High Level Trigger Algorithms for Muon Isolation

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Abstract

We present an algorithm to reject muons from decays of pions, kaons and $b, c$ quarks while preserving high efficiency for muons from heavier objects. It is shown that the proposed algorithm can be used in High Level Triggers (HLT) and that a significant rejection factor can be achieved.

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1 Introduction

At the LHC, several interesting processes have one or more high-$p_T$ muons in the final state. This feature can be exploited in the on-line trigger and for the off-line selection by setting a threshold on the $p_T$ of reconstructed muons.

The generator-level rate as a function of the muon $p_T$ for the main sources of muons at the LHC is shown in Fig. 1. In the low-$p_T$ regime it is dominated by muons from $K$ and $\pi$ decays, while muons from $b$ and $c$ decays are dominant in the intermediate-$p_T$ range. The contribution from $W$ and $Z$ decays becomes the most important above $p_T \sim 30$ GeV/c.

In order to preserve a good efficiency for signal channels, the $p_T$ threshold of the single muon trigger should be as low as 15–25 GeV/c [1]. In this range the $b, c$ component of muon rates is the largest source of background in the trigger. The rejection of these muons is difficult since they are real and prompt, i.e. produced close to the interaction point.

A way to separate muons from $b, c$ decays from those from signal events relies on the fact that $b$ and $c$ quarks are produced in jets while muons from heavy object decays (like $W\rightarrow\mu\nu$) are isolated – i.e. not surrounded by other particles, except for those from pile-up collisions.

The rejection of unwanted events in the trigger chain should be as fast as possible. In order to get adequate rejection fulfilling the constraints of available time and data bandwidth, the CMS trigger selection is subdivided in several steps (levels), each one taking a decision using only part of the available data. The CMS trigger consists of a First Level Trigger (L1) [2] implemented using dedicated programmable hardware and a software High Level Trigger (HLT) system running on a farm of commercial processors. While the L1 can only access data from calorimeters and muon detectors with limited granularity, at the HLT step the fine granularity data from all CMS subdetectors can be accessed and analysed. The HLT is further divided in two selection steps: the Level-2 (L2), taking a decision using data from muon detectors and calorimeters, and the Level-3 (L3), including also tracker data.

An algorithm for L1 muon isolation based on the energy measured in calorimeters is described in [3]. Its effectiveness for the reduction of the muon rate after L1 is limited since a significant fraction of the muons passing even high L1 $p_T$ thresholds have actually very low $p_T$ [4], due to the combined effect of non-Gaussian tails in the $p_T$ resolution of L1 muons and the steeply falling behaviour of the $p_T$ spectrum of minimum bias events. In addition, low-$p_T$ muons, as those from $K, \pi$ decays, are usually accompanied by low energy jets, which leave little or no deposit in the calorimeters, and are therefore not efficiently rejected by isolation algorithms.

The aim of this note is to present efficient muon isolation algorithms to be applied in the HLT selection of CMS.

2 Simulation

The details of the simulation chain are described elsewhere [5]. The generation of events is done with the PYTHIA [6] Monte Carlo generator version 6.158 interfaced to the CMKIN [7] package. The detector simulation is performed with CMSIM 125 [8] – a simulation program based on GEANT3 [9] which includes a detailed description of the CMS geometry. The detector response, i.e. the hits, are then passed to ORCA 6.0.2 [10], where the events are digitised with minimum bias pile-up corresponding to $10^{23} \text{cm}^{-2}\text{s}^{-1}$ or $2 \times 10^{23} \text{cm}^{-2}\text{s}^{-1}$ luminosities (taken to represent the high and low luminosity modes of LHC operation) and stored in an object database. Detector inefficiencies (e.g. electronic readout inefficiencies for the silicon detectors) are taken into account in this step. The event reconstruction and analysis is performed with ORCA 6.2.3 in the framework described in [11].

At LHC startup, the trigger electronics will not be installed in the forward Cathode Strip Chamber (CSC) station ME1/1a, thus limiting the Level-1 trigger acceptance for muons to $|\eta| < 2.4$. However, the algorithms presented in this note have been implemented over the full acceptance of the muon system ($|\eta| < 2.4$), because they can be used also for off-line analyses.

Three types of events containing muons are used in this study: rare (e.g. Higgs) signal samples; $W, Z, t$ production, and minimum bias events. Events in these samples are filtered in the various production steps to ensure that all contain at least one triggerable muon. Additionally, the minimum bias sample is produced in three bins of muon $p_T$: 1-4 GeV/c, 4-10 GeV/c, and above 10 GeV/c. The samples used are listed in Table 1.

1) CMS working group for Physics Reconstruction and Selection (PRS) activities related to the CMS muon detectors.
Table 1: Signal and background samples used. Only events containing muons in the acceptance of the detector are selected for the processing. The Minimum Bias (MB) sample is produced in three bins of $p_T^\mu$. Details on the generation and simulation of these samples can be found in [5].

<table>
<thead>
<tr>
<th>Physics process</th>
<th>Generated events</th>
<th>Selected events</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H \rightarrow WW \rightarrow 2\mu 2\nu$</td>
<td>12346</td>
<td>10000</td>
</tr>
<tr>
<td>$H \rightarrow \tau\tau \rightarrow \mu + X$</td>
<td>31052</td>
<td>10000</td>
</tr>
<tr>
<td>$W \rightarrow \mu + X$</td>
<td>569618</td>
<td>50000</td>
</tr>
<tr>
<td>$Z/\gamma^* \rightarrow \mu\mu$</td>
<td>2268510</td>
<td>50000</td>
</tr>
<tr>
<td>$t\bar{t} \rightarrow \mu + X$</td>
<td>46229</td>
<td>20000</td>
</tr>
<tr>
<td>MB, $p_T^\mu ≥ 1-4 \text{ GeV/c}$</td>
<td>1357640</td>
<td>278884</td>
</tr>
<tr>
<td>MB, $p_T^\mu ≥ 4-10 \text{ GeV/c}$</td>
<td>54709859</td>
<td>404992</td>
</tr>
<tr>
<td>MB, $p_T^\mu ≥ 10 \text{ GeV/c}$</td>
<td>30375111</td>
<td>110513</td>
</tr>
</tbody>
</table>

The parameters of the isolation algorithms presented in this note are optimised using a procedure which consists in maximising the rejection of the events in a reference background sample while keeping the efficiency on a reference signal channel with isolated muons above a predefined nominal value. In this study, the direct $W \rightarrow \mu\nu$ decay was chosen as reference signal since it contains well isolated muons with a $p_T$ spectrum in the range relevant to the expected HLT $p_T$ thresholds (see Fig. 2). The reference background sample is composed of events with muons coming from minimum bias collisions. Both the reference signal and the reference background samples should include only events with muons above typical trigger thresholds, since the performance of isolation algorithms depends on the $p_T$ of the muon and no optimisation is needed for low-$p_T$ muons which should be discarded anyhow by the trigger. Therefore only events with muons with transverse momentum above 16 (22) GeV/c are considered for the optimisation at low (high) luminosity. It should be noted that this selection is applied to the $p_T$ of the Monte Carlo muons, and not to the $p_T$ measured by the trigger. This is necessary since, for whatever $p_T$ threshold, the L1 and L2 trigger selection is highly contaminated by the feed-through of low-$p_T$ muons [4], that must not be included in the reference signals.

