Expected Magnetic Field Quality of the LHC Septum Magnets used for Injection (MSI) and for Extraction to the Beam Dump (MSD)

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Abstract

The two-dimensional magnetic field of the LHC steel septum magnets has been evaluated with the computer code FLUX2D. The results of this analysis are compiled in this note. In the case of the injection septum magnet MSI the field is computed for nominal excitation ($I = 950\,\text{A}$), whereas the field of the extraction septum magnet MSD corresponds to its maximum excitation ($I = 880\,\text{A}$). An estimate of the effect of the Permalloy™ shielding, required to screen the circulating beam from the stray field in the septum hole, and of the mechanical tolerances on the pole geometry on the field in the gap are obtained analytically. The results in this revised note refer to the final design of the septum magnets (optimised shims, reduced septum thickness of the MSD), whereas those in the original note correspond to the preliminary design.

1. The Model

1.1 Magnet Version

The septum magnets considered in this note correspond to those described in [1]. The injection septum magnet assemblies MSI consist of five 4 m long modules; the two closer to the injection point are of MSIA type whereas the other three are of MSIB type [2].

The two extraction septum magnet assemblies MSD are composed each of fifteen 4.46 m long modules. These are of three different types: 5 modules of type MSDA, MSDB and MSDC. A drawing (reduced size) of the cross-section of each different module MSIA, MSIB, MSDA, MSDB and MSDC is attached in the Appendix (page 30 ff).

1.2 Model for FLUX2D

The five different modules were modelled with the finite element software FLUX2D [3] by admitting the following simplifications: the coils were represented by one single square copper region without cooling holes and the shape of the outside boundary was assumed to be a rectangular square to which the Dirichlet condition was applied (i.e. no stray flux outside the magnet). The number of elements (nodes) into which the problem was subdivided has been chosen such as to guarantee a sufficiently high precision (Table 1).
1.3 Russian Steel “21848”

The steel proposed by the manufacturer (IHEP, Protvino) to meet our requirements [4] is a low carbon steel (C<0.2‰, Si<3‰, Mn<3‰) with a coercive force $H_c<48$ A/m and a maximum relative permeability $\mu = 4800$. It is called “Steel 21848” and its magnetic properties are summarised in Figure 1.

For field strengths in the iron above about 7 Oersted, the permeability $\mu$ and the field strength $H$ [Oe] can be related by the following analytical formula, whose constants are determined by linear regression (see also $\mu(H)$-curve in Figure 1):

$$\mu^{1.101726}H = 30071.9477 \text{ Oersted}$$

(1)

This expression allows to extrapolate the B(H)-curve beyond the measured values. The saturation curve describing the material properties in the model is shown in Figure 2. Derived from the measured points (indicated by a small square) and based on what in the manual they call “third order splines”, it is constructed from the module CSLMAT of the software package FLUX2D.

2. Required good field region

The five 4 m long modules of the MSI, all separated by 45 cm from each other, occupy a total installation length of 21.8 m. The 3 upstream modules of type MSIB are installed at a distance between 8.9 m and 21.8 m with respect to the exit of the MSI, whereas the 2 downstream modules of type MSIA occupy the last 8.45 m of space. The nominal horizontal position (deflection) of the beam to be injected with respect to the circulating beam along the MSI is shown in Figure 3. It can be seen, that the “good field region” for the centre particle has to extend over a range between 0 mm and 15.6 mm for the downstream MSIA modules, and between about 17 mm and 119 mm for the upstream MSIB modules. Additional clearance representing the required number of beam sigmas must of course be added to these figures.

By analogy, the nominal vertical position (deflection) of the extracted beam with respect to the circulating beam along the MSD is plotted in Figure 4. The fifteen 4.46 m long modules are installed over a total length of 73.2 m, allowing for a 45 cm free space between the individual modules. The required range for the good field region lies between 0 mm and 8 mm for the upstream MSDA modules occupying the space between 0 m and 24.1 m, between 8 mm and 34 mm for the centre MSDB modules located between 24.55 m and 48.65 m, and between 34 mm and 80 mm for the downstream MSDC modules installed between 49.1 m and 73.2 m after the entrance of the MSD.

