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Commissioning the rotation three-coil mole for the magnetic measurements of the MQW series magnets for the LHC accelerator.

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Abstract

For the magnetic measurements of the firsts MQW series magnets, a three coils mole was used. The MQW are resistive twin-aperture quadrupole magnets built for the LHC cleaning insertions. This paper describes the field measuring system, the data acquisition system and the off-line analysis program.

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1 Installing the magnet

To position the magnet on the measurement stand a crane is used to descend it onto three jacks. These jacks serve to align the magnet. The jacks have two horizontal movements possibilities (longitudinal and transversal) and they can also move the magnet vertically.

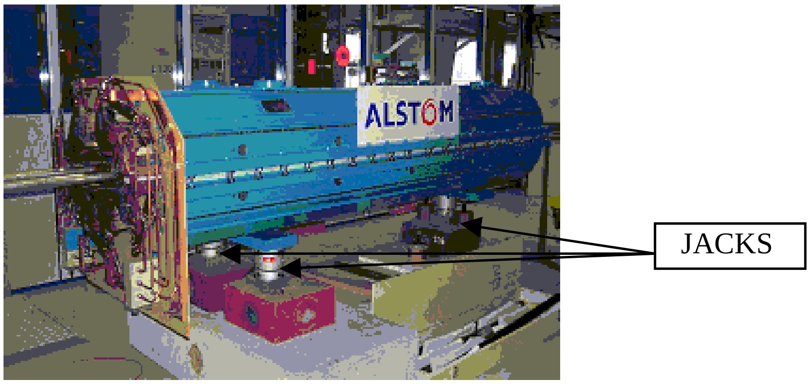


Fig. 1 MQW magnet on the measuring stand.

1.1 Alignment of the magnet itself

An electronic levelmetre is used to measure the inclination of the magnet in both longitudinal and transverse directions.

The magnet has dedicated reference points. These reference points, located on both extremities on top of the magnet are used for longitudinal alignment. Two flat plates are positioned on these reference points. The levelmetre placed on the flat plates shows the longitudinal inclination of the magnet.

To level the magnet transversally, rectified Ø44mm bars are positioned in the apertures and two flat plates are placed on top of them. In this case the levelmetre shows the horizontal inclination of the magnet.

The jacks are adjusted to have minimum inclination and minimum twist angle on both sides of the magnet.

Figure 2 shows the positioning of the levelmetre.

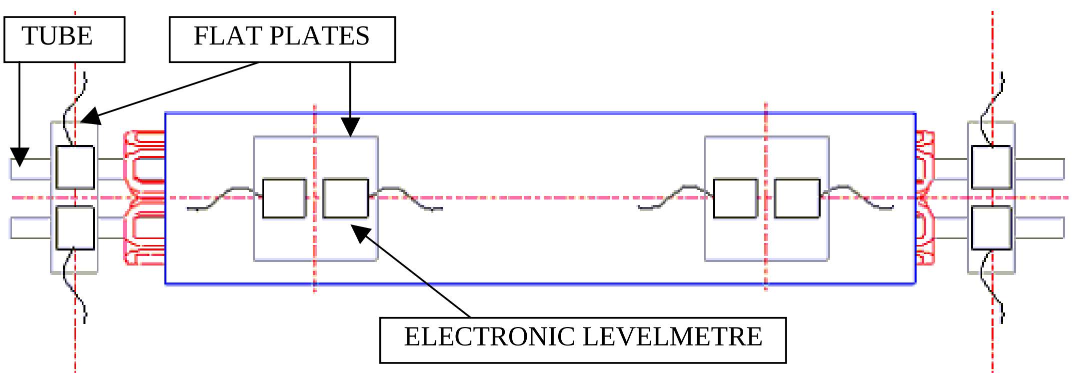


Fig. 2 Positioning of the electronic levelmetre when levelling the MQW magnets.

1.2 Alignment of the magnetic measurement stand to the magnet

The measurement stand is installed on the non-connection side of the magnet. It consists of a beam assembled on a chariot. The chariot is supported by three jacks. The front face of the chariot is parallel to the end side of the magnet. On the top of the beam there is a guideline onto which the motor and the mole supports are installed. The motor and these supports can slide along this guideline. The axis of the motor and the centre of the magnet aperture, which is to be measured, must be aligned. For that, we use a laser located inside the magnet and centred in one of the apertures at the connection end. The motor is positioned on the guideline far away from the magnet. A Taylor-Hobson sphere is placed on a sliding support at the same high of the axis of the motor. The laser beam light has to be aligned with the axis of the motor and hence the centre of the Taylor-Hobson sphere, wherever the sphere is along the guideline. So, the stand is displaced until we attain this alignment condition.

To measure the second aperture the beam is displaced transversally until the axis of the motor and the centre of the second aperture are aligned. We can do it with the help of a screw. Figures 3 and 4 show the measurement stand.