The performance of the optimised algorithms within a realistic trigger selection is shown in Section 4, using the full minimum bias sample to estimate the rate reduction and the signal channels $W \rightarrow \mu\nu$, $H \rightarrow WW \rightarrow 2\mu 2\nu$ ($M_H = 180 \text{ GeV/c}^2$), $H^{SU}/SY \rightarrow \tau\tau \rightarrow \mu\mu_{jet} + X$ ($M_A = 200 \text{ GeV/c}^2$, $\tan\beta = 20$), $Z/\gamma^* \rightarrow \mu\mu$ ($M_{Z/\gamma^*} > 2 \text{ GeV/c}^2$), $t\bar{t} \rightarrow \mu + X$ (where the muon comes from a direct $W$ decay) as benchmarks for signal efficiencies.

3 The Algorithm

The general principle of isolation algorithms is the analysis of the detector response in a region around the direction of the object under study. While the following considerations apply to any type of isolated objects, e.g. leptons and photons, we will focus on the case of muons.

Since non-isolated muons are accompanied by jets, while isolated ones have only uncorrelated soft particles from pile-up in their proximity, a muon can be defined as isolated by comparing the measurement of some quantity (e.g. the transverse energy deposited in the calorimeters or the sum of transverse momenta of tracks) in a cone around the muon with a predefined threshold. The measured quantity will be called detector measurement in the following. The cone axis is chosen according to the muon direction with a procedure that is tailored to the specific properties of each algorithm. The geometrical definition of the cone is given by the condition $\Delta R ≤ \Delta R_{MAX}$, where $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$, $\Delta\eta$, $\Delta\phi$ being the distances in pseudorapidity and azimuthal angle between the deposit and the cone axis. The muon itself contributes to the detector measurement inside the cone. This contribution (which we call veto value) can be subtracted to improve the discriminating power of the isolation algorithm (see Fig. 3).

The threshold on the detector measurement in the cone can be $\eta$-dependent in order to guarantee a flat efficiency in $\eta$, and must be higher than the average pile-up contribution but low enough to efficiently reject jet originated deposits.

Several quantities can be used as detector measurement in the cone. The algorithms presented here use the transverse energy deposit in calorimeters and the transverse momenta of the tracks reconstructed in the tracking system. In the first case – calorimeter isolation – the transverse energy measured in the towers of the hadronic calorimeter (HCAL) is combined with the reconstructed transverse energy deposit in the electromagnetic calorimeter (ECAL).
Since it is based on the calorimeters, this algorithm becomes less effective at high luminosity as more pile-up will be collected.

The CMS tracking system consists of a pixel detector with very fine granularity, surrounded by silicon strip detectors. The pixel detector in its baseline design consists of 3 cylinders of pixel detectors surrounding the beam pipe, supplemented by two disks in each endcap. The disks and cylinders are arranged so that each track coming from the nominal beam crossing point with pseudorapidity $|\eta| \leq 2.5$ crosses at least 3 layers of pixel detectors, thus allowing track reconstruction even without the aid of the silicon strip detectors. While pixel-only reconstruction algorithms are very fast, the reconstruction with the full tracking system (where the average number of measurements per track is 12) is more robust, efficient and accurate. The typical full tracker transverse momentum resolution for low-$p_T$ tracks ($\mathcal{O}(1 \text{ GeV})$) is $\sigma(p_T)/p_T = 1\%$, to be compared with the 8% resolution obtained with the pixel-only reconstruction. We therefore define two isolation algorithms using reconstructed tracks: pixel isolation, which relies on tracks reconstructed using the pixel detector alone, and tracker isolation which takes advantage of the full tracker detector. For both algorithms the sum of transverse momenta of tracks reconstructed inside the cone is used as detector measurement. Only tracks originating from the same collision vertex as the muon are used in the sum, so that these algorithms are less sensitive to pile-up than calorimeter isolation.

The multi-tiered structure of the CMS trigger allows for progressively more powerful muon isolation algorithms at each trigger level as more information becomes available. At L2, data from calorimeters are already available and the calorimeter isolation can be applied. It is also possible to process data from the pixel detector and to apply pixel isolation to the L2 candidate. At L3, candidates reconstructed using the full tracker information can have applied both pixel and tracker isolation. More than one isolation algorithm can be applied to the same muon in the different HLT steps; the best strategy (i.e. which algorithms are included in the HLT chain and in which order) should be determined by a trade-off between algorithm rejection power, efficiency and speed.

The isolation algorithms consist of two logical steps: the extraction of the signal deposited in a cone around the muon, which is specific to the detector used, and the actual isolation cut, i.e. the comparison of the cone content with the threshold. Both the size of the cone and the threshold can be optimised in order to maximise the rejection of the background while keeping the efficiency on the reference signal above a predefined value. Such predefined values of the reference signal efficiency (which we will refer to as nominal efficiency in the following) can be used as a unique parameter to tune the algorithm performance for a particular study, i.e. to choose the balance between the background selection and the efficiency on the signal of interest.

### 3.1 Optimisation of the Algorithms

The optimisation of an algorithm consists of determining the optimal size of the cone and the value of the threshold. In order to find the optimal cone sizes we define a set of cones of different size (Table 2). Using the reference signal sample (direct $W \rightarrow \mu\nu$ decays), the $\sum E_T$ or $\sum p_T$ inside all these cones is determined during the extraction step.

