Effect of Shear Deformation and Relaxation of Support Conditions on Buckling of Pressurized Pipelines containing Expansion Bellows

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Keywords: Mechanical stability, pressurized pipelines, bellows, shear deformation, support conditions

Summary

Mechanical stability of pressurized expansion bellows and tube-bellows-tube interconnects is considered. Effect of shear deformation on buckling pressure of bellows is shown by using Engesser's model. A generalized equivalent column concept is developed in order to study the effect of relaxation of support conditions on stability of interconnects containing expansion bellows. Two general modes of buckling are discussed: column instability of well-supported bellows (I) and buckling of tube-bellows--tube interconnect if the support-bellows distances are large enough (II). A dramatic drop of critical pressure in the transition region from mode I to mode II is shown.

1. Introduction

Mechanical instability of pressurized bellows expansion joints occurs either as a local effect (a limited number of convolutions involved: in-plane squirm or root bulge) or as a global phenomenon (all convolutions involved) called column buckling. The latter was studied by Haringx (1952) in his excellent paper based on the rectangular shape of bellows convolutions. In order to compute the buckling pressure for bellows Haringx proposed a concept of equivalent column defined by its bending stiffness equal to that of the considered bellows. The equivalent column shall be subjected to an axial force equivalent to the total axial resultant applied at the bellows ends. Analysis of the classical eigenvalue problem for the column with given boundary conditions leads to an estimate of the buckling pressure. This fundamental concept has been adopted by many authors (Axelrad, 1976, Wilson, 1984, Broyles, 1989, Skoczen, 1992) since it is a logical extension of a similar concept developed for buckling of pressurized tubes (Haringx, 1952). Finally the idea was incorporated in the EJMA Standards (1958-1993) showing the way of calculating bellows expansion joints against buckling. Since the buckling pressure is directly proportional to the bellows axial rate, simplified formulas for the initial axial and bending stiffness of bellows with U-shaped corrugations were provided.

Haringx assumed that the global buckling is entirely determined by the flexural stiffness of bellows convolutions. However, the global reaction forces (at the bellows ends) reduced in the postbuckling state to the centers of bellows segments show among the others also shear components. Hence, a certain amount of postbuckling shear deformation may exist, which sustains the hypothesis that buckling pressure may depend on the shear stiffness. This particular effect is studied in the present paper.

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It is well known that inappropriate guidance of tubes in pipelines equipped with expansion bellows may lead to a premature mechanical instability of the interconnects. Relaxation of the support conditions is usually provoked either by an excessive guidance-bellows distance or by insufficient angular guidance. It is of major interest for pipeline designers to know what is the critical guidance-bellows distance leading to interconnect instability. To solve this problem a generalized equivalent column concept has been developed.

2. Generalized Equivalent Column Concept

An expansion joint interconnect in a pipeline composed of two supported tubes and a bellows is considered (Fig. 1a)). It is assumed that the bellows of convoluted length $L_b$, accompanied by tubes of the same inner diameter $D_n$, is guided at distances $L_1$, $L_2$, respectively. Since the tubes have their transversal and flexural stiffness the bellows may be considered as elastically supported. Hence, the convolutions represented by an equivalent column are suspended on a system of springs, as shown in Fig. 1b).

![Fig. 1: a) Bellows expansion joint interconnect. b) The equivalent column with elastically supported ends.](image)

The angular stiffness parameters $k_1$, $k_2$ and the relative transversal stiffness $k_3$ shall be calculated as for clamped tubes (cantilever beams) provided that the guidance is tight enough (only axial movements in the support). The stiffness parameters are given by the following simple formulas:

$$k_1 = \frac{EI_1}{L_1}, \quad k_2 = \frac{EI_2}{L_2}, \quad \text{(1)}$$

$$k_3 = \frac{c_1 c_2}{c_1 + c_2}, \quad \text{(2)}$$
where
\[ c_1 = \frac{3EI_1}{L_1^3}, \quad c_2 = \frac{3EI_2}{L_2^3}. \]  

A solution to the eigenvalue problem for such a column with flexural effects only was shown by Zyczkowski (1991).

