fraction of the pixel dimension by using charge sharing information. This will be discussed in more detail in Sections 5 and 6.

To enhance these effects in the forward regions, the endcap “fan blades” are rotated by 20° about the radial axes of the blades (see Fig. 7) to produce similar effects. There is geometrical charge sharing in the azimuthal direction and a combination of geometrical and Lorentz-drift induced sharing in the radial direction.
3 Description of PIXELAV

PIXELAV incorporates the following elements: an accurate model of charge deposition by primary hadronic tracks (in particular to model delta rays); a realistic electric field map including many semiconductor-related effects; an established model of charge drift physics including mobilities, Hall Effect, and 3-d diffusion; a simulation of radiation damage and charge trapping effects including charge induction; and finally, a simulation of electronic noise, response, and threshold effects.

3.1 Charge Deposition

Charge deposition in the sensor is modelled using the “exact” $\pi - e$ elastic cross section of Bichsel [2] which is shown as a function of electron energy in Fig. 8. The integrated cross section for scattered electron energies between 10 eV and 1 MeV is used to determine the pion mean free path. The actual distance from one interaction to next is chosen from an exponential distribution with this mean free path. The energies of the scattered electrons are taken from the correct distribution shown in Fig. 8. The direction of each scattered electron with respect to the primary pion is constrained by two-body kinematics. The scattered electrons or “delta rays” lose energy producing electron-hole pairs as they propagate. The total number of electron-hole pairs is chosen from a Poisson distribution where the mean number of pairs is determined assuming that it takes 3.68 eV to produce a pair. The length of the delta-ray track and the distribution of pairs along it are determined from the electron energy range relationship shown in part (b) of Fig. 9 which has been taken from an excellent review article by Damerell [4]. The delta-rays are propagated until they lose all energy or they leave the sensor. This procedure generates the correct distribution of deposited charge in the sensor. Note that the Landau and Vavilov distributions are approximations to the one generated by this algorithm.
3.2 Electrostatic Simulation

Before it is possible to simulate charge transport in a pixel sensor, it is essential to determine the electric fields in a pixel cell. Calculating the field distribution within a pixel cell involves the simultaneous solution of Poisson’s equation and carrier continuity equations for electrons and holes. This fairly sophisticated problem has been solved by the semiconductor industry and specialized codes are available to the paying customer. We have obtained a code called Atlas (no connection to our LHC colleagues) from Silvaco International. The code is capable of both 2-d and 3-d simulation. The geometry of a pixel cell (100-150 $\mu$m in two transverse dimensions and 250-300 $\mu$m in depth) is not conducive to 2-d analysis and requires a full 3-d simulation to correctly model even the gross features of the intrapixel electric field. This dramatically increases the size of the lattice needed to model the potential and also increases the cpu time needed for a convergent result (a 6-cpu-minute 2-d simulation requires about 24 cpu hours when done in 3-d).

In order to reduce lattice size and cpu time, the pixel cell was assumed to have four-fold symmetry. This assumption has the consequence that the small gap in each pstop ring is not simulated (the gaps are included to allow charge from disconnected pixels to leak to ground). Four different n-side implant geometries were considered and are shown in Fig. 10. For simplicity, all cell dimensions are multiples of 6 $\mu$m which makes the use of a $3 \times 3 \times 3$ $\mu$m lattice convenient. The baseline 150$\times$150 $\mu$m two-ring pstop design was compared with a one-ring pstop design and with smaller one-ring pixel cells. The area of the n+ implant in the two-ring design is only 23% of the total pixel area whereas the area of the implant in the one-ring design is 64% of the total pixel area. This difference leads to improved depletion depth of the one-ring design with respect to the two-ring design at fixed voltage.

The electric field configuration for each of these structures depends strongly upon the effective doping density $N_{eff}$ of the bulk region. The effective doping density changes with radiation exposure and is related to the depletion depth/voltage and the resistivity of the bulk material,

\[
V_{dep} \simeq \frac{e}{2\varepsilon_{s}\varepsilon_{0}}N_{eff}\ell^{2}
\]

\[
\rho = \frac{6 \times 10^{15} \, \Omega \cdot \text{cm}^{2}}{N_{eff}}
\]

where $V_{dep}$ is the depletion voltage, $\varepsilon_{s} \simeq 11.7$ is the dielectric constant of silicon, $\varepsilon_{0} = 55.4 \, e/(V \mu m)$ is the permittivity of space, $\ell$ is the thickness of the depleted region, and $\rho$ is the resistivity of the bulk material. Note that equation 1 is valid for a transversely large diode structure and becomes less accurate when applied to pixel struc-
Figure 10: n-side implant geometries

Figure 11: The effective doping density is plotted as a function of charged hadron exposure (from Ref. [3]). Curves for ordinary silicon, carbon-enriched silicon, and oxygenated silicon are shown. The full depletion voltage for a 300 µm thick sensor is also shown on the right scale.

Atlas is provided with a number of tools to extract 2-d and 3-d information. The 3-d tools do not function properly. Unfortunately, the “inexpensive” academic license that we hold ($5k for a 2-year license for Atlas and it’s process simulation companion called Athena) does not allow us access to technical support. It was necessary to decode the undocumented master structure files generated by the simulation to extract the potential function at all points on the lattice. The function was then numerically differentiated to extract a full electric field map.
3.3 Charge Transport

The electrons and holes produced by the primary hadron drift under the influence of the internal electric field and the external magnetic field to the n+ and p+ implants. The physics of charge transport in semiconductors is nicely reviewed by Kaufmann and Henrich [5]. Within the simulation, the charge carriers are transported by numerically integrating the equations of motion,

\[
\frac{d\vec{x}}{dt} = \vec{v}
\]

\[
\frac{d\vec{v}}{dt} = \frac{e}{m^*} \left[ q\vec{E} + qr_H \vec{v} \times \vec{B} - \vec{v} \mu(E) \right]
\]

where \(m^*\) is the effective mass, \(\mu(E)\) is the mobility, and \(r_H\) is the Hall factor of the particle. The effective masses of electrons and holes in Si are 0.260 \(m_e\) and 0.241 \(m_e\), respectively. The mobilities of electrons and holes are taken from the parameterizations given in Ref. [5] and are shown in Fig. 12 along with the saturated drift velocities \(v = \mu(E)E\) in units of \(v/c\). The Hall factors are of order unity: 1.15 for electrons and 0.90 for holes.

The presence of a drag term (characterized by the mobility) in equation 4 makes the numerical integration somewhat problematic. In the absence of such a term, the integration step size need only be kept small as compared with the distance over which the fields change. It can thus be fairly “large”. The drag term causes the carrier speeds to exponentially saturate in a characteristic time \(t^* = m^* \mu/e \leq 0.1\) ps. If the step size is larger than this, the velocity change caused by the drag term becomes unphysically large and the numerical solution becomes unstable. In order to use the largest step size possible, the equations are simultaneously integrated using a fifth-order Runge-Kutta technique (because it has excellent convergence properties). The optimal step size \(t^*\) is computed for each carrier at each electric field lookup.

The small step sizes make the simulation very cpu-intensive. The initial coding of PIXELAV required approximately 5 cpu hours on an 800 MHz Pentium III class processor to simulate the electron transport for one small-\(\eta\) barrel or forward pixel hit. (Cases for which the track is nearly normal to the sensor walls. High-\(\eta\) barrel hits can take 4-5 times longer.) The cpu time per event was reduced by a factor of 9 by reducing the number of electric field lookups from one per Runge-Kutta step to one per 100 steps. The typical distance interval between lookups is less than 1 \(\mu m\) which is still small as compared with distances over which the fields vary.