<table>
<thead>
<tr>
<th>Cone</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta R_{MAX}$</td>
<td>0.02</td>
<td>0.045</td>
<td>0.09</td>
<td>0.13</td>
<td>0.17</td>
<td>0.2</td>
<td>0.24</td>
<td>0.28</td>
<td>0.32</td>
<td>0.38</td>
<td>0.45</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Since some detector effects (resolution, noise) are $\eta$-dependent, to get a flat $\eta$ efficiency on the reference signal the optimisation is done independently in each of the pseudorapidity bins listed in Table 3, which correspond to the $\eta$ segmentation of the calorimeter towers.

For each cone and each $\eta$-bin, the threshold corresponding to a set of nominal efficiency values on the reference signal (typically 75%,...90%,...95%,...97%,...) is determined. The optimal cone for a given nominal efficiency is defined as the one which gives the maximal rejection on the reference background sample in the full $\eta$ range. The result of this optimisation procedure is that for any predefined nominal efficiency a cone size is chosen, with thresholds defined in bins of pseudorapidity.

It is important to mention that the optimisation is affected by two factors: the LHC luminosity and the $p_T$ range of muons used. The luminosity affects the average energy deposited per unit of $\Delta R$ in each event, so that the thresholds for a given nominal efficiency must be scaled with luminosity. This dependence is not critical: if the actual luminosity is lower than the value used for the optimisation, e.g. due to the decrease of luminosity during a run, the signal efficiency will simply be higher than the nominal value. Results obtained after optimising the algorithms for the high and low luminosity LHC operating modes are presented for all algorithms described in
Table 3: The maximum $\eta$ values for the pseudorapidity bins used for the optimisation of the size and threshold of the cones. The first bin starts at $\eta=0$. Negative bins are obtained by mirror reflection.

<table>
<thead>
<tr>
<th>$\eta$-bin</th>
<th>η $\text{MAX}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.087</td>
</tr>
<tr>
<td>2</td>
<td>0.174</td>
</tr>
<tr>
<td>3</td>
<td>0.261</td>
</tr>
<tr>
<td>4</td>
<td>0.348</td>
</tr>
<tr>
<td>5</td>
<td>0.435</td>
</tr>
<tr>
<td>6</td>
<td>0.552</td>
</tr>
<tr>
<td>7</td>
<td>0.609</td>
</tr>
<tr>
<td>8</td>
<td>0.696</td>
</tr>
<tr>
<td>9</td>
<td>0.783</td>
</tr>
</tbody>
</table>

this note. As already mentioned, the rejection power is highly dependent on the muon $p_T$, which is correlated to the energy of the accompanying jet [3]. It is therefore important to optimise the threshold for the $p_T$ range of interest, in order to get the desired efficiency for useful muons (i.e. those actually selected by the trigger $p_T$ cut). However, it is still possible to use the algorithm for muons outside the optimised $p_T$ range; the corresponding signal efficiency will be slightly different than the nominal value.

3.2 Calorimeter Isolation

The calorimeter isolation algorithm uses as input the direction and momentum of the L2 reconstructed muon [4] at the vertex (i.e. the point of closest approach to the beam line in the plane transverse to the beam). The muon direction at the vertex is the best approximation of the direction of a possible accompanying jet and is used in the definition of the cone axis as described in the following.

The extraction of the energy deposits is done independently in the ECAL and the HCAL. In the case of ECAL the measured quantity is the $\sum E_T$ in the crystals around the muon direction at the vertex. In the case of HCAL, whose segmentation is much coarser than that of ECAL, the cone axis is defined instead as the centre of the tower to which the muon direction at the vertex points; the measured quantity is the $\sum E_T$ of the towers whose centre belongs to the cone. This guarantees that the same number of towers contributes to all cones of a given size at a given pseudorapidity.

In order to reject pile-up deposits, the HCAL towers with reconstructed transverse energy ($E_T$) below 0.5 GeV and the ECAL crystals with reconstructed $E_T$ below 0.2 GeV are neglected. To avoid electronic and detector noise, an additional energy ($E$) threshold of 0.12 GeV in barrel ECAL, 0.45 GeV in endcap ECAL and 0.6 GeV in HCAL is applied. These values correspond to 3 standard deviations of the nominal noise level.

To subtract the energy deposited in the cone by the muon itself, the muon trajectory is extrapolated to the boundary between ECAL and HCAL using the package GEANE [12]. The distance between the extrapolated muon position and the Monte Carlo hit produced by the muon in the ECAL or HCAL is shown in Fig. 4. In the ECAL the transverse energy of crystals in a small area of $\Delta R \leq 0.07$ around the muon extrapolation is subtracted from the cone measurement. In the HCAL the transverse energy of a single tower is subtracted, chosen as the tower with highest deposit among those whose centre lies at $\Delta R \leq 0.1$ from the muon extrapolated point.

The actual isolation variable is constructed from both HCAL and ECAL deposits in the cones using a weighting parameter $\alpha$:

$$E^{\text{WEIGHT}}_T = \alpha \cdot \sum E_T^{\text{ECAL}} + \sum E_T^{\text{HCAL}}.$$ 

The optimal weighting parameter has been found to be $\alpha = 1.5$ (see below). Typical spectra of $E^{\text{WEIGHT}}_T$ for the reference signal sample are shown in Fig. 5 for cone number 6 in different $\eta$-bins. For each cone and $\eta$-bin, the threshold is given by the point where the integral of the normalised spectrum is equal to the nominal efficiency value$^2)$. The resulting thresholds for one of the cones and for some values of the nominal efficiency are shown in Fig. 6 and 7 for the low and high luminosity cases respectively. As expected, the thresholds are higher for high luminosity and vary with pseudorapidity.

Figure 8 shows the $\eta$-averaged efficiency for each cone and nominal efficiency. The efficiency is by construction above the nominal value. We can observe that for smaller cone sizes the curves for different nominal efficiencies

$^2)$ Due to the finite size of the bins used in this procedure, the resulting signal efficiency may be above the nominal value.
tend to converge. This is due to the fact that in small cones around an isolated muon the deposit can be small or zero, so that the optimised threshold is the same regardless of the nominal efficiency.

To determine the optimal cone sizes, the same thresholds are then applied to the reference background sample. Figure 9 shows the background efficiencies obtained using different cones. The optimal cone for a given nominal efficiency is given by the minimum of each curve. The results of this optimisation are summarised in Table 4.

Table 4: Optimal cones for calorimetric isolation as a function of the nominal efficiency. Numbers correspond to the cones defined in Table 2.