2.1 Eigenvalue problem for the column with a finite shear stiffness

Ziegler (1982) has widely discussed effect of shear on buckling of straight bars and helical springs. Considering two approaches: Engesser’s (1891) approach and the so-called modified approach (Haringx, 1948-49) he showed that the first was superior for bars and columns whereas the second was suitable rather for springs. Since deformation of bellows convolutions under shear forces is close to deformation of a column the Engesser approach seems to be justified and has been applied in the present analysis.

Assume that N denotes the initial axial force (spatially fixed) acting on the column, \( \gamma_1, \gamma_2 \) denote angles of inclination of the deformed column axis due to bending and shear, respectively (Fig. 2).

![Fig. 2: Inclination angles due to bending and shear deformation components](image)

The local angle of inclination of the column axis is given by:
\[ \frac{dy}{dx} = \gamma_1 + \gamma_2, \]  

(4)
where
\[ \frac{d\gamma_1}{dx} = -\frac{M_{bg}}{c_{bg}}, \quad \gamma_2 = \frac{Q}{c_t}. \] (5)

Here \( M_{bg}, c_{bg} \) and \( Q, c_t \) denote bending moment, bending stiffness and shear force, shear stiffness, respectively. The Engesser approach applied to the elastically supported column leads to the following expression for \( \gamma_2 \):
\[ \gamma_2 = \frac{N(\gamma_1 + \gamma_2) - Q_0}{c_t}. \] (6)

where \( Q_0 \) represents the lateral force in spring \( k_3 \). Thus, the angle of inclination amounts to:
\[ \frac{dy}{dx} = \gamma_1 \left( 1 + \frac{N}{c_t - N} \right) - \frac{Q_0}{c_t - N}, \] (7)

whereas the local curvature is given as follows:
\[ \frac{d^2y}{dx^2} = -\frac{c_t}{c_t - N} \frac{M_{bg}}{c_{bg}} = -\frac{M_{bg}}{(EI)}, \] (8)

Here the substitutive flexural stiffness \((EI)_s\) is defined in terms of the “classical” bending stiffness \( c_{bg} \) and the shear stiffness \( c_t \):
\[ (EI)_s = \frac{(c_t - N)c_{bg}}{c_t}. \] (9)

Effect of shear on the value of the critical force for statically determined support modes can be evaluated from the second order differential equation (cf. Timoshenko, 1961; Ziegler, 1982; Zyczkowski 1991):
\[ \frac{d^2y}{dx^2} + \frac{N}{c_{bg} (1 - N / c_t)} y = 0 \] (10)

On the other hand, in the case of static indeterminacy it is more convenient to use a more general equation of bending with the shear deformation taken into account (see Ziegler, 1982; extension of the beam axis has been neglected):
\[ \frac{d^4y}{dx^4} + \frac{N}{c_{bg} (1 - N / c_t)} \frac{d^2y}{dx^2} = 0 \] (11)

This equation can be transformed to a compact form:
\[ \frac{d^4y}{dx^4} + \frac{N}{(EI)_s} \frac{d^2y}{dx^2} = 0 \] (12)

with the general solution given in terms of four constants:
\[ y = C_1 \sin(kx) + C_2 \cos(kx) + C_3 + C_4 \] (13)
where

\[ k^2 = \frac{N}{(EI)} \] \hspace{1cm} (14)