<table>
<thead>
<tr>
<th>Module</th>
<th>Length of module [m]</th>
<th>Space between modules [m]</th>
<th>Installation range [m]</th>
<th>Min/max deflection [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSIA</td>
<td>4</td>
<td>0.45</td>
<td>[-8.45 ; 0]</td>
<td>[0 ; 15.6]</td>
</tr>
<tr>
<td>MSIB</td>
<td>4</td>
<td>0.45</td>
<td>[-21.8 ; -8.9]</td>
<td>[17 ; 119]</td>
</tr>
<tr>
<td>MSDA</td>
<td>4.46</td>
<td>0.45</td>
<td>[0 ; 24.1]</td>
<td>[0 ; 8]</td>
</tr>
<tr>
<td>MSDB</td>
<td>4.46</td>
<td>0.45</td>
<td>[24.55 ; 48.65]</td>
<td>[8 ; 34]</td>
</tr>
<tr>
<td>MSDC</td>
<td>4.46</td>
<td>0.45</td>
<td>[49.1 ; 73.2]</td>
<td>[34 ; 80]</td>
</tr>
</tbody>
</table>

Table 2: Minimum range for the “good field region” in the different modules
3. Flux distribution and field in the septum hole and 2nd beam hole

The flux distributions in the iron and in the gap of the two modules MSIA and MSIB are plotted in Figure 5 and Figure 7. The flux intercepted by the septum hole and the 2nd beam hole is illustrated by the flux lines which are shown in Figure 6 and Figure 8 respectively. These flux lines correspond to the nominal excitation current for the MSI of 950 A. The values of the flux per unit length are summarised in Table 3 below.

Taking into account that all modules of the extraction septum magnet MSD are symmetrical with respect to the mid-plane, the flux distributions were computed only for the upper half of each module. The flux lines which correspond to the maximum excitation current of 880 A are plotted in Figure 9 to Figure 11. However, the values of the flux per unit length, produced by this current and which are listed in Table 3, correspond to the entire magnet and not only its upper half.

<table>
<thead>
<tr>
<th>Number of windings N</th>
<th>MSIA</th>
<th>MSIB</th>
<th>MSDA</th>
<th>MSDB</th>
<th>MSDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation current I [A]</td>
<td>950</td>
<td>24</td>
<td>32</td>
<td>40</td>
<td>48</td>
</tr>
<tr>
<td>Computed total flux $\Phi_{\text{tot}}$ [Wb/m]</td>
<td>0.248544</td>
<td>0.355307</td>
<td>0.321116</td>
<td>0.391438</td>
<td>0.44902</td>
</tr>
<tr>
<td>Flux through septum $\Phi_s$ [Wb/m]</td>
<td>$\sim 4 \times 10^{-5}$</td>
<td>$\sim 9 \times 10^{-5}$</td>
<td>$\sim 4 \times 10^{-5}$</td>
<td>$\sim 5 \times 10^{-5}$</td>
<td>$\sim 8 \times 10^{-5}$</td>
</tr>
<tr>
<td>Flux through 2nd hole $\Phi_H$ [Wb/m]</td>
<td>$\sim 2 \times 10^{-5}$</td>
<td>$\sim 7 \times 10^{-5}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Flux distribution in the different modules of the MSI and MSD

The field inside the cylindrical septum and 2nd beam hole (MSI) is best described by its harmonic content [5]. Using complex notation, it can be expressed as an infinite power series at any location $z = x + i \cdot y$ inside these holes:

$$\overline{B}(z) = \text{conj}(B(z)) = B_x(z) - i \cdot B_y(z) = -i \sum_{n=1}^{\infty} n \cdot c_n(z - z_0)^{n-1}$$  \hspace{1cm} (2)

The complex coefficients $c_n$ are the multipole coefficients of order $n$ with respect to the centre $z_0$ of the hole. The first 10 of these multipole coefficients are listed in Table 4 (amplitudes and phases) for the MSI, whereas those for the MSD are summarised in Table 5.