<table>
<thead>
<tr>
<th>nominal efficiency</th>
<th>0.75</th>
<th>0.80</th>
<th>0.85</th>
<th>0.90</th>
<th>0.95</th>
<th>0.97</th>
<th>0.98</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>The best cone for $2 \times 10^{33} \text{cm}^{-2} \text{s}^{-1}$</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>The best cone for $10^{34} \text{cm}^{-2} \text{s}^{-1}$</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

The same cone size is chosen for a given nominal efficiency for all $\eta$-bins. It was checked that the independent optimisation of the cone size in a few $\eta$ ranges leads to a slight improvement in the rejection factor; however we are not confident that such optimisation is significant given the available statistics, and we do not use it in the following.

With the optimised cones of Table 4 the performance of the algorithm can be studied. In Fig. 10 the isolation efficiency for the reference signal is shown as a function of the $p_T$ of the generated muon. The efficiency is slightly decreasing for low $p_T$ values. However the efficiency is rather flat for muons passing typical trigger thresholds.

The efficiency is by construction flat in pseudorapidity, as shown in Fig. 11. Similar plots can be made for the minimum bias background. Figure 12 shows that the efficiency for the reference background is flat as a function of $\eta$. However the background efficiency strongly depends on the muon $p_T$ (Fig. 13). As already mentioned, this is a general feature of isolation algorithms, due to the correlation between the $p_T$ of the muon and the energy of the accompanying jet.

The performance of the algorithm can be summarised by plotting the efficiency for minimum bias events as a function of the efficiency for signal events. The background rejection efficiency is highly dependent on the muon $p_T$ and such plots should have a cutoff for the minimal $p_T$ taken into account (lower $p_T$ values are supposed to be discarded by the muon trigger cut). The curves for a few different $p_T$ cutoff values are shown in Fig. 14.

Such plots allow the comparison of the final performance of different algorithms. In particular, the effect of different values for the weighting parameter $\alpha$ is presented in Fig. 15, which shows that $\alpha = 1.5$ is close to optimal in most of the cases.

### 3.3 Pixel Isolation

The pixel isolation algorithm uses as detector measurement the $\sum p_T$ of tracks reconstructed in a cone around the muon by the pixel detector alone, neglecting the $p_T$ of the muon itself.

The pixel reconstruction algorithm [13] looks for pixel hits compatible with tracks with transverse momenta as low as 1 GeV/$c$. The track candidates are used to fit primary vertices; track candidates with no association to reconstructed vertices are rejected. The algorithm returns a list of vertices with the corresponding tracks and their momenta. Vertices are sorted by the $\sum p_T$ of tracks assigned to them, allowing the identification of the primary vertex of the hardest interaction (usually the most interesting one in the bunch crossing).

The main drawback of this isolation algorithm is that the pixel reconstruction relies on the reconstruction of three hits out of the three layers available, and therefore it is very sensitive to detector, geometrical and electronic read-out inefficiencies. Moreover, it will not be available in staging scenarios where only two pixel layers will be installed.

#### 3.3.1 Pixel Isolation with Level-3 Muons

An isolation algorithm using pixel tracks is powerful if applied on L3 muons, which are reconstructed with enough precision to provide a precise estimate of the muon vertex (i.e. the point of closest approach of the muon trajectory)

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3) Note that the nominal efficiencies are obtained convoluting these curves with the actual $p_T$ spectra of the reference signal (Fig. 2).
to the beam line). It is therefore possible to require that all pixel tracks contributing to the energy measurement in the cone come from the same primary vertex as the muon.

Figure 16a shows the distance $\Delta R$ between the direction of the muon and the direction of the closest pixel track. The peak for $\Delta R < 0.015$ indicates a very good matching between the L3 reconstructed muon and the corresponding pixel track (hereafter called pixel muon). Figure 16b shows the distance along the beam line (the $Z$ coordinate in the CMS reference frame) between the vertex associated to the pixel muon and the L3 muon vertex. A cut of $|\Delta Z| < 0.2$ cm for pixel tracks contributing to $\sum p_T$ has been set.

A set of thresholds was created with the same procedure described in the previous section for the calorimeter isolation algorithm. Typical spectra of $\sum p_T$ are shown in Fig. 17 for cone number 6 in few selected $\eta$-bins. The thresholds assigned to the set of predefined cones are shown in Fig. 18 and 19 for a few different nominal efficiency values. The relative difference between high and low luminosity is smaller than in the case of calorimeter isolation, due to the requirement that the contributing tracks come from the same primary vertex.

The efficiencies for reference signal events are shown in Fig. 20. For small cones, the lack of tracks in the muon neighbourhood makes the determination of a threshold impossible for any nominal efficiency, and the curves for different nominal efficiencies overlap. This effect is more pronounced here than in the case of calorimeter isolation.

Following the procedure already described for calorimeter isolation, the computed thresholds are used to determine the best cone for each nominal efficiency value (Fig. 21). The results are summarised in Table 5.

Table 5: Optimal cones for pixel isolation as a function of the nominal efficiency. Numbers correspond to the cones defined in Table 2.

<table>
<thead>
<tr>
<th>nominal efficiency</th>
<th>0.75</th>
<th>0.80</th>
<th>0.85</th>
<th>0.90</th>
<th>0.95</th>
<th>0.97</th>
<th>0.98</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>The best cone for $2 \times 10^{23} cm^{-2} s^{-1}$</td>
<td>11</td>
<td>10</td>
<td>9</td>
<td>9</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>The best cone for $10^{24} cm^{-2} s^{-1}$</td>
<td>10</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

In Fig. 22 and 23 the resulting efficiencies for the reference signal sample are shown as a function of the pseudorapidity and $p_T$ of the generated muon. As discussed earlier, the algorithm ensures by construction that the resulting efficiency is above the nominal efficiency and that it is flat in $\eta$. The efficiency is also quite flat as a function of $p_T$ except for a small efficiency drop at high luminosity for low nominal efficiency values and low-$p_T$ muons.

The efficiency for the reference background (Fig. 24) is quite flat as a function of pseudorapidity, except for an increase at high $\eta$ values. The reason is that the pixel reconstruction requires the presence of three pixel hits for each track, while very forward tracks do not cross all three pixel layers when their vertex is displaced from the detector centre. As for calorimeter isolation, and for the same reason, the background efficiency depends strongly on the $p_T$ of the muon, as shown in Fig. 25.