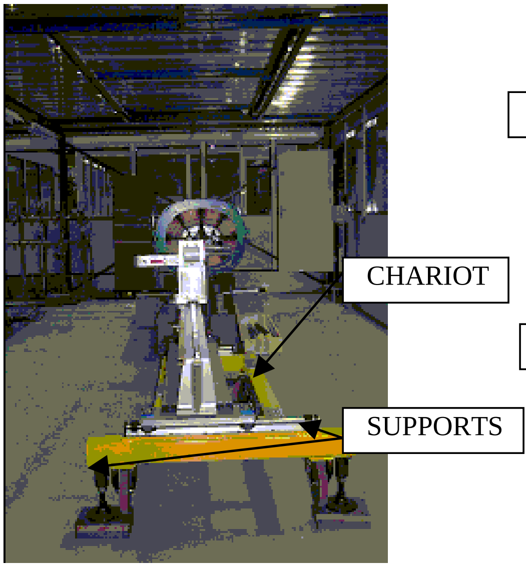


Fig. 3 Magnetic measurement stand as seen from the non-connection site.

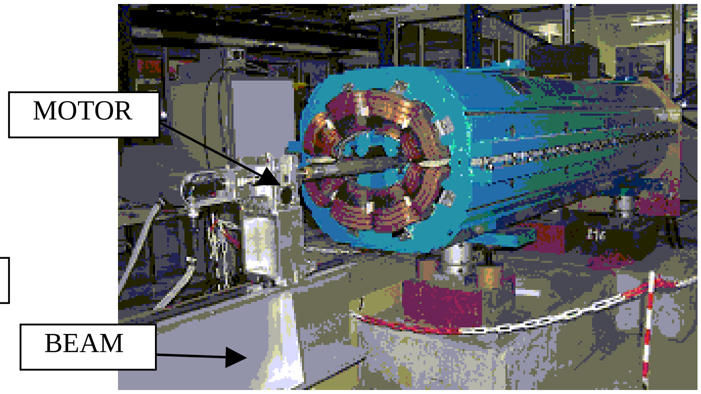


Fig. 4 Magnetic measuring stand as seen from the non- connection site.

2 Magnetic measurements

2.1 Measurement objective

The main objective of measuring the magnet is to ensure its field quality, starting from injection current up to 115% of the nominal current.

The second objective was to compare the measurement results with the 2D and 3D model simulations [1].

2.2 Hardware description

This chapter describes the mole, the data acquisition and the software of the magnet measurement installation.

2.2.1 Mole description

A mole composed of three rotating radial coils has been used for the magnetic measurements of the first MQW magnets. This mole was initially intended for the measurement of super-conducting LHC dipole magnets. The effective length of the coils is 748.05 mm, their effective coil surface is approximately 1.6 mm^2 and the effective outer diameter is 42 mm.

2.2.2 Data acquisition

The hardware architecture [2] is shown in Figure 4.