The solution is determined by the following four boundary conditions:

\[ y(0) = 0 \] \hspace{1cm} (15)
\[ k_1 y_1'(0) - (EI) y^*(0) = 0 \] \hspace{1cm} (16)
\[ k_2 y_2'(L_0) - (EI) y^*(L_0) = 0 \] \hspace{1cm} (17)
\[ (EI) y^*(L_0) + Ny^*(L_0) - Q_0 = 0 \] \hspace{1cm} (18)

or in a more convenient form:

\[ y(0) = 0 \] \hspace{1cm} (19)
\[ y'(0) = \psi_1 L_0 y^*(0) - \frac{y(L_0)}{\vartheta_1 L_0 \eta} \] \hspace{1cm} (20)
\[ y'(L_0) = -\psi_2 L_0 y^*(L_0) - \frac{y(L_0)}{\vartheta_2 L_0 \eta} \] \hspace{1cm} (21)
\[ y(L_0) = \vartheta_2 L_0 \eta \left[ y^*(L_0) + k^2 y'(L_0) \right] \] \hspace{1cm} (22)

where

\[ \vartheta_1 L_0 \eta = \frac{c_1}{k_3} \left( 1 - \frac{N}{c_i} \right) \] \hspace{1cm} (23)
\[ \vartheta_2 L_0 \eta = \frac{c_{be}}{k_3} \left( 1 - \frac{N}{c_i} \right) \] \hspace{1cm} (24)
\[ \psi_1 L_0 = \frac{c_{be}}{k_1}, \quad \psi_2 L_0 = \frac{c_{be}}{k_2} \] \hspace{1cm} (25)

Substituting (13) into (19), (20), (21), (22) one obtains a set of four linear homogenous algebraic equations:

\[ [A] \Gamma = 0 \] \hspace{1cm} (26)

where

\[ \Gamma = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} \]

The condition of non-triviality of the solution leads to the equation:

\[ \det[A] = 0 \] \hspace{1cm} (27)
which, eventually, takes the following form:

\[
2(1 + \xi) - \left[ 2(1 + \xi) + (\psi_1 + \psi_2)(1 - \eta \nu_2 X^2) X^2 \right] \cos(X) + \\
\left[ -1 + (1 + \xi)(\psi_1 + \psi_2) + X^2(\psi_1 \psi_2 + \eta \nu_2 - \eta \nu_2 \psi_1 \psi_2 X^2) \right] X \sin(X) = 0
\]  

(28)

where

\[
\eta = 1 - \frac{N}{c_i}, \quad \xi = \frac{\nu_2}{\nu_1} X^2,
\]

(29)

and the eigenvalues are represented by:

\[
X = kL_n.
\]

(30)

It is worth pointing out that if \( c_i \to \infty \) consequently \( \xi \to 0 \) and \( \eta \to 1 \) so that Eq. (28) reduces to a similar equation presented by Zyczkowski (1991) for the flexural deformation only.

### 2.2 Definition of bending and shear stiffness of bellows

Since a bellows constitutes a set of identical axisymmetric segments both bending and shear stiffness have to be defined first with respect to a single segment. The bending stiffness (bellows angular rate) may be calculated with a sufficient precision on the basis of the EJMA Standards (1993):

\[
c_{bg} = \frac{f_{sw} D_m^2 q}{8},
\]

(31)

where \( D_m \) denotes the bellows mean diameter, \( q \) is the bellows pitch and \( f_{sw} \) stands for the bellows segment axial elastic rate:

\[
f_{sw} = 1.7 \frac{D_p E_j t_p^3 n}{w^3 C_f}.
\]

(32)

The following notation has been used (after EJMA Standards):

- \( E_j \) - elastic modulus of the bellows material,
- \( t_p \) - thickness of one ply corrected for manufacture thinning,
- \( n \) - number of plies (in case of multiply bellows),
- \( w \) - convolution depth,
- \( C_f \) - factor depending on \( q, w \) and \( t_p \).

Since there is no commonly agreed formula for the shear stiffness of bellows convolutions a finite element model was used to provide the necessary information on the amount of shear deformation under transversal loads. Shear stiffness is computed as:

\[
c_s = \frac{F}{\gamma_2}
\]

(33)
where $F$ denotes transversal force applied to bellows segment (single convolution). Finally, it is assumed that both the bending and the shear stiffness calculated for a single segment and distributed uniformly over the whole bellows length define the response of the equivalent column.