The background efficiency obtained as a function of the signal efficiency is shown in Fig. 26.

### 3.3.2 Pixel Isolation with Level-2 Muons

The pixel reconstruction algorithm can also be used at L2, using the L2 muon to define the cone axis. However the L2 muon reconstruction does not provide a precise enough measurement of the muon vertex (see Fig. 27), to be used to select the pixel tracks that contribute to the cone.

An estimate for the primary vertex of the “most interesting event” in the bunch crossing is given by the pixel reconstruction algorithm itself. In Fig. 28 the distance along the beam axis between the Monte Carlo primary vertex of the event containing the triggering muon and the vertex reconstructed by the pixel reconstruction algorithm on minimum bias samples is shown for the minimum bias sample. The peak at zero corresponds to correct associations. However, the association efficiency is not satisfactory (even at low luminosity), showing that this estimate cannot be used effectively. Another possibility could be to estimate the position of the muon vertex using the pixel track closest to the muon. In Fig. 29 the $\Delta R$ distance between the direction of the L2 muon and the direction of the closest pixel track, obtained from the minimum bias sample, is presented. This distance is rather large ($\Delta R \leq 0.2$) due to the limited resolution of the L2 muon. Even though the pixel track found using this procedure usually points to the correct primary vertex, the efficiency of the association is again too low. We therefore conclude that it not possible to use efficiently the primary vertex constraint on the tracks contributing to the cone for L2 pixel isolation.

The optimisation procedure for pixel isolation with L2 muon differs therefore from what is described in the previous section because of the lack of the primary vertex constraint and because the pixel muon is searched in a much
wider $\Delta R$ around the direction of the L2 muon. The results are shown in Fig. 30. The background rejection at low luminosity is comparable with what obtained with pixel isolation at L3. However, at high luminosity the rejection is smaller due to the missing vertex constraint. This algorithm will not be discussed further in the following.

### 3.4 Tracker Isolation

The tracker isolation algorithm uses the $\sum p_T$ of fully reconstructed tracks in a cone around the direction of the L3 muon, neglecting the contribution from the muon itself.

Tracks are reconstructed using regional tracking, i.e. track seeds are created using pairs of pixel hits in a region of interest defined by a vertex constraint, by constraints on the track direction at the vertex and by the minimum transverse momentum for the tracks to be reconstructed. The direction of the tracks is constrained to be in a region of width $\Delta \eta \times \Delta \phi$ (determined by the size of isolation cone) around the cone axis, defined as the direction of the L3 muon. The vertex constraint is specified by the radius $r$ and the half-width $\Delta z$ of the cylinder around the beam line which contains the impact point of the L3 muon. The transverse and longitudinal displacement of the L3 muon vertex from the primary vertex is shown in Fig. 31, which allows to choose the values $r = 0.1 \text{ cm}$ and $\Delta z = 0.2 \text{ cm}$.

Another important parameter is the minimal transverse momentum required for the tracks contributing to the isolation cone. The optimal value for this parameter has been found to be $0.8 \text{ GeV}/c$ (see below). Since very high quality reconstruction of the tracks contribution to the isolation cone is not essential, the track fitting was stopped as soon as five hits were included in the fit. This allows a significant speedup of the algorithm. However, in the case of low-$p_T$ particles there is some probability of reconstructing ghost tracks. In order to reject them the value of the $\chi^2$ of the track fit was required to be less than 8 for the tracks with only two pixel hits supplemented by three silicon strip hits. This requirement is not necessary if one more hit is used in the track fit.

Once tracks are reconstructed, the same algorithm used for the pixel isolation with L3 muon is used. Typical spectra of $\sum p_T$ are shown in Fig. 32 for cone number 6 in a few selected $\eta$-bins. The thresholds assigned to the set of predefined cones are shown in Fig. 33 and 34 for a few different nominal efficiency values. The thresholds are similar to those used by the L3 pixel isolation algorithm.

The efficiencies for reference signal events are shown in Fig. 35. As for the case of pixel isolation, for small cones the lack of tracks in the muon neighbourhood makes the determination of a threshold impossible for any nominal efficiency, and the curves overlap.

Following the procedure already described, the computed thresholds are used to determine the best cone for each nominal efficiency value (Fig. 36). The results are summarised in Table 6.

<table>
<thead>
<tr>
<th>nominal efficiency</th>
<th>0.75</th>
<th>0.80</th>
<th>0.85</th>
<th>0.90</th>
<th>0.95</th>
<th>0.97</th>
<th>0.98</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \times 10^{33} \text{cm}^{-2}s^{-1}$</td>
<td>11</td>
<td>10</td>
<td>9</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$10^{34} \text{cm}^{-2}s^{-1}$</td>
<td>10</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

In Fig. 37 and 38 the resulting efficiencies for the reference signal sample are shown as a function of the pseudorapidity and $p_T$ of the generated muon. The algorithm ensures by construction that the resulting efficiency is above the nominal efficiency and that it is flat in $\eta$. As for the case of the L3 pixel isolation algorithm, the efficiency is also quite flat as a function of $p_T$.

The efficiency for the reference background (Fig. 39) is more flat as a function of pseudorapidity than in the case of the pixel isolation algorithm. The reason is that with full tracker reconstruction tracks can be reconstructed with only two pixel hits, while in the case of pixel reconstruction three hits out of the three pixel layers are necessary.

The background efficiency as the function of the $p_T$ of the muon is shown in Fig. 40. The summary of the algorithm performance, i.e. the background efficiency obtained as a function of the signal efficiency, is shown in Fig. 41.

As already mentioned, the performance of this algorithm is influenced by the cut on the $p_T$ of tracks contributing to the isolation cone. The effect of varying this cut is shown in Fig. 42, where the efficiency for the reference signal is reported as a function of the minimal $p_T$ of contributing tracks for a few different values of nominal efficiency. This plot was obtained repeating the threshold optimisation for each of the $p_T$ cut values used. The dependence is
not very strong, nevertheless the best performance is obtained for $p_T$ cuts in the range 0.7–0.9 GeV/c. Hence, the value 0.8 GeV/c was chosen.

4 Overall Performance

The performance of the isolation algorithms in terms of inclusive trigger rate reduction, efficiency on benchmark signal channels and timing is presented in the following.

As a general remark, the performance of isolation algorithms is affected by the luminosity. The presence of pile-up forces the thresholds to be higher for the same nominal efficiency; this affects mostly the calorimeter isolation, while for tracking algorithms it is reduced by the requirement that contributing tracks come from the same primary vertex as the muon.

