On the global and local mechanical stability of the LHC

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Summary

The Large Hadron Collider (LHC) will be the first CERN accelerator entirely operating in superfluid helium below 2 K. The superconducting magnets and their interconnections constitute a discontinuous, nearly 27 km long, pressure vessel, subdivided in 8 octants. The main dipole and the quadrupole magnets are elastically supported in the LHC Arc Cryostat System and will be subjected to considerable interconnection forces when cooling down the LHC. The present note addresses the problem of mechanical stability of the LHC both on the global and on the local level. In both cases, a bifurcation buckling analysis has been carried out and the corresponding safety factors were assessed.

1. Introduction

The Large Hadron Collider (LHC) being built at CERN is a proton storage ring based on high field superconducting magnets operating in superfluid helium below 2 K and under 0.13 MPa internal pressure. Before bringing the beams of protons into collisions, the collider needs to be cooled from room temperature down to its 1.9 K operational temperature. The cool-down process consists in pumping cold helium at the rates of 30/60/100 g/s into the sequence of magnets by using high performance compressors. During the primary stage of cool-down, the system works under an inner pressure that might exceed 1 MPa thus developing considerable forces in the zones of interconnections (Fig. 1). Also, when at low temperature, the accelerator can be subjected to an incidental pressure rise due to a quench (resistive transition) with an estimated peak pressure of 1.5 MPa. For both the above listed reasons, the design pressure for the LHC cold mass enclosure has been specified as 2 MPa.
In view of the large interconnection forces developed during cool-down and warm-up, as well as due to the quench of superconducting magnets, the global mechanical stability of the LHC and its possible coupling with the local stability of the LHC interconnections must be assessed. In case of transverse buckling, the integrity of the system might be compromised, thus leading to helium leaks and contamination of the insulation vacuum. The present technical note is aimed at verifying the global and the local mechanical stabilities of the LHC Arc and the possible coupling between them.

The following general assumptions are made with respect to global stability of the LHC Arc:

- difference between the absolute values of global axial forces developed in the interconnections at the beginning and at the end of cool-down shall be minimised,
- the global non-equilibrated axial forces due to cool-down or magnet quench developed between two neighbouring interconnections shall not exceed 30000 N (the maximum admissible shear force defined for the central cold foot),
- the forces induced by the LHC interconnections should not lead to a global mechanical instability of the LHC Arc,
- the safety factor against global instability (buckling) shall be at least 10,
- the maximum elastic transverse displacement in the string of magnets (corresponding to the LHC Arc) under design pressure of 2 MPa shall not exceed 0.1 mm.

The local mechanical stability of the LHC interconnections shall satisfy the following general criteria:

- the axial forces developed by each bellows expansion joint shall not induce a local buckling of the corresponding interconnect,
- the total distance between the extremities of each bellows expansion joint and the corresponding guiding points on the magnet heads shall not exceed half the critical value (safety factor of 2),
- the definition of stability of each expansion joint (column buckling + in-plane squirm) shall respect the safety factors imposed by the EJMA Standards [1],
- it is assumed that the mechanical stability of the stainless steel bellows expansion joints at cryogenic temperatures is higher than at room temperature (higher modulus of elasticity, smaller amount of plastic deformation), thus giving some additional safety margin.
2. Interconnection forces

The majority of the LHC bellows expansion joints (cryogenic lines M1, M2, M3, X) are installed in the LHC interconnections with an initial prestress of 16 mm. Thus, a global compressive force exists in any cross-section of the collider already at the beginning of cool-down. This force increases considerably when pressurising the accelerator to the cool-down initial pressure of around 1 MPa. At this stage, the interconnection forces include the prestress of bellows expansion joints and all the pressure induced forces in all the cold mass interconnection bellows (all contributions amount to the global cross-sectional resultant). The forces evolve when cooling down the accelerator mainly due to the following effects:

- evolution of prestress in the bellows expansion joints (resulting from the shrinkage of magnets),
- evolution of internal pressure in the system (it decreases from initial 1 MPa to 0.13 MPa operational pressure).