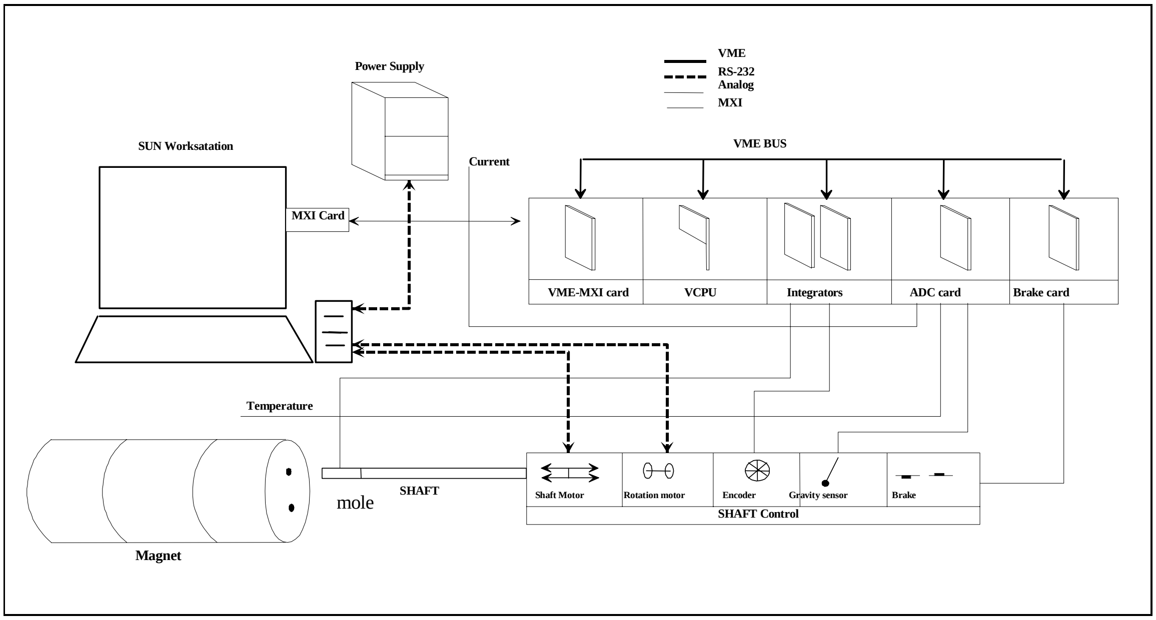


Fig. 4 Hardware architecture

The control unit of the hardware architecture is a SUN workstation equipped with an SB2020-MXI card for the data acquisition. Via an RS-232 serial port the workstation controls directly the shaft motor to level the mole with its 3 measuring coils to 0° horizontal position. This position is measured with a gravity sensor.

Via a second serial port the workstation controls a motor to rotate the 3 measuring coils. In order to avoid an over-ranged signal, the rotation speed of the coils has to be adapted to the excitation current in the magnet. To prevent the mole from vibrating during measurements, pneumatic breaks keep the mole in position.

The SB2020-MXI card on which is used on the SUN workstation is also connected to a VME crate using a VME-MXI Interface Module. The VME crate is equipped with integrators (Integrators AT680-2030 CERN build) to pick up the output signals from the three measuring coils. To measure the angular position of the measuring coils on the shaft an angular encoder is mounted. Its signal goes also to the integrators.

Since the measurement installation is used also for other applications, like the measuring of the LHC dipoles, not all options are used for our application. This includes the shaft sensor, temperature sensor and the control of the magnet power supply via the serial port RS-232.

Each output signal of the measuring coils, absolute and compensated, has a separated integrator. For a higher accuracy of the measurements, integrators with an amplifier (gain 100) are used for low current measurements. At high currents no amplifiers (gain 1) are needed. This gives a total of four integrators.

To power the magnet a power supply with the maximum current of 6500A was available. For higher accuracy at low current measurements we used the power supply with the maximum current of 350A.

2.2.3 Software

The software used to run and control the measurement installation is LABView. The program was developed by the CERN LHC-IAS group and contents more than 300 sub-routines. The program has also an option for online analysis, which was for our application not used.

2.3 Measurement procedure

The magnet is 3100 mm long, and the harmonics coils are 748.05 mm long, hence to cover the full magnetic field we defined 5 mole positions along the magnet. Figure 5 shows the 5 positions along the mole. In order to reach the positions we need two extension shafts with a length of about 1500 mm. The extension shafts and the mole were provided by the LHC/MMS group.

Fig. 5 Positioning of the mole along the MQW magnet.

A measurement consists of the reading of an absolute and a compensation signal as delivered by the rotating coils over a complete forward and backward of 360° rotation. At each position five measurements are done and an average is calculated for the analysis.

The electrical connections determine the magnetic polarity. Both apertures can operate with the same polarity, focusing/focusing (FF), or in opposite polarities, defocusing/focusing (DF). Measurements are done in both apertures at injection current (40A), at 200A, at nominal current (710A) and at 115% of the nominal current (810A) for the DF mode. For the FF mode, measurements were done at 40A, 200A and 600A.

In order to draw the BH-curve of the magnet, measurement from 10A up to 850A in steps of 50A were done. Both apertures were measured at mole position three.

Before starting the measurements, the magnet is demagnetised. The demagnetisation curve goes up to 300A. Figure 6 shows the demagnetisation curve.

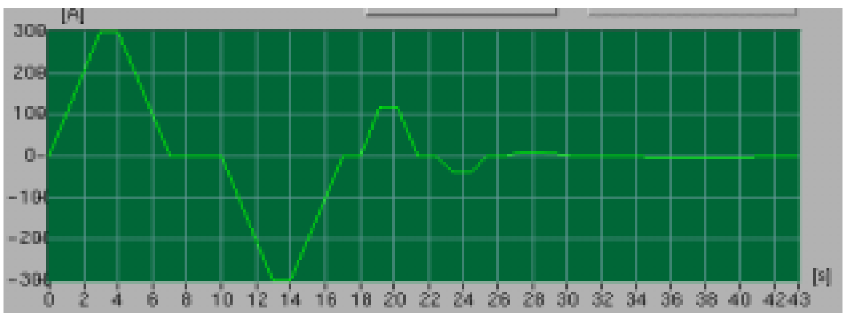


Fig. 6 Demagnetisation curve

2.4 Conventions

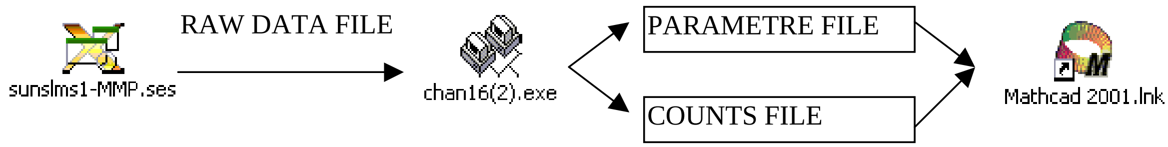
The LHC magnetic convention [3,4] have to be taken in account when analysing the measurements. We use as a measuring coil the one is on the right when the mole is installed in the magnet and looking at the magnet towards its non-connexion end. Also looking at the magnet towards the non-connexion side, the forward rotation of the coils is counter-clock wise and the backward rotation is clockwise. An easy sign transformation translates these results to the standard which is looking at the magnet towards the connection end.