<table>
<thead>
<tr>
<th>MSIA</th>
<th>MSIB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_n$ [T/m$^{n-1}$]</td>
<td>$\angle c_n$ [°]</td>
</tr>
<tr>
<td>n</td>
<td>1</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>4.9017$\times 10^{-4}$</td>
</tr>
<tr>
<td>2</td>
<td>5.5742$\times 10^{-3}$</td>
</tr>
<tr>
<td>3</td>
<td>7.0529$\times 10^{-2}$</td>
</tr>
<tr>
<td>4</td>
<td>9.5596$\times 10^{-1}$</td>
</tr>
<tr>
<td>5</td>
<td>1.5495$\times 10^{1}$</td>
</tr>
<tr>
<td>6</td>
<td>3.4495$\times 10^{2}$</td>
</tr>
<tr>
<td>7</td>
<td>8.6797$\times 10^{2}$</td>
</tr>
<tr>
<td>8</td>
<td>1.9986$\times 10^{3}$</td>
</tr>
<tr>
<td>9</td>
<td>4.1808$\times 10^{3}$</td>
</tr>
<tr>
<td>10</td>
<td>8.3209$\times 10^{3}$</td>
</tr>
</tbody>
</table>

Table 4: Multipole coefficients of the unshielded field in the septum and 2nd beam hole of the MSI
Table 5: Multipole coefficients of the unshielded field in the septum hole of the MSD

<table>
<thead>
<tr>
<th>n</th>
<th>c_n [T/m^n]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>i·5.204316E·10^{-4}</td>
</tr>
<tr>
<td>2</td>
<td>-i·6.794307E·10^{-3}</td>
</tr>
<tr>
<td>3</td>
<td>i·8.334089E·10^{-2}</td>
</tr>
<tr>
<td>4</td>
<td>-i·8.165534E·10^{-1}</td>
</tr>
<tr>
<td>5</td>
<td>-i·1.236052E·10^{0}</td>
</tr>
<tr>
<td>6</td>
<td>i·2.942517E·10^{2}</td>
</tr>
<tr>
<td>7</td>
<td>-i·9.07551E·10^{3}</td>
</tr>
<tr>
<td>8</td>
<td>i·2.573914E·10^{4}</td>
</tr>
<tr>
<td>9</td>
<td>-i·5.198916E·10^{5}</td>
</tr>
<tr>
<td>10</td>
<td>i·7.032297E·10^{7}</td>
</tr>
</tbody>
</table>

The multipole coefficients of the field in these holes were obtained by FFT of the computed vector potential on a concentric circle of 28 mm radius as described in [5]. The absolute value of the magnetic induction on these circles is plotted in Figure 12 to Figure 16. Near the iron (r = 32 mm), the maximum values are roughly 20% higher and the minimum values are slightly lower (see Table 6).

In the presence of a cylindrical 1mm thick copper vacuum chamber which is surrounded with a Permalloy™ shielding of 0.9 mm thickness, the flux through these holes increases by a factor of approximately 1.5 – 2.5 (MSI) and 3.6 (MSD) respectively. The difference of these flux enhancement factors is due to the different (inner and outer) diameters of the vacuum chambers, which are Ø = 40/44 mm in the case of the MSI and Ø = 46/50 mm for the MSD (see Table 8 in chapter 6).

Assuming that this increased flux is equally split into the two half cylinders of the shielding, an approximate value for the maximum flux density in the Permalloy™ shielding can then easily be estimated from the numbers listed in Table 3. Since this shielding is far from being saturated, the magnetic field inside the shielded vacuum chamber should not exceed a few 10^{-5} T.

4. Dipole field in the gap

The relative difference between an ideal dipole field B_0 and the computed field B = B_x + i·B_y on the mid-plane of the gap and on two parallel planes at ± 5 mm distance is plotted in Figure 17 to Figure 21.

For the MSI, the co-ordinate system is chosen such that the x-axis is in the middle of the 25 mm gap and parallel to the pole faces. Its origin x = 0 corresponds to the x-location of the centre of the septum.
Its orientation is positive in the direction of the injected beam. The normalising field $B_0$ corresponds to the $B_y$ component of the B-field at $x = y = 0$.

In the case of the MSD, the origin of the co-ordinate system coincides with the centre of the 44 mm wide gap and the x axis corresponds to the symmetry axis of the MSD. The normalising field $B_0$ corresponds to the field in the centre of the gap.