The global interconnection forces in all phases of the cool-down process are shown in Fig. 2.

![Fig. 2: LHC MB-MB interconnections: global forces (F_p=16.8 kN, F_g1=39.3 kN, F_g+=34.6 kN)](image)

It is worth pointing out that \( |F_{g1}| - |F_{p}| \) shall be minimised in order to optimise the interconnection peak forces (one of the design objectives). This condition is fulfilled for all the MB-MB interconnections, however, it does not hold for the MB-SSS and SSS-MB interconnections (the SSS has a different length than MB).
3. **General mechanical stability of the LHC**

3.1. **Eigenvalue buckling analysis of the LHC Arc**

The LHC cold mass constitutes a system of mechanically decoupled pressure vessels separated by means of the mechanical compensation system (bellows expansion joints). The magnets are supported inside the vacuum vessel on the composite cold feet that have their proper shear and bending flexibility. Thus, they form an elastic discrete system of pressure vessels on elastic foundation (Fig. 3 and 4). Such a system is susceptible to buckling when pressurised to its corresponding critical pressure. In order to evaluate the critical pressure, a simplified analytical model of the LHC has been developed (Fig. 5) and the eigenvalue analysis, leading to the verification of the critical bifurcation pressure, has been carried out.

![Fig. 3: Elastically supported cold-mass with the interconnections](image)

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The model is based on the assumption of infinite rigidity of the cold mass when compared to the interconnections and the cold supports. Consequently, it is assumed that the entire energy of elastic deformation is concentrated in the interconnections (bellows expansion joints) and in the feet. The rigid body motion of the cold mass inside the cryostat due to assembly tolerances is not taken into account in the present analysis. In order to account for the combined contribution of the feet and the cryostat, the elastic (Winkler’s type) foundation was introduced. The stiffness of the elastic foundation (see Fig. 5) is equivalent in terms of the elastic energy to the sum of all the components that deform under pressure loads (cold mass, cold feet, vacuum vessel).

![Fig. 4: LHC cell subjected to inner pressure](image)

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![Fig. 5: The corresponding mathematical model](image)

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The elastic multi-link model consisting of \( n \) segments and laying on the above defined elastic foundation leads to the following general set of equations:
\[ P = \frac{c}{L_D} \frac{B_i + \frac{kL_D^2}{3} C_i}{A_i} = \frac{L}{M} ; A_i = A_i(\delta_i) ; B_i = B_i(\delta_i) ; C_i = C_i(\delta_i) \] (1)

where:
- \( c, k \) denote the elastic equivalent stiffness of the interconnections and the elastic foundation,
- \( L_D \) denotes the length of dipole magnet (the length of SSS is approximately half that of MB),
- \( P \) stands for the global axial force,
- \( \delta \) are the transverse horizontal displacements of extremities of the consecutive magnets.

In order to minimise the load a set of \( n-1 \) linear, partial differential equations are solved:

\[ \frac{\partial L}{\partial \delta_i} - P \frac{\partial M}{\partial \delta_i} = 0 \text{ for } i = 1, \ldots, n-1 \] (2)

which leads to a set of linear homogenous algebraic equations:

\[ [H] \Gamma = 0 \] (3)

where:

\[ [H] = [H(\lambda)] \text{ and } \Gamma = (\delta_1, \ldots, \delta_{n-1}) \] (4)

The condition of non-triviality of the solution:

\[ det[H] = 0 \] (5)

leads finally to the characteristic polynomial of the order \( n-1 \) for the matrix \( H \), that can further be solved for eigenvalues \( \lambda \). The eigenvector \( \Gamma \) can be interpreted directly as a mode of instability of the LHC cold mass. The eigenvalues \( \lambda \) are linked to the critical buckling load by the following equation:

\[ P_{cr} = \lambda kL_D^2 \] (6)

It is worth pointing out that the critical load depends in a direct way on the stiffness of the elastic foundation \( k \) and in an indirect way (via the eigenvalues \( \lambda \)) on the ratio \( c/k \). In other words, it is mainly the combined stiffness of the cold feet and the cryostat (represented by the parameter \( k \)) that contribute directly to the general mechanical stability of the LHC Arc.