3 Analysis of measurements

The data file contains the absolute and the compensation signals. For the measurement analysis we do not look at the bucked signal and the harmonics are computed directly from the absolute signal.

3.1 Software description

Once the measurements are done, the Lab-View program (MMP) generates a raw data file. This file contains the integrator counts at each angular interval and the time interval between two subsequent trigger pulses from the encoder. This file is treated with a program called CHAN16 [5]. The two new output files are ready to be treated with the MATHCAD program [6], which calculates the harmonic coefficients. The expressions used in this program are derived mainly from Ref. [7].



This MATHCAD program was originally written to treat measurements made with a five-coil mole. Hence, coil sensitivity coefficients had to be modified.

Integral harmonics are obtained by a weighted sum with respect to the main quadrupole component. The gradient at the central of the magnet is calculated by averaging the gradient at the inner positions of the magnet (positions 2, 3 and 4).

For the first analysis, the magnet is supposed to be a pure quadrupole. This means that the magnetic centre is found where the dipole is zero. Feed-down correction is necessary to compute the harmonics in the magnet centre. The non-normalised harmonics must be rotated in the reference frame of the main field, and normalised to main field afterwards.

The algorithm developed using MATHCAD is presented in the Annex.

3.2 Magnetic Centre

Initially the program calculated the magnetic centre foreseeing that there was no dipole field. As there is a dipole moment due to the cross talk between apertures, the program had to be modified. We know from numerical simulations that some higher order harmonics are zero. We use this consideration to calculate the magnetic centre [8].

Very high order unallowed terms are not sensitive to small construction errors in the magnet, and hence must be zero. We select a sufficiently high order allowed term (m = 10) and calculate the magnetic centre by requiring the neighbouring non-allowed term (m-1) to be zero.

At the outer positions the magnetic centre is deviated due to the influence of the connection ends. Hence, the centre location is calculated from the integral harmonics at the inner positions. The integral harmonics at the inner positions are obtained by a weighted sum with respect to the main quadrupole component.

The new total integral harmonics are calculated with respect to the new magnetic centre.

3.2.1 Example of effects

Theses examples are based on the measured integrated harmonics for the right aperture of the MQW001 magnet (first series magnet) at 40 A, DF mode.

The first example presents the measured integrated harmonics considering that it was a pure quadrupole (m=2). Values of the real and imaginary components of the harmonics are represented in the table 1 and figure 7. Notice that the values of the dipoles are decreased 10 times in the graphs for reasons of clearness of presentation.

The second case considers the nine-order harmonic to be zero. That means that the high order allowed term is the ten (m=10). The real and the imaginary components of the compensated harmonics are shown in table 2 and figure 8.

n	Re	Im
1	2.50	-71.6
2	10000	0.00
3	-0.81	4.80
4	2.08	-0.37
5	3.07	0.11
6	0.82	0.21
7	-0.80	-0.51
8	0.18	0.24
9	-0.07	0.05
10	0.77	0.03
11	-0.12	0.08
12	-0.03	0.08
13	0.02	-0.02
14	-0.13	0.12
15	0.00	0.03

Table 1 Integrated harmonics "pure quadrupole assumption"

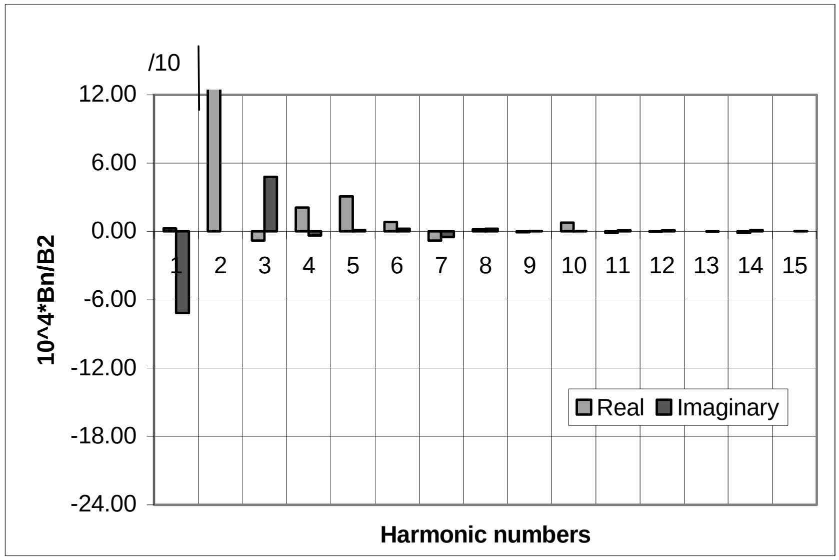


Fig. 7 Integrated Harmonics of the MQW001 magnet - 40A DF - Right aperture – m=2, "pure quadrupole assumption"