The maximum field error within the “good field region” is listed in Table 7 below, in which $B_{\|}$ refers to the field component parallel to the direction of the desired dipole field and $B_{\perp}$ to the one perpendicular to it.

In the range of possible beam positions in the gap of the MSI, the variation of these field components amounts to less than $\pm 0.12 \%e$ for the MSIA and, with respect to the mean dipole field, less than $\pm 0.6 \%e$ for the MSIB.

In the case of the MSD, these variations are less than about $\pm 0.08 \%e$ for the MSDA and $\pm 0.1 \%e$ for the MSDB. For the MSDC, the variation of the $B_x$ component stays within $\pm 1 \%e$ whereas the component perpendicular to it varies within 3 \%e of $B_0$.

<table>
<thead>
<tr>
<th>Range of “good field region” [mm]</th>
<th>MSIA</th>
<th>MSIB</th>
<th>MSDA</th>
<th>MSDB</th>
<th>MSDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computed value of $B_0$ [T]</td>
<td>0.75717</td>
<td>1.12893</td>
<td>0.797362</td>
<td>0.991382</td>
<td>1.162162</td>
</tr>
<tr>
<td>Maximum value of $\Delta B_{|}/B_0$ [%e]</td>
<td>+0.17</td>
<td>+1.32</td>
<td>+0.08</td>
<td>+0.17</td>
<td>+0.76</td>
</tr>
<tr>
<td>Minimum value of $\Delta B_{\perp}/B_0$ [%e]</td>
<td>−0.05</td>
<td>+0.16</td>
<td>−0.08</td>
<td>−0.03</td>
<td>−0.89</td>
</tr>
<tr>
<td>Maximum value of $B_{\perp}/B_0$ [%e]</td>
<td>+0.22</td>
<td>+0.41</td>
<td>+0.14</td>
<td>+0.27</td>
<td>+3.15</td>
</tr>
<tr>
<td>Minimum value of $B_{|}/B_0$ [%e]</td>
<td>−0.02</td>
<td>−0.08</td>
<td>0</td>
<td>+0.05</td>
<td>+0.38</td>
</tr>
</tbody>
</table>

Table 7: Maximum relative field errors in the considered “good field region” within $\pm 5$mm from the midplane

The field at any location $z = x + iy$ in the gap can also be expressed in terms of its harmonic content with respect to any location $z_0$ inside this region (see equation 2). The real and imaginary parts of $c_n$ correspond to the multipole strengths of the normal and the skew 2n-pole respectively. The radius of convergence for this series expansion is limited by the presence of a singularity (current and/or a material boundary) and can therefore not exceed half the gap size.

The multipole coefficients were therefore computed for different centre locations $z_0$ on the mid-plane ($y = 0$) of the gap. Their moduli and phases (up to the decapole) are plotted in Figure 22 to Figure 26 as a function of the $x$ or $y$ co-ordinate of the centre location $z_0$, depending on whether the MSI or MSD is considered. The origin and orientation of the co-ordinate system is the same as described above.

5. Saturation

The saturation characteristics $B_0 = B_0(I)$ were computed for the modules MSIA, MSIB, MSDA and MSDC. $B_0 = |B(0)|$ is the total inductance in the centre $z = 0$ of the co-ordinate system defined in chapter 4, i.e. on the mid-plane at the same $x$ co-ordinate as the centre of the septum hole in the case of the MSI and in the centre of the gap in the case of the MSD.

It can be seen that the MSIA operates well below saturation which starts at about 20 kA (Figure 27), whereas the MSIB is energized up to the threshold of saturation (Figure 28). At nominal total excitation of $N \cdot I = 22.8$ kA, the field reduction amounts to about $7 \%e$ compared to the linear part.
Figure 29 shows that up to its nominal maximum current, the MSDA is still excited within the linear part of the saturation characteristic. At 28160 A, the field reduction due to saturation amounts to only about 1.4 ‰ which is negligible. On the other hand, the field drop in the MSDC excited at its nominal maximum current of N·I = 42.24 kA is already in the order of 3.0 % (Figure 30). For the MSDB, the saturation characteristic can be interpolated to be somewhere between these two extremes.