The above given set of equations was solved for \( n-1 \) corresponding to the length of the LHC cell (\( n=8 \)). The primary buckling mode of the LHC cell is shown in Fig. 6. It corresponds to a bifurcation buckling pressure 220 times higher than the design pressure for the LHC cold mass (2 MPa). Another primary buckling mode computed for the LHC subsector (2 cells) is shown in Fig. 7. In both cases, it is the SSS, situated centrally in the string of magnets, which shows the largest transverse displacement.
Further analysis was carried out for the gradually increasing number of cells (2, 3, ...) in order to obtain a basis for the extrapolation to the length of the LHC Standard Arc (23 cells). The curve presenting the critical buckling load as a function of the number of half-cells is shown in Fig. 8.

\[ P_{cr} = a_1 + \frac{a_2}{j} + \frac{a_3}{j^2} + \frac{a_4}{j^3} \]  

(7)

where \( j \) denotes the number of half-cells. The extrapolation was based on the computations made for 1 cell, 1.5 cells, 2 cells and 3 cells (on the flat part of the curve). The final safety factor for the LHC Arc, with respect to the design pressure, equals to 190.
3.2. Effect of imperfections (misalignment of the LHC Arc)

The high safety factor against global instability of the LHC Arc does not mean that there is no transverse motion of the accelerator when pressurised to its design or test pressure. In order to verify the maximum transverse displacements of the pressurised cold mass (due to elastic deformation of the LHC Arc Cryostat System), the following assumptions are made:

- the system is fully elastic (no inelastic deformation in the interconnections),
- the transverse displacements and rotations are small when compared to the size of the LHC Arc,
- the sensitivity to imperfections relates to misalignment corresponding to the primary mode of instability.

For the fully elastic systems, the following equation holds:

\[ P(f) = \left(1 - \frac{f_0}{f}\right) P_{cr} \]  (8)

where \( f_0, f \) denote the initial imperfection and the transverse deformation under load \( P \), respectively.

This linear equation is shown in the form of a graph in Fig. 9.

\[ P(f) \]

\[ P_{cr} \]

\[ 1 \]

\[ f_0 \]

\[ f \]

\[ f_0/f \]

\[ 1 \]

\[ f \]

**Fig. 9: Effect of imperfections \( f_0 \) on the transverse deformation \( f \) of the LHC Arc**

Eq. 8 can be presented in a slightly different form showing an increment of transverse deformation under the load \( P \):

\[ \Delta f = f_0 \frac{1}{P_{cr}/P-1} \]  (9)

For a typical misalignment of 4 mm, given the ratio: \( P_{cr}/P = 190 \), one obtains the increment of transverse deformation:

\[ \Delta f = 0.02 \text{ mm} \]  (10)

Thus, the LHC Arc can be considered mechanically stable.
4. Local mechanical stability of the LHC interconnections

4.1. Stability diagram and critical guiding distance

The global mechanical stability of an accelerator is a necessary but not a sufficient condition of local stability of the interconnections. It provides suitable boundary conditions for the pressurised interconnections. Similarly, an interconnect can get locally unstable without provoking a general instability of the accelerator. Therefore, the problem of local instabilities has to be treated separately, with the global stability as a necessary condition for further analysis.

The most critical interconnections from the point of view of local stability are the main bus-bar channels M1, M2, M3, as well as the thermal shield interconnect E (Cf. [2]).