n	Re	Im
1	90.5	-219.9
2	10000	0
3	-0.77	4.69
4	2.20	-0.55
5	3.13	0.06
6	0.74	0.26
7	-0.77	-0.51
8	0.17	0.24
9	-0.01	-0.05
10	0.78	0.05
11	-0.11	0.10
12	-0.03	0.08
13	0.03	0.02
14	-0.12	0.12
15	0.00	0.03

Table 2 Integrated harmonics "nine-order harmonic to zero"

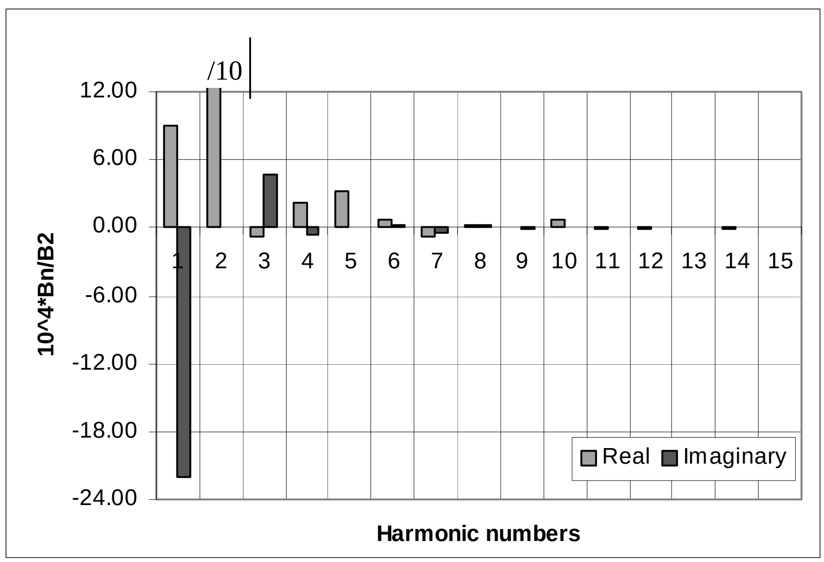


Fig. 8 Integrated Harmonics of the MQW001 magnet - 40A DF - Right aperture – m=10, "nine-order harmonic to zero"

Comparing both cases we see that there is a big difference between the dipole values. This effect it also confirmed at 710A. We can consider that the absolute value for the quadrupole and the rest of harmonics do not vary significantly.

1 Experience gained

1.1 Experience gained on the hardware side

During the measurements campaign the two main problems were the centring of the mole and adding the extension shafts for measurements.

1.1.1 Centring of the mole

For an accurate measurement of the multipoles in the quadrupole the correct centring of the mole is important. As written above in this report the mole slides inside a supporting tube for the measurements, which is fixed in the magnet's aperture. The aperture diameter is 46mm and the diameter of the support tube is 45mm. The first design of this fixture was not accurate enough. Modifications were done in such a way to hold the support tube with five 0.4mm thick rings, each 50mm long. This results in an offset of 0.2mm. 0.2mm is also the construction tolerances for the magnet. With tighter rings, e.g. 0.45mm thick, the support tube would not slide into the magnet's aperture.

The analysis of the measurements corrects this offset, as shown in chapter 3.2 Magnetic Centre.

Modifications were also done in the centring the mole itself in side the support tube. For centring four feet are used, two on the mole and two on the first extension shaft. Since we did not always use the same extension shaft for the measurement, the mole had sometimes only two feet and was thus inclined in the support tube. Adding four 1.5mm high feet to the mole solved this problem.

All this changes resulted in more reliable mesurements.

1.1.2 Adding extension shafts

To slide the mole inside the magnet to the different measurement positions we had to use extension shafts. Adding an extension shaft could provoke mechanical friction to the rotation of the measuring coils. The result would be a high torque at the first steps of the forward and backward rotation and an over-ranging on the integrators. Such measurements are useless for the analyses.

On the one hand the construction with the mole and extension shafts has to be stiff to ensure the correct positioning of the measurement coils and to avoid them from vibrating. On the other hand frictions must be kept small otherwise they would prevent the measuring coils from rotating. It was decided not to modify the design of such shafts.