6. Flux enhancement due to Permalloy™ shielding

The presence of a Permalloy™ shielding inside the cylindrical septum-hole (or 2nd beam hole) reduces the magnetic resistance of this hole by a factor F which roughly corresponds to the ratio of the mean path length of the flux lines in the air before and after shielding.

In order to get an estimate of this factor, the following assumptions are made:

1. Without shielding the field in the hole (Ø = D) is assumed to be a homogeneous dipole field (left figure). The mean path length is then obtained by dividing the circular area by its diameter.

2. With the presence of a concentric, Permalloy™ shielding (Ø = d), the flux lines are assumed to be radial (right figure), which corresponds to infinite permeability of the surrounding iron and the shielding material. In this case the (mean) path length is simply given by the difference between the outer and inner diameter, and the enhancement factor amounts to

\[
F = \frac{\pi \cdot D}{4 \cdot (D-d)}
\]

The reduction of the magnetic resistance of the septum hole and/or 2nd beam hole causes the magnetic flux through this hole to increase by practically the same factor F. This can be shown by applying network theory to the following equivalent electrical circuit in which the “current” \( \Phi_{\text{tot}} \) corresponds to the total magnetic flux and \( \Phi_S \) and \( \Phi_H \) to the flux through the septum and 2nd beam hole, respectively:

\[
R_G = \text{magnetic resistance of gap and part of the yoke} \\
R_S = \text{magnetic resistance of septum hole} \\
R_H = \text{magnetic resistance of 2nd beam hole} \\
R_i = \text{magnetic resistance of some additional iron in flux path through hole} \\
R_{Fe} = \text{magnetic resistance of the yoke through which the remainder of the flux is carried and which is not already included in } R_G
\]

According to Kirchhoff’s law, the following equation relates the unknown magnetic resistance of the different paths with the magnetic flux through this path:

\[
(\Phi_{\text{tot}} - \Phi_S - \Phi_H) \cdot R_{Fe} = \Phi_S \cdot R_S = \Phi_H (R_H + R_i)
\]

Assuming that \( R_S = R_H = R \), the normalised magnetic resistance \( R/R_{Fe} \) and \( R/R_{Fe} \) can then easily be computed by substituting the values for the different fluxes (Table 3) into the following formulae which are derived from (4):
\[
\rho = \frac{R}{R_{Fe}} = \frac{\Phi_{tot} - \Phi_{S} - \Phi_{H}}{\Phi_{S}}
\]
\[
\rho_i = \frac{R_i}{R_{Fe}} = \frac{\Phi_{S} - \Phi_{H}}{\Phi_{H}} \cdot \rho
\]

Assuming further that due to the presence of a Permalloy\textsuperscript{TM} shielding:

- the value of \( \rho \) (\( R_S \) and \( R_H \)) is reduced by a factor \( F \) as described above (Eq. 3): \( \rho_\mu = \rho/F \)
- the magnetic resistance \( R_{Fe}, R_i \), and \( R_G \) of the other paths remains unchanged, and that
- the increase of the total flux \( \Phi_{tot} \) is a 2\textsuperscript{nd} order effect and can be neglected (\( R_S = R_H >> R_{Fe} \)),

it is then easy to calculate the redistribution of \( \Phi_{tot} \) due to the reduction of the magnetic resistance of the hole(s). The new values of \( \Phi_S \) and \( \Phi_H \) are obtained by solving the following two equations for these two unknowns:

\[
(\Phi_{tot} - \Phi_S - \Phi_H) = \Phi_S \cdot \rho_\mu = \Phi_H (\rho_\mu + \rho_i)
\] (5)

The fluxes through the septum and 2\textsuperscript{nd} beam hole being less than 1 \( \% \) of the total flux, their increase due the presence of a shielding is practically given by the ratio of the magnetic resistance of the corresponding path without and with shielding (see Table 8), i.e.

\[
\frac{\rho_\mu}{\rho_\mu} = F \quad \text{in the case of the septum hole}
\]
\[
\frac{\rho + \rho_i}{\rho_\mu + \rho_i} = F \quad \text{in the case of the 2\textsuperscript{nd} beam hole}
\]