Each 100 steps, an offset is added to each coordinate of the carrier position to account for diffusion. These offsets are randomly chosen from a Gaussian distribution of rms \(\ell_D\),

\[
\ell_D = \sqrt{D \Delta t}
\]

where \(\Delta t\) is the total time interval for the 100 steps and \(D\) is the diffusion constant which is taken from the Einstein relation,

\[
D = \frac{kT}{e \mu}.
\]

Simulating charge transport in a pixel sensor does not require the tracking of each secondary electron or hole. It was found that tracking only one carrier in ten still retained most of the information. Even with these improvements,
the simulation of high-eta barrel hits was marginal due to limited statistics. A final speed improvement was to customize the code for the Motorola/IBM G4 processor which has several vector co-processing units. This was quite natural since the position of each electron is specified by a 4-vector (three spatial coordinates and drift time) and it’s velocity is specified by a three vector. The G4 vector length is 128 bits which permits “simultaneous” operations on four single precision floating point numbers. Since number of steps in a typical trajectory integration is a few tens of thousands, single precision arithmetic is adequate for the task. The vectorization of code reduced the cpu-time per event by an additional factor of 7. After vectorization, a 500 MHz G4 7410 processor which is roughly the equivalent of an 800 MHz Pentium III for scalar operations was able to simulate a single small-eta hit in less than 30 cpu seconds. A 5 GHz Pentium III or a 7 GHz Pentium IV would be required to perform the same calculation with unvectorized code.

### 3.4 Signals and Trapping

The electrons and holes created by the passage of a primary hadron induce signals on the anodes and cathode of a sensor array. The sensor array is approximately a parallel plate capacitor as shown in Fig. 13. A (signed) charge $q$ present in the bulk silicon induces an infinite series of image charges of both signs as shown in the Figure. In the limit that the transverse dimension of the capacitor $R$ is much larger than its thickness $t$, the total charge induced on the anode $Q_{tot}$ is proportional to the distance $z$ of the primary charge from the cathode (p+ side)[6],

$$Q_{tot} = -q \frac{z}{t}. \quad (7)$$

It vanishes when $z = 0$ (and the charge overlaps with an opposite-sign image) and increases to $-q$ when $z = t$. This causes a net charge of $-q$ to be displaced from the preamplifier leaving a charge of $+q$ to be integrated. The transverse charge distribution on the anode is quite broad when the primary charge is at small $z$ (the dashed curve in Fig. 13) and narrows as the charge approaches the anode side of the detector (the solid curve in Fig. 13). The narrowing of the induced distribution is shown quantitatively in Fig. 14. The total charge induced on the entire anode and on a 150×150 μm “pixel” (transversely centered on the charge) are shown as functions of $z$. Note that there is relatively little charge induced on the pixel until the charge is within a distance of half the pixel dimension (75 μm). The charge induced on a single neighboring pixel is also shown and peaks at roughly the same distance. It then decreases to zero as the charge reaches the anode plane. It is clear that in the absence of trapped charge in the bulk, it is not necessary to consider charge induction on the implants. The drift times (2-20 ns) are less than the integrating time of the preamplifier (30-40 ns) and one need only count the charges hitting each anode.

Radiation exposure damages the silicon lattice creating traps for both electrons and holes. The charge carriers are captured for periods of time that are long as compared with the integrating time of the preamplifiers and are not detected with full efficiency. Kramberger et al. [7] have recently measured the effective trapping times $\tau_{eff}$ for electrons and holes in radiation-damaged silicon. They express the probability $dP_{trap}$ for the trapping of a charge carrier in time $dt$ as follows,

$$dP_{trap} = \frac{dt}{\tau_{eff}} = \beta(T)\Phi_{eq} \cdot dt \quad (8)$$

where $\beta(T)$ is a function of temperature $T$ and $\Phi_{eq}$ is the hadronic flux. Their measurements of the trapping rate $1/\tau_{eff}$ as a function of $\Phi_{eq}$ for neutron, pion, and proton exposures are shown Fig. 15. It is clear that the trapping probability increases linearly with flux as suggested by equation 8. The trapping probability is found to be independent of resistivity (doping) and oxygenation. The trapping caused by proton and pion exposure is similar and worse than that caused by neutron exposure. Annealing reduces (improves) the trapping rate for electrons and increases (worsens) the trapping rate for holes by only about 30%. The slopes $\beta$ at $T = 263K$ are given in Table 1. The temperature dependence of the $\beta_e$ and $\beta_h$ functions is shown in Fig. 16. The solid curves are a fit to the form,

<table>
<thead>
<tr>
<th>hadron</th>
<th>$\beta_e [10^{-15} \text{ cm}^2/\text{ns}]$</th>
<th>$\beta_h [10^{-15} \text{ cm}^2/\text{ns}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>neutron</td>
<td>4.1±0.1</td>
<td>6.0±0.2</td>
</tr>
<tr>
<td>pion</td>
<td>5.7±0.2</td>
<td>7.7±0.2</td>
</tr>
<tr>
<td>proton</td>
<td>5.6±0.2</td>
<td>7.7±0.2</td>
</tr>
</tbody>
</table>

$$\beta(T) = \beta(T_0) \left(\frac{T}{T_0}\right)^\kappa \quad (9)$$

8
Figure 13: Several of the image charges generated by a real (solid dot) charge in the bulk silicon. The same-sign images are shown as solid dots and the opposite-sign images are shown as open dots. The induced charge distribution on the anode (n+) side of the detector is shown schematically as the dashed (small $z$) and filled (larger $z$) curves.

Figure 14: The magnitudes of the induced charge on the entire anode and on a 150×150 µm pixel are plotted versus $z$ for a 270 µm thick sensor.

where the exponents $\kappa$ are universal for all incident particle types. The best fit yielded the following values,

$$\kappa_e = -0.86 \pm 0.06 \quad \kappa_h = -1.52 \pm 0.07$$

These expressions were used to incorporate the trapping of charge carriers into PIXELAV. After each 100 Runge-Kutta steps, the ratio $\Delta t/\tau_{eff}$ [the same $\Delta t$ used in equation 5] is tested against a random number. If the ratio was larger than the random number, propagation of that charge carrier is halted. After all charge carriers have reached the boundary of the detector or have been trapped, the program counts the number of electrons that have been collected by each n+ implant and then calculates the additional charge induced on each pixel by trapped electrons and holes. The induced charge is calculated by approximating the detector as a parallel plate capacitor with a rectangularly segmented anode. This geometry has a quasi-analytic solution (a sum over an infinite number of images which can be truncated). Note that the simulation of a new sensor does not involve trapping ($\tau_{eff}^{-1} = 0$): only electrons need be transported and no charge induction calculation is necessary. These simulations are approximately 3 times faster than those involving radiation-damaged material because the transport of the holes is slower than the transport of the electrons by a factor of two (the required number of Runge-Kutta steps is inversely proportional to mobility which is approximately a factor of two smaller for holes than it is for electrons).

An example of charge trapping is shown in Fig. 17 for a normal (unoxigenated) sensor that has been irradiated by $6 \times 10^{14}$ h/cm$^2$ and is operated with a bias voltage of 500 V. The numbers of trapped electrons and holes are plotted versus the position across the diode. Since the detector is just depleted, the holes move very slowly near the p+ implant (where the electric field is small) and are trapped with high probability. The electrons have higher mobilities and see larger electric fields near the n+ implant. Therefore, only about 40% of the electrons are trapped. Since the charge induced on the n+ implants by “distant” trapped charge is small [see Fig. 14] and since the holes and electrons induce charge of opposite sign, only the excess of electrons near the n+ implants has much effect on the measured signals (those above readout threshold) where they typically contribute 5% to the total signal.
Figure 15: The inverse trapping time $1/\tau_{\text{eff}}$ for electrons and holes is shown as a function of hadronic flux for neutron, pion, and proton exposures (from Ref. [7]).

Figure 16: The quantity $1/(\tau_{\text{eff}} \Phi_{\text{eq}})$ is shown for electrons and holes as a function of temperature (from Ref. [7]).