3. Numerical Analysis of Deformation of Typical Bellows Segments

Numerical study has been carried out by using the finite element code ANSYS (1997). The model (Fig. 3a) was based on 8-node structural shell elements, suitable for curved 3-D surfaces. The configurations chosen for the numerical tests are shown in Table 1. Material data for stainless steel of grade 316 L, often used for bellows convolutions, was applied ($E = 195$ GPa, $\nu=0.3$, at ambient temperature).

Typical shear deformation of bellows segment subjected to a transversal force is shown in Fig. 3b. Both the bottom and the top edges of the segment are guided in the planes perpendicular to the bellows axis. Significant local bending of the shell wall is observed all around the segment.

![Bellows convolution subjected to a transversal force - FE model.](image)

![Shear deformation shown in the plane of symmetry.](image)

**Fig. 3:** a) Bellows convolution subjected to a transversal force - FE model. b) Shear deformation shown in the plane of symmetry.

Complementary analysis (based on the same model) led to verification of the axial and the bending stiffness. The numerical and analytical results (formulas 28, 29) referring to single segments of different profile parameters are shown in Table 2 (analytical values are given in the brackets).

**Table 1: Bellows configurations chosen for numerical study**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_m$</td>
<td>80 mm</td>
</tr>
<tr>
<td>$q$</td>
<td>8 mm</td>
</tr>
<tr>
<td>$w$</td>
<td>4.5; 6; 8; 10 mm</td>
</tr>
<tr>
<td>$t$</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>$n$</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 2: Stiffness components calculated for segments of different convolution depth

<table>
<thead>
<tr>
<th>Convolution depth W [mm]</th>
<th>Axial stiffness $f_{iu}$ [N/mm]</th>
<th>Bending stiffness $c_{bg}$ [Nmm/rad]</th>
<th>Shear stiffness $c_t$ [N/rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>3926.5 (3762)</td>
<td>25531429 (27052692)</td>
<td>1449275.5</td>
</tr>
<tr>
<td>6</td>
<td>1875.5 (1900)</td>
<td>12566250 (14150611)</td>
<td>1136363.5</td>
</tr>
<tr>
<td>8</td>
<td>892 (871.5)</td>
<td>6186461.5 (6794989.5)</td>
<td>751879.5</td>
</tr>
<tr>
<td>10</td>
<td>496 (469.5)</td>
<td>3590357 (3828345.5)</td>
<td>549450.5</td>
</tr>
</tbody>
</table>

4. Effect of Shear on Buckling of Clamped-Clamped Short Bellows

Combining (8), (14) and (30) the following formula for the buckling force (pressure) can be derived:

$$ N_{cr} = \frac{c_{bg} \chi}{1 + \frac{c_{bg}}{c_t} \chi} = \frac{N_{0cr}}{1 + \frac{c_{bg}}{c_t} \chi}, $$

(34)

where

$$ \chi = \frac{X^2}{L_o^2}, $$

(35)

and $N_{0cr}$ represents buckling of a column with infinite shear stiffness. Hence, the shear effect is determined by the factor:

$$ \frac{c_{bg}}{c_t} \chi, $$

(36)

which for a clamped-clamped bellows is equal to:

$$ \frac{c_{bg}}{c_t} \frac{4\pi^2}{L_o^2}. $$

(37)

Effect of shear deformation on buckling of bellows composed of 5, 10 and 20 convolutions is shown in Fig. 4. It is particularly pronounced for short bellows (small number of convolutions) and large $\frac{q}{2w}$ (close to 1). The reduction in critical force (pressure) may reach 30% and more if $\frac{q}{2w}$ approaches 1 (semicircular profile of convolutions - toroidal bellows).
5. Relaxation of Support Conditions - Extended Buckling Analysis