<table>
<thead>
<tr>
<th></th>
<th>MSIA</th>
<th>MSIB</th>
<th>MSDA</th>
<th>MSDB</th>
<th>MSDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi_S/\Phi_{tot} ) (no shielding) [%]</td>
<td>0.1609</td>
<td>0.2533</td>
<td>0.1246</td>
<td>0.1277</td>
<td>0.1782</td>
</tr>
<tr>
<td>( \Phi_H/\Phi_{tot} ) (no shielding) [%]</td>
<td>0.0805</td>
<td>0.1970</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho = R/R_{Fe} ) (no shielding)</td>
<td>6212</td>
<td>3946</td>
<td>8027</td>
<td>7828</td>
<td>5612</td>
</tr>
<tr>
<td>( \rho_i = R_i/R_{Fe} )</td>
<td>6212</td>
<td>1127</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diameter of hole(s) [mm]</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>Diameter of shielding [mm]</td>
<td>44</td>
<td>44</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Factor F</td>
<td>2.5133</td>
<td>2.5133</td>
<td>3.5904</td>
<td>3.5904</td>
<td>3.5904</td>
</tr>
<tr>
<td>( \rho_\mu = \rho/F ) (with shielding)</td>
<td>2472</td>
<td>1570</td>
<td>2236</td>
<td>2180</td>
<td>1563</td>
</tr>
<tr>
<td>( \Phi_S/\Phi_{tot} ) (with shielding) [%]</td>
<td>0.4044</td>
<td>0.6363</td>
<td>0.4469</td>
<td>0.4583</td>
<td>0.6390</td>
</tr>
<tr>
<td>( \Phi_H/\Phi_{tot} ) (with shielding) [%]</td>
<td>0.1151</td>
<td>0.3703</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Increase of ( \Phi_S ) due to shielding</td>
<td>2.5126</td>
<td>2.5120</td>
<td>3.5876</td>
<td>3.5876</td>
<td>3.5864</td>
</tr>
<tr>
<td>Increase of ( \Phi_H ) due to shielding</td>
<td>1.4303</td>
<td>1.8797</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8 : Flux and magnetic resistance in the different paths of the equivalent network
7. Multipoles introduced by lack of parallelism between poles

The influence of a non ideal pole geometry on the field quality in the gap can roughly be estimated with the following “worst case” considerations. According to the specification [1], each of the two poles has to be parallel to the mid-plane within a tolerance (strip) \( p \approx 0.02 \text{ mm} \) over the whole pole width \( w \). Assuming that

a) both poles are absolutely straight but tilted with a maximum angle \( \alpha = \arctan(p/w) \) as indicated schematically in the sketch below,

b) the pole is not saturated (permeability \( \mu >> 1 \)) such that the pole surface can be considered to be an equipotential surface \( \Phi = \text{constant} \),

then the field which satisfies these boundary conditions can be derived from a complex logarithmic potential [5] of the complex co-ordinate \( z = x+iy \) with its origin in the centre of the gap:

\[
\Omega(z) = \Phi(z) + i\Psi(z) = -iB_0D\ln(z+D) \quad (6)
\]

<table>
<thead>
<tr>
<th>MSI</th>
<th>MSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap height ( 2h ) [mm]</td>
<td>25</td>
</tr>
<tr>
<td>pole width ( w ) [mm]</td>
<td>230</td>
</tr>
<tr>
<td>parallelism ( p ) [( \mu \text{m} )]</td>
<td>20</td>
</tr>
<tr>
<td>tilt angle ( \alpha ) [( \mu \text{rad} )]</td>
<td>87</td>
</tr>
<tr>
<td>distance ( D ) [m]</td>
<td>143.75</td>
</tr>
<tr>
<td>(</td>
<td>c_i/B_0</td>
</tr>
<tr>
<td>(</td>
<td>c_i/B_0</td>
</tr>
<tr>
<td>(</td>
<td>c_i/B_0</td>
</tr>
</tbody>
</table>

Table 9 : Maximum (normalised) multipole errors introduced by lack of parallelism between poles

The field lines which correspond to \( \Psi(z) = \text{const} \), are concentric circles with their centre at the intersection \( z = -D \) of the poles extended to infinity, and the (conjugate complex) field vector can be expressed as:

\[
\overline{B}(z) = B_x - i \cdot B_y = \frac{d\Omega(z)}{dz} = -i \cdot B_0 \frac{D}{z + D} \quad (6)
\]