### 3.5 Electronics and Readout Simulation

For each event, the simulation outputs the coordinates of the pion entry and direction, the generated number of electron-hole pairs, and two sets of signals for a $13 \times 5$ pixel array. The first set includes only collected electrons and the second set includes collected electrons and induced signals from trapped charge. The simulation of the electronics and readout system is performed by a separate, analysis code. This permits the electronics simulation to be varied without the repetition of the (cpu-costly) charge transport simulation.

The electronics simulation is quite simple. First, a random noise signal of 500 electrons rms is added to each pixel signal. Then, a function which simulates the analog response of the readout chip (ROC) is applied to the total signal. Two versions of the ROC analog response function are shown in Fig. 18. The solid curve is a fit to the response function measured [8] for a RICMOS IV implementation of the readout chip which is shown as circular points. There are no measurements above input charges of 20,000 electrons where the response begins to exhibit non-linearity. The data are fit to a second order polynomial which is extrapolated to complete saturation at 33,000 electrons. This is probably a conservative “guessimate” of the true ROC response which is likely to be more linear.
than the fit function. It is probably adequate for the much of pixel barrel but may be less than ideal for the forward endcaps which have intrinsically less charge sharing. A hypothetical, more linear response function for the forward detectors is shown as the dashed curve which saturates at about 50,000 electrons. Finally, the simulated output of the ROC is “digitized” assuming that the Front End Digitizer (FED) transmits 6 bits of pulse height information and that the saturated pulse height corresponds to the digitizer maximum.

Figure 17: The numbers of trapped electrons and holes are plotted versus the position across a normal (unoxegenated) sensor that has been irradiated by $6 \times 10^{14}$ $h/cm^2$ and is operated with a bias voltage of 500 V.

4 Tuning and Checking

4.1 Lorentz Drift

A number of studies and checks of the Lorentz drift were performed by simulating carrier transport in an idealized detector. In simplest approximation, a charge carrier will drift at an angle $\theta_L$ (the Lorentz angle) with respect to the electric field direction. A small angle approximation for the Lorentz angle is

$$\theta_L \simeq \frac{er_H e B \sin \theta_{EB}}{e E} = r_H \mu(E) B \sin \theta_{EB}$$

Figure 18: The analog response function of the readout chip. The solid curve is a fit to the measured response function [8] which is shown as circular points. The dashed curve is a modified response function for the forward detectors.
where $\theta_{EB}$ is the angle between the electric and magnetic fields. The total transverse displacement, $\Delta y$ is then given by integrating $\sin \theta_L$ over the drift path,

$$\Delta y = \int_{\text{path}} \sin \theta_L d\ell \simeq \int_{\text{path}} r_H \mu(E) B \sin \theta_{EB} dz$$  \hspace{1cm} (11)$$

and we see that the total displacement is approximately determined by the integral mobility or integral electric field across the diode. Since mobility decreases with electric field [see Fig. 12], larger Lorentz angles and displacements are expected for smaller fields (voltages).

To study the Lorentz-drift induced transverse motion of the electrons, an electric field configuration corresponding to a single large diode (with a single large n+ implant) was simulated. This provides a transversely uniform electric field and avoids the focusing of electrons to the edges of the n+ implants (which distorts the transverse distribution making it more difficult to interpret). Two doping densities were considered: $-3 \times 10^{12} \text{ cm}^{-3}$ (n- doping) which corresponds to a new sensor and $+3 \times 10^{12} \text{ cm}^{-3}$ (p- doping) which corresponds to an oxygenated sensor that has been exposed to a charged hadron flux of $6 \times 10^{14} \text{ h/cm}^2$. Both of these cases have a full depletion voltage of approximately 170 V. At the same voltage, the electric field profiles across the diodes [see Figures 3 and 4] are mirror images of each other (the magnitudes of the fields are parity reversed, the directions of the fields are the same). This implies that an electron drifting from the p+ implant to the n+ implant sees the same integral electric field along it's passage. Note however, that the integral field seen by electrons originating at other points across the diode differs in the two cases. A plot of several electron trajectories in both simulated diodes operated at 180 V is shown in Fig. 19. The magnetic field is 4 T and is orthogonal (in the $-x$ direction) to the electric field (in the $-z$ direction). Note that the trajectories of the electrons originating at the p+ implant ($z = 0$) do indeed impact the n+ side in approximately the same place. For electrons originating nearer the n+ side, the new (n- doped) sensor has smaller fields and larger Lorentz angles. The $\Delta y$ distribution of the electrons at the n+ side is shown in Fig. 20 for the two cases. The vertical scale is the fraction of the deposited signal collected in a $\Delta y$ bin. Since the threshold for the readout of an adjacent pixel is approximately 10% of the deposited signal (for a normally incident track), the 10% integrals are shown as vertical lines. Even though the distributions have the same endpoint, it is clear that operationally, the new sensor has a broader $\Delta y$ distribution (125 $\mu$m vs 110 $\mu$m) and would have better resolution. Neither of the distributions is “flat” which would result from using a constant Lorentz angle to model the E×B drift. It should be clear that a real reconstruction algorithm could not be based upon a constant Lorentz angle model.

![Figure 19: The trajectories of electrons drifting in new ($n_{eff} = -3 \times 10^{12} \text{ cm}^{-3}$) and irradiated oxygenated ($n_{eff} = +3 \times 10^{12} \text{ cm}^{-3}$) transversely large single diodes. The magnetic field is 4 T and is orthogonal (in the $-x$ direction) to the electric field (in the $-z$ direction).](image1)

![Figure 20: The final $\Delta y$ distribution of the electrons for the two cases shown in Fig. 19. The track is normally incident at $y = 0$. The integrated charge to the right of the vertical lines is just at readout threshold.](image2)
Note that the use of unoxygenated silicon would have required a much larger voltage for full depletion (480 V). The electric field would still be small at the p+ implant, but at the n+ implant it would increase to a value 2.8 times larger than in the oxygenated case. This would cause a further narrowing and loss of resolution in the y direction.

A final and very significant effect on Lorentz drift in irradiated sensors is caused by trapping. Electrons produced near the p+ implant have the longest drift distances and the largest transverse displacements. Unfortunately, they are also the most likely to be trapped. This is shown for an irradiated diode operating at 180 V in Fig. 21. The upper histogram is the “irradiated” histogram shown in Fig. 20 and the lower (dashed) one is the same distribution after trapping has been included. Note that “trapped” histograms integrate to only 50-60% of the deposited signal. The vertical lines separate the regions containing 10% of the deposited charge. It is clear that the distribution becomes significantly narrower. As the voltage is increased, the electric fields increase causing the Lorentz angles and the width of the distribution to become smaller. However, the larger fields also increase the drift velocities causing the probability of trapping to become smaller. These competing effects are illustrated in Fig. 22 which shows the ∆y distribution for the irradiated diode operated at 220 V. The “untrapped” distribution is indeed narrower (97.6 µm) than it was at 180 V (110 µm) but after trapping is included, the 220 V distribution is wider (59.4 µm) than is the 180 V distribution (50 µm). The untrapped and trapped values of ∆y (with the 10% threshold requirement) are listed in Table 2 for several values of bias voltage. After applying the threshold condition, the ∆y distribution has maximum width near 220 V and becomes narrower at higher voltages.

Figure 21: The ∆y distribution of electrons in an irradiated oxygenated diode ($n_{eff} = +3 \times 10^{12} \text{ cm}^{-3}$) operated at 180 V with no trapping and with trapping. The magnetic field is 4 T and is orthogonal (in the $-z$ direction) to the electric field (in the $-z$ direction).

Figure 22: The ∆y distribution of electrons in an irradiated oxygenated diode ($n_{eff} = +3 \times 10^{12} \text{ cm}^{-3}$) operated at 220 V with no trapping and with trapping. The magnetic field is 4 T and is orthogonal (in the $-x$ direction) to the electric field (in the $-z$ direction).