4.1 Rate Reduction

The effect of isolation algorithms on L2 and L3 inclusive trigger rates is shown in Fig. 43. The L2 rate is above the generator rate for any trigger threshold, due to the feed-through of low-$p_T$ muons which contaminate the L2 $p_T$ spectrum [5]. As already mentioned, isolation algorithms are not effective against these muons; therefore the L2 calorimeter isolation algorithm gives only a small reduction of the inclusive L2 rate. The L3 rate is not affected by low-$p_T$ contamination, and L3 isolation algorithms show a significant rejection.

While the application of several isolation algorithms in sequence provides some additional rate reduction, the overall rejection is lower than the product of the rejection factors obtained with individual algorithms, since the quantities measured by the different isolation algorithms are not independent. In particular, no significant gain is given by the combined use of both the pixel and tracker isolation algorithms at L3. Hence, if the CPU timing is critical a good compromise is to apply the calorimeter isolation algorithm at L2 and either the pixel or tracker isolation algorithms at L3.

The overall rate rejection is significant for thresholds below 30 GeV/c. Above that value the rate is dominated by isolated muons coming from $W$ decays, as shown in Fig. 44, where the contributions of the different sources of muons to the L3 rate at high luminosity before and after the application of the isolation algorithms is presented. It can be noted that isolation algorithms strongly suppress the contributions from $b$, $c$, $K$, and $\pi$ decays.

4.2 Signal Efficiency

The efficiency of each isolation algorithm on the reference signal is by construction equal to the nominal efficiency. To study the correlation between the signal efficiencies, the algorithms were applied in sequence, setting the nominal efficiency of each to 97%. The total efficiency on the reference signal is given in Table 7. Since tracker and pixel algorithms exploit the presence of the same tracks around the muon, their signal inefficiencies are rather correlated. However the signal inefficiency of calorimeter isolation is largely uncorrelated to those of the pixel and tracker algorithms. This can be explained by the fact that the sources of inefficiency in the calorimeter and tracker algorithms are different. For example, pile-up in the calorimeters can come from an interaction vertex far from the one producing the muon, so that it does not affect the region around the muon in the tracking system. Noise and ghosts, which affect the calorimeter and tracker algorithms respectively, are also independent.

Table 7: Total efficiency of combinations of isolation algorithms on the reference signal. Algorithms included are L2 calorimeter isolation (“Calo”), L3 pixel isolation (“Pixel”) and L3 tracker isolation (“Tracker”), all set to 97% nominal efficiency.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Calo + Pixel</th>
<th>Calo + Tracker</th>
<th>Pixel + Tracker</th>
<th>Calo + Pixel + Tracker</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$ (Low luminosity)</td>
<td>0.947</td>
<td>0.947</td>
<td>0.960</td>
<td>0.937</td>
</tr>
<tr>
<td>$\varepsilon$ (High luminosity)</td>
<td>0.946</td>
<td>0.946</td>
<td>0.960</td>
<td>0.935</td>
</tr>
</tbody>
</table>

Finally, the efficiency of isolation algorithms has been tested on few benchmark channels simulated at high luminosities (see Section 2). A realistic trigger chain, consisting in both a single muon and a symmetric di-muon selection, was simulated. The acceptance of the muon trigger was limited to $|\eta| < 2.1$ to account for the missing trigger electronics in the forward CSC station ME1/1a. The L1 $p_T$ thresholds were 20 GeV/c for the single muon and 5 GeV/c for the di-muon trigger. Higher $p_T$ thresholds were used in the HLT selection, i.e. 31 GeV/c for
the single muon and 10 GeV/c for the di-muon trigger. Calorimeter isolation was applied at L2, and pixel and tracker isolation were applied at L3, setting the nominal efficiency of all algorithms to 97%. In the case of di-muon triggers, only one of the two muons was required to pass the isolation requirements. The efficiency of the isolation algorithms for the different channels is reported in Table 8.

Table 8: Efficiency of isolation algorithms in the HLT selection of some benchmark channels at high luminosity. The last column refers to the cumulative efficiency obtained applying the three isolation algorithms together. In the samples $t\bar{t}\rightarrow\mu+X$ and $H\rightarrow\tau\tau\rightarrow\mu+X$, only events where the muon is produced respectively by a direct $W$ decay and a direct $\tau$ decay inside the detector acceptance are considered. The statistical error on such numbers is of the order of $0.002 \sim 0.006$, depending on the sample.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Calorimeter</th>
<th>Pixel</th>
<th>Tracker</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W\rightarrow\mu\nu$</td>
<td>0.971</td>
<td>0.972</td>
<td>0.973</td>
<td>0.938</td>
</tr>
<tr>
<td>$Z/\gamma^*\rightarrow\mu\mu$</td>
<td>0.973</td>
<td>0.974</td>
<td>0.981</td>
<td>0.942</td>
</tr>
<tr>
<td>$t\bar{t}\rightarrow\mu+X$</td>
<td>0.913</td>
<td>0.938</td>
<td>0.934</td>
<td>0.875</td>
</tr>
<tr>
<td>$H\rightarrow WW\rightarrow 2\mu 2\nu$</td>
<td>0.964</td>
<td>0.964</td>
<td>0.955</td>
<td>0.920</td>
</tr>
<tr>
<td>$H\rightarrow\tau\tau\rightarrow\mu+X$</td>
<td>0.964</td>
<td>0.986</td>
<td>0.981</td>
<td>0.951</td>
</tr>
</tbody>
</table>

4.3 Algorithm Timing

The time available for running HLT algorithms in the on-line trigger is limited. The CPU processing time is therefore a relevant parameter for the feasibility of algorithms to be included in the trigger chain.

The CPU time spent by the isolation algorithms was measured simulating the full HLT chain on a subsample of events from the minimum bias and $W$ samples with proper composition and muon $p_T$ spectrum. At each trigger level, single muon trigger thresholds of 10 and 18 GeV/c were assumed for high and low luminosity, respectively. The measurement was performed on a commercial computer equipped with an Intel Pentium III processor running at 1 GHz in a controlled environment. The standard CMS software for ORCA 6.2.3 was used.

The average CPU time spent by the three algorithms is shown in Table 9. The time given for calorimeter isolation does not include the time spent in the GEANE propagation to calorimeters (see Sec. 3.2), since in the on-line selection this information will be available from the L2 muon track fit. The time spent in reconstructing HCAL and ECAL towers in the simulation is also not included.