During the measurement campaign we have learned how to add the shafts without provoking bad measurements. Additionally, the mechanical construction got abraded and a teeny mechanical play occurred, which eased the measurements.

1.2 Experience gained on the analysis

1.2.1 Raw data file

The Lab-View program (MMP) generates a raw data file, which includes the parameters of each measurement and the integrator counts. Five measurements are done at each position. The data of these five measurements is merged and saved giving the run number as a file name. This file is transferred via FTP to Windows and treated with the CHAN16 program.

Eventually, some lines are added at the end of the file when it is saved in the Lab-View format. These extra-lines must be removed. The file has to be verified before being transferred to Windows because these lines provoke errors when opening the file with the CHAN16 program. We still do not know the reason of this feature.

1.2.2 Recalibration of the mole

The mole was recalibrated. The coil surfaces and the barycentre values were slightly different from the values of the first calibration. These values had to be corrected in the analysis program since they influence the coil sensitivity coefficients.

1.2.3Measuring coils configuration

The measuring coils can be connected with different configurations. The coil sensitivity coefficients are related to the coil configuration. These coefficients have to be in concordance with the measuring coils configuration.

1.2.4 LHC conventions

The results of the magnetic field analysis are based on the measuring conventions that we decided to apply. These conventions are different from the LHC conventions; hence the integrated harmonics have to be translated to the general LHC conventions.

2 Conclusions

We use an electronic levelmetre to align the magnet proper. The alignment of the measuring stand to the magnet is successfully done with help of a laser and a Taylor-Hobson sphere.

The hardware architecture was applied before by the LHC-MMS group to measure the LHC quadrupole magnets at room temperature. We used the same structure. The same software is also used, with some modifications, to compute the harmonic coefficients at a given longitudinal position of the magnet.

The existence of transverse and torsional vibrations in the rotational motion of a coil can produce spurious harmonics. In this magnet, it is essential to buck out the quadrupole and the dipole terms to obtain accurate harmonics from the observed signal. This will be solved using the new 5 coils mole, where the coil design incorporates the ability to buck the dipole and the quadrupole components.

References

[1] Modelling in 2D of Magnetic Field of the MQW Magnet for the Series Production. E. Boter – SL-Note-2002-010-MS.

[2] Magnetic Measurements Program VME-SUN V. 5 (Saclay) - H. Reymond - CERN LHC/IAS/LS, 23 July 1998.

[3] LHC magnet polarities. P. Proudloch, P. Burla. LHC-DC-ES-0001 rev2.1.

[4] Field error naming conventions for LHC magnets. R.Wolf. LHC-M-ES-0001 rev 3.0.

[5] CHAN16 – converter program. V. Remondino LHC/MMS.

[6] Private communication. Vittorio Remondino.

[7] Procedures for Field Quality Measument of the LHC Magnets - Part I: Harmonics. L.Bottura, LHC/MTA-IN-97-007.

[8] CAS - Measurement and alignment of accelerator and detector magnets. Anacapri, Italy. April 1997.

[9] Magnetic Measurements of the LHC quadrupole magnets at room temperature. J.Billan, P.Galbraith, A.Musso, V. Remondino. LHC-MMS. LHC Project 2000-08-20.