Table 2: Widths of electron distributions of an irradiated oxygenated diode ($n_{eff} = +3 \times 10^{12} \text{ cm}^{-3}$) operated at various voltages. The x-widths refer to the central 84% of an η = 1 hit.

<table>
<thead>
<tr>
<th>Voltage</th>
<th>No trap: ∆y (90%)</th>
<th>W/ trap: ∆y (90%)</th>
<th>W/ trap: ∆x (84%)</th>
</tr>
</thead>
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<tr>
<td>180 V</td>
<td>110.0 µm</td>
<td>50.0 µm</td>
<td>181.9 µm</td>
</tr>
<tr>
<td>220 V</td>
<td>97.6 µm</td>
<td>59.4 µm</td>
<td>190.4 µm</td>
</tr>
<tr>
<td>260 V</td>
<td>87.6 µm</td>
<td>57.6 µm</td>
<td>199.5 µm</td>
</tr>
<tr>
<td>300 V</td>
<td>80.0 µm</td>
<td>56.0 µm</td>
<td>204.1 µm</td>
</tr>
<tr>
<td>500 V</td>
<td>54.8 µm</td>
<td>41.8 µm</td>
<td>217.6 µm</td>
</tr>
</tbody>
</table>

The complete story also requires that we consider the geometrical sharing of charge in the orthogonal direction [see Fig. 5]. The presence of trapping causes a loss of charge carriers drifting from the p+ side of the detector. As is shown in Fig. 23 for an η = 1 barrel hit, this effect shortens one end of a cluster. The cluster lengths
$\Delta x$ are calculated by subjecting both ends to an 8% integrated charge requirement (which corresponds to a 2500 electrons). They are listed for several voltages in Table 2. It is clear that width (and resolution) improve with increasing voltage. Since the forward pixel arrays always involve geometrical charge sharing even in the radial direction (where it is enhanced by Lorentz drift), we expect that the FPix optimal operating voltages will differ from those of the barrel.

Figure 23: The $\Delta x$ distribution of electrons in an irradiated oxygenated diode ($n_{\text{eff}} = +3 \times 10^{12} \text{ cm}^{-3}$) operated at 220 V for an $\eta = 1$ barrel hit with no trapping and with trapping. The thresholds are set at 8% of the signal which corresponds to 2500 electrons.

4.2 Atlas Measurements

At the Vertex “99 meeting, F. Ragusa (Atlas Collaboration) presented a measurement of charge sharing in irradiated Atlas 400×50 micron detectors [9]. The results were characterized by an effective Lorentz angle $\theta_L$ that was measured by determining the projected angle that minimized the cluster size as shown in Fig. 24.

Figure 24: Atlas charge-sharing measurement technique determines the cluster size as a function of the track angle. The angle that minimizes the cluster size is the “Lorentz angle”. Figure from Ref. [9].

In order to cross check PIXELAV and to better understand the physics of magnetically-induced charge sharing in irradiated silicon detectors, the Atlas measurements were simulated. Unfortunately, many of the details of these measurements were not documented in Ref. [9]: the initial resistivity of the material was not specified, the temperature of the sensors was not given, the thresholds of the electronics (important for cluster size) were not specified, the Hall factor is uncertain at the 10-20% level (this is an uncertainty affecting all of our simulations), the Atlas sensors used in the test were reported to have charge collection problems (they have 100% efficiency in the simulation), the data selection criteria are not specified, and the finite angular spread of the test beam is unknown (the simulation had no angular beam spread). The simulation was operated assuming that initial effective doping density was $n_{\text{eff}} = -1.7 \times 10^{12} \text{ cm}^{-3}$ and the p-stop rings were replaced by a large p-spray implant ($n_{\text{eff}} = +3 \times 10^{12} \text{ cm}^{-3}$). All other uncertain quantities were taken to be the same as those used for
simulating CMS sensors. The plots of measured cluster width versus track angle that were shown in Ragusa’s original presentation are reproduced in Fig. 25. The (blue) solid circular data points are overlaid with (red) solid squares which show the results of the simulation. The upper plot shows the measurements before irradiation and the lower plot after an exposure of $5 \times 10^{14}$ h/cm$^2$. Note that simulated points are absolutely normalized and that the cluster size scale is different for the two plots. The data and simulation are in reasonable agreement and show that charge sharing is reduced after radiation exposure. Note also that the simulation parameters have not been adjusted to fit the measurements.

Figure 25: The cluster width in Atlas 400×50 $\mu$m sensors is plotted against track angle for new (upper plot) and irradiated (lower plot) detectors [9]. The measurements are shown as (blue) solid circular data points and are compared with a fit to a simplified model (shown as the solid curve). The (red) solid squares show the results of the PIXELAV simulation.

The simulated data were fit to the form,

$$w = \sqrt{a^2 + b (\tan \theta - \tan \theta_L)^2}$$  \hspace{1cm} (12)

where $w(\theta)$ is the cluster width at track angle $\theta$ and $a$, $b$ are constants determined by the fit. The measured and simulated values are listed as functions of hadron flux in Table 3. The agreement between the measured and simulated results is again quite reasonable. Note that the microscopic Lorentz angles of the drifting electrons are given by the equations of motion. As was discussed in Section 4.1, the microscopic Lorentz angles are smaller in type-inverted silicon because the average electric field across the diode is larger. We have also seen that trapping is a significant effect and makes the diode effectively thinner by removing electrons from the low-field, large-Lorentz-angle, p+ side of the detector (which also reduces the charge sharing after irradiation). The simulation quantitatively supports the Atlas finding that irradiation reduces charge sharing, however, one should keep in mind that a single effective Lorentz angle cannot be used to fully specify the charge sharing functions.

Table 3: The measured and simulated values of the effective Lorentz angle $\theta_L$ as functions of the hadronic flux.

<table>
<thead>
<tr>
<th>Flux [10$^{14}$ h/cm$^2$]</th>
<th>Measured $\theta_L$ [deg]</th>
<th>Simulated $\theta_L$ [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.1 ± 0.1 ± 0.6</td>
<td>10.0</td>
</tr>
<tr>
<td>$5 \times 10^{14}$</td>
<td>3.0 ± 0.5 ± 0.2</td>
<td>4.0</td>
</tr>
<tr>
<td>$1 \times 10^{15}$</td>
<td>3.2 ± 1.2 ± 0.5</td>
<td>3.1</td>
</tr>
</tbody>
</table>
5 Reconstruction of Simulated Data

The propagation of tracks through the barrel or forward pixel arrays produces clusters of “hit” pixels (those with signals above the readout threshold). Fig. 26 shows several barrel clusters corresponding to tracks having pseudorapidities of $\eta = 0.25, 1.25,$ and 2.0. The actual projected “footprint” of the track is shown as the dotted line and the track position at the center of the hit is shown as an X. The detected signal (in units of $10^3$ electrons) is shown in each square. Note that the x ($z$ in cylindrical coordinates) and y ($\phi$ in cylindrical coordinates) topologies of the cluster are correlated (the cluster always switches rows near it’s center). As is suggested by these examples, there is considerable fluctuation in the individual pixel signals. The examples shown in Fig. 26 are relatively well behaved ones. A sampling of $\eta = 1.25$ hits is shown in Fig. 27. One can see that the topology varies with position of the hit (which is good) but is also subject to large fluctuations.

![Figure 26](image1.png)

Figure 26: Several barrel clusters corresponding to tracks having pseudorapidities of $\eta = 0.25, 1.25,$ and 2.0. The actual projected “footprint” of the track is shown as the dotted line and the track position at the center of the hit is shown as an X. The detected signal (in units of $10^3$ electrons) is shown in each square.

![Figure 27](image2.png)

Figure 27: A sampling of $\eta = 1.25$ hits. The actual projected “footprint” of the track is shown as the dotted line and the track position at the center of the hit is shown as an X. The detected signal (in units of $10^3$ electrons) is shown in each square.