![Fig. 10: Elastically supported column – model of the LHC interconnections](image)

The interconnections M1, M2, M3 (containing the quadrupole, the dipole and the spool piece bus bars) are susceptible to buckling since they are subjected to the same design pressure as the cold mass (2 MPa). Also, the layout of the interconnections is such that the bellows expansion joints are situated close to the cold mass head and the portion of the interconnect on the opposite side of bellows is rather long (Fig. 11). This induces a weak support of the expansion joints on one side. For this particular reason, a more precise analysis of the mechanical stability of the LHC interconnections was needed.

![Fig. 11: Layout of the M1, M2, M3 interconnections (bus-bar lines)](image)

The mathematical model used in this analysis was described in [3], [4] and [5]. The model is based on the equivalent column concept (Fig. 10). The bellows expansion joint is modelled as a flexible and elastically supported column. The parameters of the elastic support are evaluated as a function of \( \frac{L_1}{L_B} \) and \( \frac{L_2}{L_B} \) for each interconnect separately. Finally, a stability diagram is plotted and the so-called critical length \( (L_1 + L_2)_{cr} \) is evaluated. It corresponds to such
a sum of guidance distances on both sides of the bellows expansion joint that provokes a radical change of instability mode and a dramatic reduction of the critical buckling pressure (Fig. 12).

Thus, each interconnect shall (approximately) satisfy the following criterion:

$$L_1 + L_2 \leq 0.5 \left( L_1 + L_2 \right)_{cr}$$

(11)

which is equivalent to an approximate safety factor of 2 with respect to the critical guiding distance. This factor is equal to 1.8 for the cold mass interconnections M1, M2, M3. Thus, given the global mechanical stability of the accelerator and the very small angular displacements of the magnets, the conditions of local stability are also fulfilled.

4.2. Sensitivity to imperfections

The sensitivity to imperfections can be approximately determined on the basis of the same principle as for the LHC Arc. It is assumed that the deformations and rotations due to instabilities are small when compared to the length of the interconnect (at least in the primary phase of buckling). Moreover, the elastic response of the system is assumed (all the local plastic strain fields are neglected), however, the bellows expansion joint axial rate is equivalent to the so-called secant modulus (identified by using the extreme points on the elasto-plastic hysteresis). Also, Mode I instability is considered (interconnect is far from the critical length).

Under the assumptions presented above the Eq. 8 as well as the Fig. 9 are valid also for the local analysis. Since in the case of M1, M2, M3 interconnections:

$$\frac{P_{cr}}{P} = 11$$

(12)
It turns out, from Eq. 9, that

\[ \Delta f = f_0 \frac{1}{10} \]  

It means that the increment of transverse deformation under the design pressure of 2 MPa is equal to 10% of the initial imperfection. Hence, if the initial imperfection is of around 1 mm (Mode I curvature – see Fig. 12) the increment of transverse deformation amounts to:

\[ \Delta f = 0.1 \text{ mm} \]  

Again, the interconnections M1, M2, M3 in the LHC Standard Arc can be considered mechanically stable.

5. Conclusions

- The LHC Standard Arc is globally stable. The maximum transverse displacements of the cold mass, due to elastic deformation of the LHC Arc Cryostat System during the transient phases (cool-down, warm-up), are around 0.02 mm.
- The global mechanical stability of the LHC provides good boundary conditions for the local stability of the LHC cold mass interconnections.
- Local mechanical stability of the cold mass interconnections M1, M2, M3 is determined by two contributions: safety factor against column buckling equal to 11 (minimum required: 6) and safety factor against relaxation of boundary conditions equal to 1.8 (minimum required: 1.5; optimum: 2).
- The sensitivity analysis with respect to imperfections shows that the interconnections M1, M2, M3 in the LHC Standard Arc are mechanically stable.
- The global and local mechanical stability of the LHC insertion zones needs to be carefully investigated. A special analysis shall be dedicated to the zones with dummy magnets in the continuous vacuum vessel, which are potentially less stable than the LHC Standard Arc.

6. References