ANNEX

$$\mathrm{Harmonic}(M, IN, \kappa, type) :=$$

$$
\begin{aligned}
& N \leftarrow M_3 \\
& \Delta t_f \leftarrow IN^{\langle 4 \rangle} \cdot 10^{-6} \\
& \Delta t_b \leftarrow IN^{\langle 5 \rangle} \cdot 10^{-6} \\
& \text{if } type = 0 \\
& \quad \Delta\psi_f \leftarrow IN^{\langle 0 \rangle} \\
& \quad \Delta\psi_b \leftarrow IN^{\langle 1 \rangle} \\
& \quad G_{int} \leftarrow M_1 \\
& \text{if } type = 1 \\
& \quad \Delta\psi_f \leftarrow IN^{\langle 2 \rangle} \\
& \quad \Delta\psi_b \leftarrow IN^{\langle 3 \rangle} \\
& \quad G_{int} \leftarrow M_2 \\
& t_{f_0} \leftarrow 0 \\
& t_{b_0} \leftarrow 0 \\
& KF \leftarrow 50000 \cdot G_{int} \\
& \psi_{f_0} \leftarrow 0 \\
& \psi_{b_0} \leftarrow 0 \\
& \text{for } k \in 1..N-1 \\
& \quad t_{f_k} \leftarrow t_{f_{k-1}} + \Delta t_{f_{k-1}} \\
& \quad t_{b_k} \leftarrow t_{b_{k-1}} + \Delta t_{b_{k-1}} \\
& \quad \psi_{f_k} \leftarrow \psi_{f_{k-1}} + \frac{\Delta\psi_{f_{k-1}}}{KF} \\
& \quad \psi_{b_k} \leftarrow \psi_{b_{k-1}} + \frac{\Delta\psi_{b_{k-1}}}{KF} \\
& \Delta t_{fave} \leftarrow \frac{t_{f_{N-1}} + \Delta t_{f_{N-1}}}{N} \\
& \Delta t_{bave} \leftarrow \frac{t_{b_{N-1}} + \Delta t_{b_{N-1}}}{N} \\
& \delta\psi_f \leftarrow \psi_{f_{N-1}} + \frac{\Delta\psi_{f_{N-1}}}{KF} \\
& \delta\psi_b \leftarrow \psi_{b_{N-1}} + \frac{\Delta\psi_{b_{N-1}}}{KF} \\
& \text{for } k \in 1..N-1 \\
& \quad \psi_{f_k} \leftarrow \psi_{f_k} - \frac{\delta\psi_f}{N} \cdot \frac{t_{f_k}}{\Delta t_{fave}} \\
& \quad \psi_{b_k} \leftarrow \psi_{b_k} - \frac{\delta\psi_b}{N} \cdot \frac{t_{b_k}}{\Delta t_{bave}} \\
& c_f \leftarrow \mathrm{CFFT}(\psi_f) \\
& c_b \leftarrow \mathrm{CFFT}(\psi_b) \\
& S1 \leftarrow \mathrm{last}(\kappa) \\
& R_{ref} \leftarrow 0.017 \\
& \text{for } s \in 1..S1 \\
& \quad C_{f_s} \leftarrow \frac{2 \cdot s \cdot R_{ref}^{\,s-1}}{\kappa_s} \cdot c_{f_s} \\
& \quad C_{b_s} \leftarrow \frac{2 \cdot s \cdot R_{ref}^{\,s-1}}{\kappa_s} \cdot c_{b_s} \\
& C_{ave} \leftarrow \frac{C_f - C_b}{2} \\
& C_{ave}
\end{aligned}
$$

$$\mathrm{Param}(P, JMM) :=$$

$$
\begin{aligned}
& \text{for } i \in 0..12 \\
& \quad MM_i \leftarrow P_{(JMM \cdot 13)+i} \\
& MM
\end{aligned}
$$

$$\mathrm{Increm}(INCR, JMM) :=$$

$$
\begin{aligned}
& \text{for } i \in 0..359 \\
& \quad \text{for } k \in 0..5 \\
& \quad \quad IN_{i,k} \leftarrow INCR_{(JMM \cdot 360)+i,k} \\
& IN
\end{aligned}
$$

Coil sensitivity calculation

Coil sensitivity

Coil Lenght: $L := 0.74805$

Number of turns: $N := 200$

Surfaces and dimensions

area (m**2) $S := \begin{pmatrix} 1.59127 \\ 1.59197 \\ 1.59166 \end{pmatrix}$ Baricenter (m) $d := \begin{pmatrix} 0.01377 \\ -0.00005 \\ -0.01381 \end{pmatrix}$

Width (m) $W := \frac{S}{N \cdot L}$ Left radius (m) $R_l := d - \frac{W}{2}$ Right radius (m) $R_r := d + \frac{W}{2}$

$n := 0..15$

$\kappa_{a_n} := N \cdot L \cdot \left[\left(R_{r_0}\right)^n - \left(R_{l_0}\right)^n \right]$

$S_l := \mathrm{last}(\kappa_a)$

File to create with chan16.exe: for positive currents par*.prn and coun*.prn

All results are referred to a current of: $I_{def} := 40$

Average positive harmonic computation at POS1

$PAR := READPRN(\text{"parPOS1.prn"})$ $COUNT := READPRN(\text{"counPOS1.prn"})$

$elem := \mathrm{last}(PAR)$ $Ifin := \frac{elem + 1}{13}$

$i := 0..Ifin - 1$ i= files number, s harmonic number

$I_i := PAR_{(13 \cdot i + 11)}$ $\alpha_i := PAR_{(13 \cdot i + 8)}$

$A_i := \mathrm{Harmonic}\left(\mathrm{Param}(PAR, i), \mathrm{Increm}(COUNT, i), \kappa_a, 0\right)$ Absolute Harmonic on + I

$s := 1..S_l$ Rotation according to gravity (inclinometer reading PAR(7))

$$AS_{s,i} := \left(A_i\right)_s \cdot e^{-j \cdot s \cdot \frac{\alpha_i}{1000}}$$

Normalization respect to specified current Idef (current reading PAR(13))

$$A_{ave} := \frac{I_{def}}{Ifin} \cdot \left[\sum_{i=0}^{(Ifin-1)} \frac{AS^{\langle i \rangle}}{I_i} \right] \qquad A_{stdev} := \sqrt{\overrightarrow{\frac{1}{Ifin} \cdot \sum_{i=0}^{Ifin-1} \left(\left| AS^{\langle i \rangle} - A_{ave} \right| \right)^2}}$$

$$Ip_{ave} := \frac{\sum_{i=0}^{Ifin-1} I_i}{Ifin}$$

$Ip_{ave} = 39.8954$ Average current measured

$$\alpha p_{ave} := \frac{\sum_{i=0}^{Ifin-1} \alpha_1}{Ifin}$$

$\alpha p_{ave} = -0.1649$ Average angle read by inclinometer (mrad)