The reconstruction of all pixel hits begins with the (possible) application of a correction for non-linear readout chip response followed by a clustering algorithm. The linearity correction is discussed in Section 6.2. The simplest clustering algorithm is to group all pixels above readout threshold that share a common side. As one can see from Fig. 27, fluctuations and the finite readout threshold can also lead to valid cases of corner adjacency. They are also included in the clustering algorithm.

The reconstruction of the hit center from the cluster differs for “short” and “long” clusters. The term “short” refers to the case when the $x$ or $y$ projection of the cluster is 1-2 pixels in size. “Long” clusters refer to projections that are at least 3 pixels in size. All hits in the forward disks produce small clusters as do the $y$ projections of barrel hits.
5.1 Short Clusters

The reconstruction of “short” clusters is relatively straightforward. To reconstruct $x$ or $y$, a sum over column or rows in the orthogonal coordinate is first performed (the cluster is projected onto the the appropriate axis). In two pixel clusters, the fraction of charge shared with neighbors is a function of the hit position. Fig. 28 shows the fraction as a function of the $x$ coordinate of a forward pixel hit where $x = 0$ is the pixel center. There is a central plateau where no charge is shared with neighbors and regions near the walls where charge is shared. Even though there is purely geometrical charge sharing in this projection, the points $x_1$ and $x_2$ where the sharing begins are not symmetrical about $x = 0$. This is a consequence of a second-order Lorentz-drift induced effect (it vanishes when the magnetic field is set to zero) and is present in the simulation because the correct equations of motion are integrated.

Figure 28: The cluster charge fraction shared with neighbors is a function of the $x$ position of a forward pixel hit in a 3T magnetic field.

For the purpose of hit reconstruction, it is convenient to define as the fraction $f_j$ ($j = x, y$) of the total cluster signal detected in the second pixel in an ordered direction. The relationship between the coordinate and $f_j$ for forward hits in a $126 \times 126 \mu m$ sensor is shown in Fig. 29 for clusters in $x$ ($\phi$) and $y$ ($r$) directions. The $x(y)$ coordinates from one pixel center to the next are shown (instead of one wall to the next). The $f = 0$ and $f = 1$ intercepts are used to define $x_2(y_2)$ and $x_1(y_1)$ which are signed quantities referenced to the local pixel center [as shown in Fig. 28]. It is clear that charge sharing in the $y$ direction is enhanced in first-order by Lorentz drift and thus manifests a larger asymmetry than the second-order one in the $x$ direction. After irradiation, the detectors are operated with higher average electric fields reducing the Lorentz drift and trapping becomes a significant effect. Since trapping suppresses the signal from one side of the junction, it makes the $x$ function very asymmetric and makes the $y$ function more symmetric.

A useful parameterization of the relationship between hit position and $f_j$ is given by the following expressions

$$
x = \begin{cases} 
x_c + x_2 + (w + x_1 - x_2) \cdot f_x^\alpha & 0 < f_x < 1 \\
x_c + (x_1 + x_2) / 2 & f_x = 0
\end{cases} \tag{13}
$$

$$
y = \begin{cases} 
y_c + y_2 + (w + y_1 - y_2) \cdot f_y^\alpha & 0 < f_y < 1 \\
y_c + (y_1 + y_2) / 2 & f_y = 0
\end{cases} \tag{14}
$$

where: $x_c(y_c)$ is the position of the center of the first (or only) hit pixel; $w$ is the size of the pixel; and $\alpha$ is an exponent. Whe only a single pixel is hit, $f = 0$ and the center of the plateau between $x_1(y_1)$ and $x_2(y_2)$ is used.

The solid lines in Fig. 29 are fits of equations 13 and 14 to the data with the exponent conatined to $\alpha = 1$. It is clear that the charge sharing in the forward sensors is approximately linear before before and after irradiation. The larger Lorentz angles present in the barrel detectors produce less linear charge sharing. A similar plot for $\eta = 0.5$ barrel clusters is shown in Fig. 30. The $y$ ($\phi$) plot shows that the Lorentz drift causes a completely asymmetric and moderately non-linear relationship between $y$ and $f_y$. The exponent varies from $\alpha \simeq 1.08$ before irradiation to $\alpha \simeq 0.90$ after irradiation due to the changed electric field profile and to trapping. Although the $y$-sharing function is independent of $\eta$, it will be shown in Section 6 that the $y$-resolution is not independent of pseudorapidity because of delta ray emission. Note also that the optimal $x (z)$ resolution of the barrel occurs near $\eta = 0.5$ since the charge is almost always shared across two pixels.
Figure 29: The fraction of signal detected in the neighboring pixel $f_{x(y)}$ is plotted as a function of the $x(y)$-coordinate of a sample of forward detector hits for a $126 \times 126 \mu m$ pixel before and after exposure to $6 \times 10^{14} \text{h/cm}^2$. The pixel center is located at $x(y) = 0$ and the inter-pixel wall is located at $x, y = 63 \mu m$. The parameters $x_1(y_1)$ and $x_2(y_2)$ are the $f_{x(y)} = 1$ and $f_{x(y)} = 0$ intercepts of a linear fit to the simulated data referenced to the centers of the pixels. The new sensor is operated at 220 V and the non-oxygenated irradiated sensor is operated at 300V.

Figure 30: The fraction of signal detected in the neighboring pixel $f_{x(y)}$ is plotted as a function of the $x(y)$-coordinate of a sample of $\eta = 0.5$ barrel hits for a $150 \times 150 \mu m$ pixel before and after exposure to $6 \times 10^{14} \text{h/cm}^2$. The pixel center is located at $x(y) = 0$ and the inter-pixel wall is located at $x, y = 75 \mu m$. The parameters $x_1(y_1)$ and $x_2(y_2)$ are the $f_{x(y)} = 1$ and $f_{x(y)} = 0$ intercepts of a power law fit to the simulated data referenced to the centers of the pixels. The new sensor is operated at 180 V and the oxygenated irradiated sensor is operated at 220V.
5.2 Long Clusters

For pseudorapidities $\eta \geq 1$, most pixel clusters in the barrel have $x$ ($z$) lengths of three or more pixels. The simple two-pixel sharing functions discussed in the last section do not describe this situation. There are a number of possible hit reconstruction algorithms that can be applied to the long-cluster case. The simplest one is to use the charge-weighted mean of all pixel centers as an estimate of the hit position. This algorithm is very sensitive to fluctuations in charge deposition and is strongly affected by large delta rays. A much better alternative is to determine the positions of the cluster ends, $x_f$ and $x_l$, and to take their mean as an estimate of the hit position,

$$
x_{\text{rec}} = 0.5 \left( x_f + x_l \right)
$$

(15)

where $x_{\text{rec}}$ is the reconstructed hit position, $x_{cf}$ and $x_{cl}$ are the coordinates of the centers of the first and last pixel in the cluster, $R_f$ and $R_l$ are the ratios of the signals in the end pixels to their neighbors (the next to end pixels), and $w$ is the pixel width. Note that $R_f$ and $R_l$ are set to unity if either is larger than 1. A plot of the hit position $x$ versus $x_{\text{rec}}$ is shown in Fig. 31 for a sample of $\eta = 1.5$ clusters in a new sensor operated at 180 V. The solid curve shows a best fit to the form $x = a + x_{\text{rec}}$ where the offset $a$ is determined to be very small ($0.1 \mu m$). After radiation exposure, trapping causes a loss of charge on one side of the cluster [see Fig. 23] causing an offset and a non-linear relationship between $x_{\text{rec}}$ and $x$. This is shown in Fig. 32 for for a sample of $\eta = 1.5$ clusters in an irradiated sensor operated at 220 V. The offset parameter grows to $a = -30.5 \mu m$ and the resolution is significantly worsened.