The above presented model of elastically supported column may be applied directly to buckling analysis of an interconnect composed of guided tubes and a bellows (see Fig.1a). The support conditions of bellows (represented by the equivalent column) depend on the guidance-bellows distances. Increase of $L_1$ and $L_2$ induces a relaxation of the support conditions which results in a radical change of the form of mechanical instability. As an example a bellows composed of 10 convolutions ($q=8$ mm, $w=6$ mm, $t=0.3$ mm) installed between 1 mm thick tubes ($D_{in}=80$ mm) is considered. Buckling analysis based on Eq. (28) reveals two basic instability modes (Fig.5 a, b), corresponding to short and long guidance-bellows distances. In the first case the buckling mode is close to deformation of a clamped-clamped column (which is an extreme case corresponding to the length of tubes equal to 0). If $L_1 = L_2$ the mode is symmetric with respect to the initial plane of symmetry of the bellows. In the second case the buckling mode allows for a limited angular displacement of the column ends as well as a limited transversal displacement of one column end with respect to the other (elastic springs). If $L_1 = L_2$ the mode is anti-symmetric with respect to the initial plane of symmetry of the bellows.

Fig. 4: Reduction in bellows buckling load (pressure) due to shear effect.
Fig. 5:  

a) I buckling mode - small guidance-bellows distance.  
b) II buckling mode - moderately large guidance-bellows distance.

Assume that the considered configuration is symmetric: $L_1 = L_2 = L$. In this case one obtains from Eq. (28) two sets of critical curves (Fig.6):

- for the flexural effects only (infinite shear stiffness, dashed lines),
- for both the flexural and the shear deformations included (solid lines).

Each set of curves corresponds to the above discussed two principal modes of instability.

The buckling pressure of bellows:

$$ p_{cr} = \frac{4N_{cr}}{\pi D_m^2} $$

is calculated as the ratio of buckling force of the equivalent column to the bellows mean cross-sectional surface ($D_m$ denotes the bellows mean diameter). The reference buckling pressure $p_{cr0}$ corresponds to the buckling pressure of a clamped-clamped bellows calculated with the flexural effects only (infinite shear stiffness). Thus, the parameter $p_{cr} / p_{cr0}$ is equal to 1 for $L = 0$ (clamped-clamped column/bellows), if the shear effects are neglected.
Fig. 6: Decrease in buckling pressure caused by change of buckling mode for small and moderately large bellows-to-support distances.

The flat horizontal curves in Fig.6 correspond to I buckling mode for which the eigenvalues $X$ are close to $2\pi$. On the other hand, the steep curves for larger $L/D_{in}$ values correspond to the II buckling mode for which the eigenvalues $X$ cover the following range: $1.4\pi \div 2\pi$. Increasing values of $L$ lead to decreasing values of $X$. The II mode curves indicate a large sensitivity of the critical load with respect to $L$.

The above presented analysis is valid for small deflections and small rotations of both the tubes and the bellows (linear buckling analysis). Also, it is valid for guidance-bellows distances that are comparable with the bellows length (configurations rather close to the first-to-second buckling mode transition zone). Stability analysis of large span tube-bellows-tube configurations shall be based on a different mechanical model. Furthermore, it is not clear how “sharp” is the transition from mode I to mode II in the real structures and whether a third intermediate form may exist.
6. Conclusions

Contributions of both the shear deformation and the relaxation of support conditions to buckling pressure decrease in the interconnects containing bellows expansion joints may be significant. Extended equivalent column model including both the above specified effects was used to carry out the buckling analysis.

Effect of shear deformation on buckling of bellows, negligible for long and deep convolutions (U-shaped profiles), turns out to be important for short and moderately flat convolutions (toroidal bellows). Reduction in buckling pressure may reach 30% with respect to the “classical” value (cf. EJMA, 1993).

A dramatic drop of buckling pressure induced by the change of buckling mode is observed for tube-bellows-tube interconnects if the guidance-bellows distances are large enough (relaxation of the support conditions). Effect of shear deformation - significant with respect to I buckling mode (well supported bellows) - turns out to be less pronounced for the II buckling mode (moderately large bellows-guidance distances).

In view of the above presented stability analysis close location of the first guidance with respect to the bellows ends imposed by EJMA (4 tube diameters) seems to be essential for the mechanical stability of the interconnect.
References

ANSYS Release 5.4, 1997, SAS IP, Inc., USA.