The field in the centre of the gap (\( z=0 \)) has only a \( B_y \) component \( B(0) = iB_y = iB_0 \) which corresponds to an ideal dipole field \( B_0 \) described by its complex potential \( \Omega(z) = \Phi(z) + i\Psi(z) = -iB_0z \). The multipole coefficients of order \( n \) with respect to \( z = 0 \) are then given by the \( n^{\text{th}} \) derivative of \( \Omega(z) \):

\[
c_n = \frac{i}{n!} \frac{d^n\Omega(z)}{dz^n} = (-i)^{n-1} \frac{B_0D}{n \cdot (z+D)^n} \quad (7)
\]

Their values, normalised with \( B_0 \) (see Table 7) and up to 4th order, are listed above. Apart from the quadrupole which is only a factor 2 to 5 smaller than the one computed numerically with FLUX2D, all the higher harmonics are several orders of magnitude smaller than the corresponding multipoles plotted in Figure 22 to Figure 26 and can therefore be neglected.
8. Summary

The design of the septum magnets described in this paper is based on the following criteria: In order to allow for maximum mechanical stability and rigidity, which is essential for magnets of this length to achieve the required precision, the classical wedge septum was replaced by a cylindrical hole, or even two holes as for the MSI. Since the field in these holes cannot be reduced below a level which would be acceptable for the circulating beam, a magnetic shielding (Permalloy™) of the vacuum chamber is necessary. Technical problems with the fabrication of such a chamber still remain to be solved.

Since the computer code FLUX2D does not allow to specify any stacking factor, all numerical results correspond to a stacking factor of 100%. For a realistic stacking factor in the order of about 97%, the field in the air would then just be about 3% lower than the computed values, whereas the field in the iron corresponds to the calculated ones.

It shall be reminded that the numbers given in section 6 for the flux increase due to the Permalloy™ shielding must be interpreted as orders of magnitude rather than as precise values. The relative high precision of the numbers in Table 8 only illustrates the good agreement between the factor F and the flux enhancement calculated with network theory.

As for the dipole field in the gap, it is shown in section 7 that the mechanical tolerances on the pole geometry are tight enough and that the corresponding field errors are tolerable. The shims of the pole profile have been optimised to improve the homogeneity with respect to the preliminary design.

9. References


† Conform to Annex Nº 1 to the Addendum Nº B4 to the Protocol to the 1993 Co-operation Agreement between THE EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN) and THE GOVERNMENT OF THE RUSSIAN FEDERATION concerning THE RUSSIAN PARTICIPATION IN THE LARGE HADRON COLLIDER PROJECT (LHC)
10. Figures

10.1 Russian “Steel 21848”

\[
\mu = \mu(H)
\]

\[
\mu = \alpha \cdot H \text{[Oe]}^{-\beta}
\]

\[
B = B(H)
\]

\[
\begin{array}{|c|c|}
\hline
H [\text{Oersted}] & \mu \\
\hline
0. & 2687 \\
0.1 & 2794 \\
0.2 & 3110 \\
0.3 & 3599 \\
0.4 & 4149 \\
0.5 & 4591 \\
0.6 & 4800 \\
0.7 & 4781 \\
0.8 & 4641 \\
0.9 & 4438 \\
1. & 4159 \\
1.1 & 3730 \\
1.2 & 3152 \\
1.3 & 2538 \\
1.4 & 1897 \\
1.5 & 1194 \\
1.6 & 516 \\
1.7 & 272 \\
1.8 & 144 \\
1.9 & 87 \\
2. & 61 \\
2.05 & 54 \\
2.1 & 24 \\
\hline
\end{array}
\]

\[\alpha = 11605.92\]

\[\beta = 0.907667\]

Figure 1: Permeability- and saturation curve of Russian “Steel 21848”

Figure 2: B(H)-curve modelled by FLUX2D
10.2 Longitudinal Positions

Figure 3: Horizontal position of the injected beam in the gap with respect to the circulating beam

Figure 4: Vertical position of the extracted beam in the gap with respect to the circulating beam
10.3 Flux in the MSIA

Figure 5: Flux lines at nominal excitation of the MSIA.