Figure 31: The $x$ position of long cluster hits at $\eta = 1.5$ is plotted against $x_{\text{req}}$ the coordinate reconstructed using Equation 15 for a new silicon sensor operating at 180 V. The solid curve is a fit to $x = a + x_{\text{req}}$.

Figure 32: The $x$ position of long cluster hits at $\eta = 1.5$ is plotted against $x_{\text{req}}$ the coordinate reconstructed using Equation 15 for an irradiated silicon sensor operating at 220 V. The solid curve is a fit to $x = a + x_{\text{req}}$.

5.3 Track Angle Information

The short cluster sharing functions for the $y$ ($r$) direction of the forward disks and for the $x$ ($z$) direction of the barrel depend upon the angle of the track. After radiation damage, the $x$ offset applied to the long clusters becomes significant and angle dependent. Therefore, a pixel hit reconstruction algorithm must have a priori knowledge of the track angle. This suggests that pixel hit reconstruction must be done in two passes: an initial reconstruction using approximate angle information from the correlation between track angle and geographical location of the hit within the detector, and a second reconstruction using actual track fit information to determine the track direction.
6 Results

6.1 Definitions

Most of the results of the PIXELAV simulation are given in terms of resolutions. One should note that resolution functions are the result of an analysis of simulated data. Resolution functions can vary a great deal in shape and are difficult to compare with one another. It is therefore convenient to characterize a resolution function in terms of a single parameter. There is no unique way to do this. Two common choices are: to fit the resolution function to a Gaussian distribution and quote the Gaussian half width as the resolution, or to quote the RMS of the distribution. These choices are illustrated in Fig. 33 for the \( \Delta y \equiv y_{rec} - y \) distribution of a set of barrel hits. The first technique measures the width of the core of the resolution function and is insensitive to the tails of the distribution yielding a value \( \sigma_y = 11.5 \) \( \mu m \). The second is very sensitive to the tails and yields a result \( \sigma_y = 20.4 \) \( \mu m \). Which quantity is more meaningful? In the absence of any additional information, the tail-sensitive RMS would be more appropriate. However, there is more information available. Because the tails are caused by signal size fluctuations (delta rays), the resolution is correlated with total signal size. This is shown in Fig. 34 for a sample of forward pixel hits in a 126\( \times \)126 \( \mu m \) sensor. The RMS \( x \) and \( y \) resolutions (shown as open squares and solid diamonds) are plotted against cluster signal size in electrons. The integrated fraction of the total sample with total charge less than the signal size is shown as the dashed-dotted curve (the scale is on the right hand vertical axis). It is clear that many of the hits are well resolved and that a reconstruction algorithm can separate the well resolved hits from the poorly resolved ones. The hit information can be weighted appropriately in a track fitting algorithm. The RMS resolutions for the entire sample (\( \sigma_x, \sigma_y = 16.8, 18.9 \) \( \mu m \)) are much larger than the values appropriate for most of the hits. An alternative approach is to quote an RMS for most of the sample, excluding hits with very large signals. We choose to adopt this approach and to quote the RMS for the 96% of the sample with the smallest signals. These values are shown as the dotted curves in Fig. 34. They are sensitive to the distribution tails but are not dominated by a small number of pathological cases.

One final thing to note is that all resolutions quoted in this document assume perfect knowledge of the charge-sharing functions. We have seen that they are sensitive to radiation damage effects and must be kept “calibrated” in actual operation. This problem is discussed in more detail in Section 6.4.

Figure 33: The \( \Delta y \) distribution for a sample of barrel hits at \( \eta = 1.25 \). The data are fit to a Gaussian distribution yielding a Gaussian half width \( \sigma = 11.5 \) \( \mu m \).

Figure 34: The RMS \( x \) and \( y \) resolutions for for a sample of forward pixel hits in a 126\( \times \)126 \( \mu m \) sensor are plotted against cluster signal size in electrons. The integrated fraction of the total sample with total charge less than the signal size is shown as the dashed-dotted curve (the scale is on the right hand vertical axis).
6.2 Readout Chip Response Effects

The large signal sizes shown in Fig. 34 are clearly inconsistent with the limited dynamic range of the readout chip shown in Fig. 18. An obvious question arises: How should one correct for the ROC non-linearity? In fact, this question should be preceded by a more fundamental one: Should one correct for the ROC non-linearity? To investigate this question, the $x$ and $y$ resolutions of a $150 \times 150 \, \mu m$ sensor were determined for three cases. The complete resolution analysis was performed: a) using the nominal response of the ROC and applying no linearity corrections, b) using the enhanced response function (more linear) and again applying no corrections, and c) using an ideal ROC with perfect linearity and no ADC granularity. The results of the simulation are listed in Table 4 as functions of $\eta$ in the barrel and for the forward detectors. It is interesting that the least linear case always produces the best resolutions and the perfect ROC produces the worst results. The effect is most striking in the $y$ direction for barrel hits. As $\eta$ increases, there are two competing effects: more charge is produced at large $\eta$ which decreases $\sigma_y$, but there is also an increased probability of large delta-ray emission which tends to worsen the resolution. At very large $\eta$, the latter effect eventually wins over the former. The nominal ROC response tends to cut-off the large signals producing better resolution and postponing the onset of the delta-ray dominance to larger $\eta$. One could obviously trim the signals from the perfect ROC offline and achieve the same effect. In fact, these results suggest that it may be advantageous to add some additional offline trimming to the nominal ROC output. It is interesting that this result also applies to the forward regions where the charge is typically shared among only 1-3 pixels.

There does not seem to be any reason to enhance the nominal ROC linearity.

Table 4: The 96% RMS resolutions are listed for: the normal ROC response (Low Dynamic Range), a response with enhanced linearity (High Dynamic Range) and a perfectly linear response with no digitization granularity. The statistical errors on the entries are approximately $0.5 \, \mu m$, however, the similar entries across each row are fully correlated (the same hits are processed with the different response functions). The results are for an unirradiated $150 \times 150 \, \mu m$ sensor operated at 180 V.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$\sigma_x$ (\mu m)</th>
<th>$\sigma_y$ (\mu m)</th>
<th>$\sigma_x$ (\mu m)</th>
<th>$\sigma_y$ (\mu m)</th>
<th>$\sigma_x$ (\mu m)</th>
<th>$\sigma_y$ (\mu m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>40.8</td>
<td>16.1</td>
<td>41.0</td>
<td>16.4</td>
<td>40.7</td>
<td>16.9</td>
</tr>
<tr>
<td>0.25</td>
<td>24.0</td>
<td>15.6</td>
<td>25.0</td>
<td>16.5</td>
<td>25.4</td>
<td>17.2</td>
</tr>
<tr>
<td>0.50</td>
<td>15.9</td>
<td>15.6</td>
<td>17.1</td>
<td>16.1</td>
<td>17.4</td>
<td>15.6</td>
</tr>
<tr>
<td>1.00</td>
<td>19.4</td>
<td>15.0</td>
<td>19.8</td>
<td>14.5</td>
<td>20.8</td>
<td>16.5</td>
</tr>
<tr>
<td>1.25</td>
<td>21.7</td>
<td>12.8</td>
<td>23.0</td>
<td>14.3</td>
<td>23.1</td>
<td>16.3</td>
</tr>
<tr>
<td>1.50</td>
<td>23.3</td>
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<td>24.3</td>
<td>13.2</td>
<td>27.3</td>
<td>18.9</td>
</tr>
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<td>2.00</td>
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<td>27.3</td>
<td>19.7</td>
<td>29.6</td>
<td>23.7</td>
</tr>
<tr>
<td>FPix</td>
<td>21.7</td>
<td>16.9</td>
<td>21.7</td>
<td>17.6</td>
<td>21.8</td>
<td>17.8</td>
</tr>
</tbody>
</table>

6.3 Baseline Detector Results

At the current time, the baseline design for the CMS pixel system incorporates a $150 \times 150 \, \mu m$, single p-stop ring sensor. We assume that oxygenated silicon works as advertised and that after an exposure of $6 \times 10^{14}$ h/cm$^2$, the effective doping density is $N_{eff} = +3 \times 10^{12}$/cm$^3$. All results use the nominal ROC response function and assume a 6-bit ADC digitization of the full scale ROC signal.