Table 9: CPU time used by isolation algorithms tuned to 97% nominal efficiency. The time reported for calorimeter isolation does not include the time spent in the GEANE propagation to calorimeters.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Calorimeter isol. (ms/L2 event)</th>
<th>Pixel isolation (ms/L3 event)</th>
<th>Tracker isolation (ms/L3 event)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low luminosity</td>
<td>25</td>
<td>65</td>
<td>190</td>
</tr>
<tr>
<td>High luminosity</td>
<td>40</td>
<td>320</td>
<td>370</td>
</tr>
</tbody>
</table>

Considering the projected improvement of the CPU speed before the LHC startup, these timings appear adequate in order to be included in the High Level Trigger chain.

4.4 Discussion

The algorithms presented in this note are optimised using a sophisticate procedure, whose goal is both to get the maximal performance and to guarantee a flat signal efficiency as a function of pseudorapidity. It is however interesting to compare the results obtained with this optimisation with the performance of a simpler algorithm. As an example, the L3 pixel isolation algorithm has been compared with a simple algorithm based on the analysis of the number of tracks in the cone around the muon. Algorithms requiring no track, no more than one track, or no more than two tracks in a cone with a size in the range $\Delta R_{MAX} = 0.09 - 0.6$ have been implemented. Only pixel tracks coming from a primary vertex reconstructed by the pixel reconstruction algorithm closer than 0.2 cm to the L3 muon vertex are taken into account. The tracks in a small cone of $\Delta R < 0.015$ around the muon are excluded.

4) Linux kernel ver. 2.2.19; code compiled with gcc ver. 2.95.2 and optimisation -O2.
to remove the contribution from the muon itself. The performance of such algorithms, in terms of background versus signal efficiency, is shown in Fig. 45, which demonstrates that the L3 pixel isolation algorithm always give a better rejection for the same signal efficiency.

5 Conclusions

Three algorithms to reject non-isolated muons are presented, based on reconstructed data from the calorimeters, the pixel detector and the full tracker. The performance of the algorithms is tunable using a parameter called “nominal efficiency”, which represents the expected efficiency on the reference signal sample, and are designed to provide flat signal efficiency as a function of pseudorapidity. It is shown that these algorithms give a considerable rate reduction in the High Level trigger, while keeping very high efficiency for isolated muons from signal events.

Acknowledgements

We are grateful to the CMS Production Team for the effort in producing the simulated data used for this study. We would also like to thank S. Arcelli and A. Fanfani for the useful discussions on issues related to this work, and M. Arneodo and D. Acosta for their valuable comments on this note.

References

[9] CERN program library long writeup W5013, GEANT3 version 3.21/13 (release 15111999) Detector Description and Simulation Tool.
Figure 1: Generated muon rate as a function of the $p_T$ threshold for a luminosity of $10^{34} \text{cm}^{-2}\text{s}^{-1}$. The separate contributions from $K/\pi$, $b/c$, $W$ and $Z$ decays are shown. (Source: [5]).

Figure 2: The $p_T$ spectrum of muons from direct $W$ decays.

Figure 3: Schematic illustration of the isolation cone. In the case of ECAL and for pixel and tracker algorithms the “cone axis” is equal to the muon direction at vertex. In the case of HCAL the cone axis it is the centre of the tower pointed by the muon direction at vertex. The “veto value” is used to subtract the contribution of the muon to the detector measurement in the cone.
Figure 4: $\Delta R$ distance between the position of the Monte Carlo muon hit in ECAL and HCAL and the extrapolated position of the muon to the ECAL/HCAL boundary.

Figure 5: Example of the spectra of $E_T^{WEIGHT}$ in the calorimeters for cone 6 for the reference signal sample at high luminosity in $\eta$-bins 1, 8, 16 and 24 (a). For comparison the distribution of $E_T^{WEIGHT}$ is shown for the reference background sample (cone 6, $\eta$-bin 8) (b).
Figure 6: Threshold values of the calorimeter isolation algorithm at low luminosity for cone 6 and nominal efficiencies 0.8, 0.9, 0.97 and 0.98.

Figure 7: Threshold values of the calorimeter isolation algorithm at high luminosity for cone 6 and nominal efficiencies 0.8, 0.9, 0.97 and 0.98.
Figure 8: Reference signal efficiency of the calorimeter isolation algorithm for different cone sizes and nominal efficiency values, at low (a) and high (b) luminosity.

Figure 9: Efficiency of the calorimeter isolation algorithm for the reference background sample for different cone sizes and nominal efficiency values, at low (a) and high (b) luminosity. The minimum of each curve indicates the optimal cone for the corresponding nominal efficiency.
Figure 10: Signal efficiency of the calorimeter isolation algorithm as a function of the $p_T$ of the muon for different nominal efficiency values, at low (a) and high (b) luminosity. The nominal efficiencies are obtained convoluting these curves with the actual $p_T$ spectra of the reference signal.

Figure 11: Reference signal efficiency of the calorimeter isolation algorithm as a function of the pseudorapidity of the muon for different nominal efficiency values, at low (a) and high (b) luminosity.
Figure 12: Efficiency of the calorimeter isolation algorithm for the reference background sample as a function of the pseudorapidity of the muon for different nominal efficiency values, at low (a) and high (b) luminosity.

Figure 13: Background efficiency of the calorimeter isolation algorithm for the full sample of minimum bias events as a function of the $p_T$ of the muon and for different nominal efficiency values, at low (a) and high (b) luminosity.
Figure 14: Efficiency of the calorimeter isolation algorithm for minimum bias events as a function of the efficiency on direct $W \rightarrow \mu \nu$ decays at low (a) and high (b) luminosity. The different curves correspond to different cuts on the $p_T$ of the generated muon; the intermediate one corresponds to the definition of the reference signal and background samples.

Figure 15: Efficiency of the calorimeter isolation algorithm on the reference background sample as a function of the ECAL-HCAL weighting parameter $\alpha$ at low (a) and high (b) luminosity.
Figure 16: Identification of the “pixel muon” at L3, for the reference background at high luminosity. (a) $\Delta R$ distance between the direction of the L3 muon and the direction of the closest pixel track. (b) Distance along the beam line between the vertex associated to the pixel muon and the L3 muon vertex.