Figure 6: Flux lines in the septum hole (left) and in the 2nd beam hole (right) of the MSIA.
10.4 Flux in the MSIB

Figure 7: Flux lines at nominal excitation of the MSIB

Figure 8: Flux lines in the septum hole (left) and in the 2nd beam hole (right) of the MSIB
10.5 Flux in the MSDA

Figure 9: Flux lines at nominal maximum excitation of the MSDA

10.6 Flux in the MSDB

Figure 10: Flux lines at nominal maximum excitation of the MSDB
10.7 Flux in the MSDC

Figure 11: Flux lines at nominal maximum excitation of the MSDC

10.8 Field in the septum hole and 2nd beam hole of the MSIA

Figure 12: Absolute value of the induction on a concentric circle of radius $r = 28$ mm
10.9 Field in the septum hole and 2nd beam hole of the MSIB

![Graph showing the field in the septum hole and 2nd beam hole of the MSIB.](image)

Figure 13: Absolute value of the induction on a concentric circle of radius \( r = 28 \text{ mm} \)

10.10 Field in the septum hole of the MSDA

![Graph showing the field in the septum hole of the MSDA.](image)

Figure 14: Absolute value of the induction on a concentric circle of radius \( r = 28 \text{ mm} \) inside the septum of the MSDA
10.11 Field in the septum hole of the MSDB

Figure 15: Absolute value of the induction on a concentric circle of radius $r = 28$ mm inside the septum of the MSDB

10.12 Field in the septum hole of the MSDC

Figure 16: Absolute value of the induction on a concentric circle of radius $r = 28$ mm inside the septum of the MSDC
10.13 Field homogeneity in the gap of the MSIA ($B_0 = 0.76$ T)

Figure 17: Relative error of the field components in the gap of the MSIA with respect to an ideal dipole field $B_0$. 

- $\Delta B_y(x)/B_0$
- $B_x(x)/B_0$
10.14 Field homogeneity in the gap of the MSIB \((B_0 = 1.13 \, T)\)

Figure 18: Relative error of the field components in the gap of the MSIB with respect to an ideal dipole field \(B_0\).
10.15 Field homogeneity in the gap of the MSDA \((B_0 = 0.8 \, T)\)

Figure 19: Relative error of the field components in the gap of the MSDA with respect to an ideal dipole field \(B_0\)
10.16 Field homogeneity in the gap of the MSDB \( (B_0 = 1.0 \ T) \)

\[ \frac{\Delta B_y(y)}{B_0} \]

\[ \frac{B_y(y)}{B_0} \]

Figure 20: Relative error of the field components in the gap of the MSDB with respect to an ideal dipole field \( B_0 \)
10.17 Field homogeneity in the gap of the MSDC \((B_0 = 1.17 \, \text{T})\)

Figure 21: Relative error of the field components in the gap of the MSDC with respect to an ideal dipole field \(B_0\)
10.18 Harmonic content in the gap of the MSIA

Figure 22: Multipole coefficients up to the decapole in the field region of possible beam positions in the MSIA
10.19 Harmonic content in the gap of the MSIB

Figure 23: Multipole coefficients up to the decapole in the field region of possible beam positions in the MSIB
Harmonic content in the gap of the MSDA

Figure 24: Multipole coefficients up to the decapole in the gap of the MSDA
10.21 Harmonic content in the gap of the MSDB

Figure 25: Multipole coefficients up to the decapole in the gap of the MSDB
10.22 Harmonic content in the gap of the MSDC

Figure 26: Multipole coefficients up to the decapole in the gap of the MSDC
10.23 Saturation in the MSIA

\[ B_0 = B_0 (I) \]

Figure 27: Saturation characteristic of the MSIA

10.24 Saturation in the MSIB

\[ B_0 = B_0 (I) \]

Figure 28: Saturation characteristic of the MSIB
10.25 Saturation in the MSDA

Figure 29: Saturation characteristic of the MSDA

10.26 Saturation in the MSDC

Figure 30: Saturation characteristic for the MSDC
11. Appendix

11.1 Cross-section of the MSIA

11.2 Cross-section of the MSIB
11.3 Cross-section of the MSDA
11.4 Cross-section of the MSDB
11.5 Cross-section of the MSDC