6.3.1 Barrel Resolutions

The 96% RMS $x$ and $y$ resolutions for the barrel are shown in Fig. 35 as functions of $\eta$ for unirradiated and irradiated detectors. As was discussed in Section 4.1, the unirradiated detector is operated at 180 V and the irradiated detector is operated at 220 V. The $x$-resolution is quite poor (close to $150 \, \mu m/\sqrt{12}$) at $\eta = 0$ where all of the charge is deposited into a single pixel and decreases to a minimum near $\eta = 0.5$ where most clusters are two pixels wide. The resolution then increases with increasing $\eta$ due to delta ray emission. After irradiation, the $x$-resolution worsens by approximately 30% due to trapping and charge loss. The $y$-resolution before irradiation decreases with increasing $\eta$ because more charge is deposited in the sensor, however, this effect competes with an increasing probability of large delta-ray emission which worsens the resolution at the largest $\eta$. The minimum 95% RMS resolution is $\sigma_y = 13 \, \mu m$ (which corresponds to a Gaussian core resolution of about 10\mu m). After irradiation, the $y$-resolution worsens by about 50% over most of the barrel.
6.3.2 Forward Pixel Resolutions

The 96% RMS forward pixel resolutions are listed in Table 5 as functions of bias voltage and \( N_{\text{eff}} \). The fractional depletion depth and the trapped fraction of the charge deposited in the depleted region are also listed. The table includes results for unirradiated sensors, irradiated ordinary silicon sensors, and irradiated oxygenated sensors. Since charge sharing in the \( y \)-direction occurs via a combination of geometrical sharing and Lorentz drift, the behavior of \( \sigma_y \) with voltage for the irradiated cases differs from that of the barrel. For unirradiated devices, the \( y \)-resolution worsens with increasing voltage (same as barrel case), but for irradiated devices, the \( y \)-resolution improves with increasing voltage because less charge is trapped. Irradiation causes approximately a 20% loss of resolution in the \( x \)-direction and a 30% loss of resolution in the \( y \)-direction. Note that these numbers are not strongly affected by delta rays. The Gaussian fit technique gives \( \sigma_x, \sigma_y = 19.3, 15.7 \mu m \) for the 180 V, unirradiated sensor which differs only by 1-2 \( \mu m \) from the 96% RMS values.

Table 5: The 96% RMS forward pixel resolutions are listed as functions of bias voltage and \( N_{\text{eff}} \). The fractional depletion depth and the trapped fraction of the charge deposited in the depleted region are also listed. The statistical errors on the entries are approximately 0.5 \( \mu m \).

<table>
<thead>
<tr>
<th>Description</th>
<th>( N_{\text{eff}} ) (cm(^{-3}))</th>
<th>Bias Voltage</th>
<th>Depletion</th>
<th>Trapped Frac.</th>
<th>( \sigma_x ) (( \mu m ))</th>
<th>( \sigma_y ) (( \mu m ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>( -3 \times 10^{12} )</td>
<td>180 V</td>
<td>100%</td>
<td>0%</td>
<td>21.6</td>
<td>16.9</td>
</tr>
<tr>
<td>New</td>
<td>( -3 \times 10^{12} )</td>
<td>220 V</td>
<td>100%</td>
<td>0%</td>
<td>20.7</td>
<td>17.3</td>
</tr>
<tr>
<td>Irr., normal Si</td>
<td>( +8.5 \times 10^{12} )</td>
<td>300 V</td>
<td>81%</td>
<td>46%</td>
<td>27.9</td>
<td>28.7</td>
</tr>
<tr>
<td>Irr., normal Si</td>
<td>( +8.5 \times 10^{12} )</td>
<td>500 V</td>
<td>100%</td>
<td>35%</td>
<td>23.6</td>
<td>22.1</td>
</tr>
<tr>
<td>Irr., oxygenated</td>
<td>( +3 \times 10^{12} )</td>
<td>220 V</td>
<td>100%</td>
<td>42%</td>
<td>25.8</td>
<td>25.0</td>
</tr>
<tr>
<td>Irr., oxygenated</td>
<td>( +3 \times 10^{12} )</td>
<td>300 V</td>
<td>100%</td>
<td>39%</td>
<td>25.4</td>
<td>23.2</td>
</tr>
<tr>
<td>Irr., oxygenated</td>
<td>( +3 \times 10^{12} )</td>
<td>500 V</td>
<td>100%</td>
<td>35%</td>
<td>22.9</td>
<td>22.3</td>
</tr>
</tbody>
</table>

6.3.3 Forward Pixel Fan Blade Angles

The charge sharing in the \( y \)-direction of the forward detectors is a combination of geometrical sharing and Lorentz drift. The 20° rotation angle of the fan blades [see Fig. 7] must have the correct sense to cause the two effects to reinforce each other. Fig. 36 illustrates this effect. Note that the electric field reverses direction on the “back” side of the blade causing the Lorentz drift to reverse which enhances charge sharing on both sides of the blade. If the fan blade angle has the wrong sense, the Lorentz drift tends to cancel the geometrical sharing as shown by the dashed arrows in Fig. 36. Given that the magnetic field has the same direction in both endcap regions of CMS, the fan angles must change sign in the two regions. This is illustrated in Fig. 37 which is a “top” view of a set of blades.
looking toward the beam axis. The sign change requires that the forward and backward endcaps have different mechanical structures. Forward and backward disks are not interchangeable which increases the engineering and fabrication costs. If the same components were used in both end regions, the blades would have parallel angles [shown by the dashed blades in Fig. 37]. This would reduce the cost of the detector but would compromise the $y$-resolution in one endcap. An obvious question is: How serious would this be?

The effect of incorrectly choosing the sign of the fan angle is shown in Fig. 38. The neighboring signal fraction is plotted as a function of $y$ for both choices. The incorrect choice significantly reduces the fraction of hits which share charge and increases the $y$-resolution from 16.9 $\mu$m for the correct choice to 30.6 $\mu$m for the incorrect choice. This cost savings measure would worsen the $y$-resolution by nearly a factor of two in the affected endcap!

### 6.4 Charge-Sharing Calibration

We have seen that the charge sharing functions that are needed to reconstruct hits in the pixel system are not stable in time. Radiation damage alters the effective doping density and also causes trapping. Both effects alter
the sharing of charge. Therefore, the calibration of the pixel system involves more than the usual calibration of electronic gains, thresholds, and pedestals; it also involves the tracking of the sharing functions and correction offsets (for the $x$-direction in the barrel). The problem is made more difficult by the fact that radiation damage is not uniform across the detector. The inner parts of the forward detectors will receive more damage than the central parts of the barrel. The innermost barrel will be worse than the outermost barrel, etc. At the current time, there is no coherent and consistent strategy for calibrating the hit reconstruction. Obviously, it would be very useful to monitor the full depletion voltage and the charge loss due to trapping of various parts of the detector as functions of time. This simulation can then be used to generate the appropriate sharing functions.