Figure 17: Example of the spectra of $\sum p_T$ of pixel tracks in cone 6 for the reference signal sample at high luminosity in $\eta$-bins 1, 8, 16 and 24 (a). For comparison the distribution of $\sum p_T$ is shown for the reference background sample (cone 6, $\eta$-bin 8) (b).
Figure 18: Threshold values of the L3 pixel isolation algorithm at low luminosity for cone 6 and nominal efficiencies 0.8, 0.9, 0.97 and 0.98.

Figure 19: Threshold values of the L3 pixel isolation algorithm at high luminosity for cone 6 and nominal efficiencies 0.8, 0.9, 0.97 and 0.98.
Figure 20: Reference signal efficiency of the L3 pixel isolation algorithm for different cone sizes and nominal efficiency values, at low (a) and high (b) luminosity.

Figure 21: Efficiency of the L3 pixel isolation algorithm for the reference background sample for different cone sizes and nominal efficiency values, at low (a) and high (b) luminosity. The minimum of each curve indicates the optimal cone for the corresponding nominal efficiency.
Figure 22: Signal efficiency of the L3 pixel isolation algorithm as a function of the $p_T$ of the muon for different nominal efficiency values, at low (a) and high (b) luminosity. The nominal efficiencies are obtained convoluting these curves with the actual $p_T$ spectra of the reference signal.

Figure 23: Reference signal efficiency of the L3 pixel isolation algorithm as a function of the pseudorapidity of the muon for different nominal efficiency values, at low (a) and high (b) luminosity.
Figure 24: Efficiency of the L3 pixel isolation algorithm for the reference background sample as a function of the pseudorapidity of the muon for different nominal efficiency values, at low (a) and high (b) luminosity.

Figure 25: Background efficiency of the L3 pixel isolation algorithm for the full sample of minimum bias events as a function of the $p_T$ of the muon and for different nominal efficiency values, at low (a) and high (b) luminosity.
Figure 26: Efficiency of the L3 pixel isolation algorithm for minimum bias events as a function of the efficiency on direct $W \rightarrow \mu \nu$ decays at low (a) and high (b) luminosity. The different curves correspond to different cuts on the $p_T$ of the generated muon; the intermediate one corresponds to the definition of the reference signal and background samples.

Figure 27: Distance along the beam line between the Monte Carlo primary vertex of the interaction containing the muon and the L2 muon vertex (reference background sample at high luminosity).
Figure 28: Reconstruction of the primary vertex by the pixel reconstruction algorithm for minimum bias events at high luminosity. (a) Distance along the beam line between the Monte Carlo primary vertex of the interaction containing the muon and the primary vertex reconstructed using the pixel reconstruction algorithm. (b) Position of the Monte Carlo primary vertex of the interaction containing the muon in the list of vertices reconstructed by the pixel reconstruction algorithm, which is sorted by the total $p_T$ of the tracks belonging to each vertex.

Figure 29: (a) $\Delta R$ distance between the direction of the L2 muon and the direction of the closest pixel track at high luminosity (minimum bias sample at high luminosity). (b) Distance along the beam line between the pixel track closest in $\Delta R$ to the L2 muon and the Monte Carlo primary vertex (reference signal at high luminosity).
Figure 30: Efficiency of the L2 pixel isolation algorithm for minimum bias events as a function of the efficiency on the reference signal at low (a) and high (b) luminosity.
Figure 31: Distance of the reconstructed L3 muon from the Monte Carlo primary vertex (reference background at high luminosity): Distance along the beam line (a) and transverse impact parameter (b).

Figure 32: Example of the spectra of $\sum p_T$ of tracker tracks in cone 6 for the reference signal sample at high luminosity in $\eta$-bins 1, 8, 16 and 24 (a). For comparison the distribution of $\sum p_T$ is shown for the reference background sample (cone 6, $\eta$-bin 8) (b).
Figure 33: Threshold values of the L3 tracker isolation algorithm at low luminosity for cone 6 and nominal efficiencies 0.8, 0.9, 0.97 and 0.98.

Figure 34: Threshold values of the L3 tracker isolation algorithm at high luminosity for cone 6 and nominal efficiencies 0.8, 0.9, 0.97 and 0.98.
Figure 35: Reference signal efficiency of the L3 tracker isolation algorithm for different cone sizes and nominal efficiency values, at low (a) and high (b) luminosity.

Figure 36: Efficiency of the L3 tracker isolation algorithm for the reference background sample for different cone sizes and nominal efficiency values, at low (a) and high (b) luminosity. The minimum of each curve indicates the optimal cone for the corresponding nominal efficiency.
Figure 37: Signal efficiency of the L3 tracker isolation algorithm as a function of the $p_T$ of the muon for different nominal efficiency values, at low (a) and high (b) luminosity. The nominal efficiencies are obtained convoluting these curves with the actual $p_T$ spectra of the reference signal.

Figure 38: Reference signal efficiency of the L3 tracker isolation algorithm as a function of the pseudorapidity of the muon for different nominal efficiency values, at low (a) and high (b) luminosity.
Figure 39: Efficiency of the L3 tracker isolation algorithm for the reference background sample as a function of the pseudorapidity of the muon for different nominal efficiency values, at low (a) and high (b) luminosity.

Figure 40: Background efficiency of the L3 tracker isolation algorithm for the full sample of minimum bias events as a function of the $p_T$ of the muon and for different nominal efficiency values, at low (a) and high (b) luminosity.
Figure 41: Efficiency of the L3 tracker isolation algorithm for minimum bias events as a function of the efficiency on direct $W \rightarrow \mu \nu$ decays at low (a) and high (b) luminosity. The different curves correspond to different cuts on the $p_T$ of the generated muon; the intermediate one corresponds to the definition of the reference signal and background samples.

Figure 42: Efficiency of the tracker isolation algorithm for the reference signal as a function of the minimal $p_T$ of the tracks contributing to the cone, for a few different values of nominal efficiency.
Figure 43: Muon rates at low luminosity (a) and high luminosity (b) at the different HLT steps, as a function of the \( p_T \) threshold. (Source: N. Neumeister, DAQ TDR).

Figure 44: Contributions of the different sources of muons to the L3 rate at high luminosity before (left) and after (right) the application of the isolation algorithms. (Source: N. Neumeister, DAQ TDR).
Figure 45: Comparison of the L3 pixel isolation algorithm with a simple algorithm based on the requirement of no track, no more than one track, or no more than two tracks in in a cone of size $\Delta R_{MAX} = 0.09 - 0.6$ around the muon.