In the absence of trapping, the full depletion voltage can be determined from a measurement of signal size versus bias voltage: the signal saturates at the full depletion voltage. Trapping can destroy the voltage plateau making the two effects difficult to untangle from one another. A simulated signal versus bias voltage measurement is shown in Fig. 39 for a sample of forward hits in an irradiated FPix sensor (exposed to $6 \times 10^{14}$ h/cm$^2$). The two sets of data shown correspond to the nominal ROC response (low dynamic range) and the enhanced response (high dynamic range). It is clear that ROC saturation does not affect the shape of the curve. The data are plotted on log-log scales to identify power law behavior. Note that below the full depletion voltage of 170 V, the charge increases in power law fashion with a slope (exponent) of $0.50 \pm 0.03$ which is exactly how the depletion depth scales with voltage (see equation 1). Near the depletion voltage some complicated behavior is observed (perhaps showing the remnant of a plateau). Well above the depletion voltage, the function behaves approximately like a power law with a smaller exponent. To investigate whether the high voltage behavior can be used to determine trapping rates, the simulation was repeated keeping the effective doping density (and depletion voltage) fixed but varying the electron and hole trapping rates to correspond to a radiation exposure of $4 \times 10^{14}$ h/cm$^2$. The resulting curve is compared with the original one in Fig. 40. At each voltage, the signal increases by 10-15%. Below the full depletion voltage, the slope is unchanged. The region near depletion voltage is also somewhat complicated. The power law behavior above depletion voltage is less well established. Even with limited statistics, it seems clear that it is possible to extract the full depletion voltage from voltage scans of heavily irradiated sensors. Given the difficulty of keeping the entire readout chain absolutely calibrated, the correlation of the absolute cluster signal size with the trapping rates is probably not a useful measure of trapping rates. However, it may be possible to use the shape of the high voltage region to extract information about trapping rates.

| Figure 39: The total cluster charge (in arbitrary units) is plotted as a function of bias voltage for a sample of hits in an irradiated FPix sensor (exposed to $6 \times 10^{14}$ h/cm$^2$). The two sets of data are for the nominal ROC response (low dynamic range) and the enhanced response (high dynamic range). | Figure 40: The total cluster charge (in arbitrary units) is plotted as a function of bias voltage for a sample of hits in an irradiated FPix sensor having an effective doping density of $N_{eff} = 3 \times 10^{12}$ cm$^{-3}$. The two sets of data are for the nominal electron and hole trapping rates (corresponding to an exposure of $6 \times 10^{14}$ h/cm$^2$) and for smaller trapping rates (corresponding to an exposure of $4 \times 10^{14}$ h/cm$^2$). |
Additional and independent calibration information can be extracted from offline analyses based upon PIXELAV simulations. Because charge sharing in the forward detectors is linear, one needs to extract only two parameters for each projection \( (x_1, x_2 \text{ and } y_1, y_2) \) to determine the sharing function. These parameters are related to the fraction of clusters (at a given angle) that have two pixels hit. The parameters \( x_1 \) and \( x_2 \) are plotted as functions of the two-pixel fraction for several samples of \( 126 \times 126 \ \mu m \) pixel clusters in Fig 41. The samples correspond to several radiation exposures (0, 1.1, 2.4, 4.4, and 6.0 in units of \( 10^{14}\) h/cm\(^2\)) and operating voltages (100 V, 220 V, 300 V and 500 V) of a non-oxygenated sensor. One can readily observe that \( x_2 \) doesn’t vary significantly with irradiation or voltage. The parameter \( x_1 \) does vary with irradiation and voltage and seems to have a fairly monotonic relationship with the two-pixel fraction. Clearly, this is very promising as a calibration aid. The advantage of using the two-pixel fraction (of all clusters) is that it is independent in lowest order of the absolute gain of the ROC and readout chain. The ratio does have a first order dependence upon these quantities because it is sensitive to the absolute size of the readout threshold. The dashed curve in Fig. 41 shows the effect of changing the threshold from 2500 electron charges to 3000 electron charges. The analogous plot for \( y_1 \) and \( y_2 \) is shown in Fig. 42. Similar behavior observed and similar conclusions follow.

![Figure 41: The parameters \( x_1 \) and \( x_2 \) are plotted as functions of the two-pixel fraction for several samples of \( 126 \times 126 \ \mu m \) pixel clusters. The samples correspond to several radiation exposures (0, 1.1, 2.4, 4.4, and 6.0 in units of \( 10^{14}\) h/cm\(^2\)) and operating voltages (100 V, 220 V, 300 V and 500 V) of a non-oxygenated sensor. The effect of modifying the readout threshold from 2500 electron charges to 3000 electron charges is shown by the dashed curve.](image)

To investigate the sensitivity of this calibration procedure to uncertainties in the absolute readout threshold, we must understand the effect of imperfect calibration on the resolution. The effective resolution including the imperfect calibration is given by the following expression,

\[
\begin{align*}
\sigma_{eff,x}^2 &= \sigma_{0,x}^2 + \left( \frac{\Delta x_1}{2} \right)^2 + \left( \frac{\Delta x_2}{2} \right)^2 \\
\sigma_{eff,y}^2 &= \sigma_{0,y}^2 + \left( \frac{\Delta y_1}{2} \right)^2 + \left( \frac{\Delta y_2}{2} \right)^2
\end{align*}
\]

where \( \sigma_0 \) is the resolution resulting from using the correct sharing function and \( \Delta x_i, \Delta y_i \) are the deviations from the correct values of \( x_i, y_i \). It is clear that one can sustain an error of 10 \( \mu m \) on \( x_1 \) or \( y_1 \) and increase the resolution by only 5 \( \mu m \) in quadrature. This is probably the maximum acceptable calibration bias. It is shown in Figures 41 and 42 that changing the readout thresholds by 20% would change \( x_1 \) and \( y_1 \) by only 5 \( \mu m \). Since \( x_2 \) and \( y_2 \) are unaffected by radiation exposure and operating voltage, this implies that an absolute knowledge of the readout threshold at the 40% level is adequate to avoid inflating the resolution.

The calibration of the barrel hit reconstruction has not been studied yet. The calibration of the short cluster reconstruction in the \( y \) direction involves the extraction of three parameters \( (y_1, y_2, \alpha) \) which probably cannot be uniquely determined from the 2-pixel ratio. Some additional information about the electric field strength will probably be required. The long cluster reconstruction is probably simpler to calibrate. As trapping causes a loss of signal from one end of the cluster, the cluster length (for a given angle) is reduced. One would expect a correlation between cluster length and bias correction. This is work that needs to be done in the not too distant future.
Figure 42: The parameters \( y_1 \) and \( y_2 \) are plotted as functions of the two-pixel fraction for several samples of 126×126 \( \mu \)m pixel clusters. The samples correspond to several radiation exposures (0, 1.1, 2.4, 4.4, and 6.0 in units of \( 10^{14} \text{h/cm}^2 \)) and operating voltages (100 V, 220 V, 300 V and 500 V) of a non-oxygenated sensor. The effect of modifying the readout threshold from 2500 electron charges to 3000 electron charges is shown by the dashed curve.

7 Conclusions

This note has described a detailed simulation of pixel sensors called PIXELAV. The simulation is designed to aid in: the fine-tuning of the pixel system design; the development of more realistic reconstruction algorithms; the tuning of the “fast” simulation to more accurately model the physical pixel system; and perhaps most importantly, the continuing calibration of the charge-sharing functions (needed for simulation and reconstruction) as the detector is radiation damaged during operation. We have seen that PIXELAV is already contributing to many of these goals. It gives a good description of charge sharing measurements made by the Atlas Collaboration and (the author hopes) helps to resolve a mild controversy with the Atlas Collaboration regarding charge sharing after irradiation. The simulation indicates that analog saturation in the readout chip does not limit but actually improves the resolution of the entire pixel system by suppressing large fluctuations. The simulation demonstrates the importance of reversing the fan blade angles in the two endcaps (it gains a factor of 2 in resolution). It provides the charge sharing functions needed to reconstruct short cluster hits. It helps select the best long pixel reconstruction algorithm and provides bias corrections after radiation-induced trapping degrades the determination of one cluster end. It can be used to provide an independent calibration of the forward pixel charge sharing functions after irradiation and will probably provide similar information for the barrel reconstruction.

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References