Average Harmonics:

$C_{ave_0} := A_{ave}$ $I_{ave_0} := Ip_{ave}$ $\alpha_{ave_0} := \alpha p_{ave}$

Coil sensitivity calculation

Coil sensitivity

Coil Lenght: $L := 0.74805$

Number of turns: $N := 200$

Surfaces and dimensions

area (m**2) $S := \begin{pmatrix} 1.59127 \\ 1.59197 \\ 1.59166 \end{pmatrix}$ Baricenter (m) $d := \begin{pmatrix} 0.01377 \\ -0.00005 \\ -0.01381 \end{pmatrix}$

Width (m) $W := \frac{S}{N \cdot L}$ Left radius (m) $R_l := d - \frac{W}{2}$ Right radius (m) $R_r := d + \frac{W}{2}$

$n := 0..15$

$\kappa a_n := N \cdot L \cdot \left[\left(R_{r_0}\right)^n - \left(R_{l_0}\right)^n \right]$

$S_l := \mathrm{last}(\kappa_a)$

File to create with chan16.exe: for positive currents par*.prn and coun*.prn

All results are referred to a current of: $I_{def} := 40$

Average positive harmonic computation at POS1

PAR := READPRN("parPOS1.prn") COUNT := READPRN("counPOS1.prn")

$elem := \mathrm{last}(PAR)$ $Ifin := \frac{elem + 1}{13}$

$i := 0..Ifin - 1$ i= files number, s harmonic number

$I_i := PAR_{(13 \cdot i + 11)}$ $\alpha_i := PAR_{(13 \cdot i + 8)}$

$A_i := \mathrm{Harmonic}\left(\mathrm{Param}(PAR, i), \mathrm{Increm}(COUNT, i), \kappa_a, 0\right)$ Absolute Harmonic on + I

$s := 1..S_l$ Rotation according to gravity (inclinometer reading PAR(7))

$$AS_{s,i} := \left(A_i\right)_s \cdot e^{-j \cdot s \cdot \frac{\alpha_i}{1000}}$$

Normalization respect to specified current Idef (current reading PAR(13))

$$A_{ave} := \frac{I_{def}}{Ifin} \cdot \left[\sum_{i=0}^{(Ifin-1)} \frac{AS^{\langle i \rangle}}{I_i} \right] \qquad A_{stdev} := \sqrt{\frac{1}{Ifin} \cdot \sum_{i=0}^{Ifin-1} \overrightarrow{\left(\left|AS^{\langle i \rangle} - A_{ave}\right|\right)^2}}$$

$Ip_{ave} := \frac{\sum_{i=0}^{Ifin-1} I_i}{Ifin}$ $Ip_{ave} = 39.8954$ Average current measured

$\alpha p_{ave} := \frac{\sum_{i=0}^{Ifin-1} \alpha_1}{Ifin}$ $\alpha p_{ave} = -0.1649$ Average angle read by inclinometer (mrad)

Average Harmonics:

$C_{ave_0} := A_{ave}$ $I_{ave_0} := Ip_{ave}$ $\alpha_{ave_0} := \alpha p_{ave}$

Average positive harmonic computation at POS2

PAR := READPRN("parPOS2.prn") COUNT := READPRN("counPOS2.prn")

$elem := \mathrm{last}(PAR)$ $Ifin := \frac{elem + 1}{13}$

$i := 0..Ifin - 1$ i= files number, s harmonic number

$I_i := PAR_{(13 \cdot i + 11)}$ $\alpha_i := PAR_{(13 \cdot i + 8)}$

$A_i := \mathrm{Harmonic}\left(\mathrm{Param}(PAR, i), \mathrm{Increm}(COUNT, i), \kappa_a, 0\right)$ Absolute Harmonic on + I

$s := 1..S_l$ Rotation according to gravity (inclinometer reading PAR(7))

$$AS_{s,i} := \left(A_i\right)_s \cdot e^{-j \cdot s \cdot \frac{\alpha_i}{1000}}$$

Normalization respect to specified current Idef (current reading PAR(13))

$$A_{ave} := \frac{I_{def}}{Ifin} \cdot \left[\sum_{i=0}^{(Ifin-1)} \frac{AS^{\langle i \rangle}}{I_i} \right] \qquad A_{stdev} := \sqrt{\frac{1}{Ifin} \cdot \sum_{i=0}^{Ifin-1} \overrightarrow{\left(\left|AS^{\langle i \rangle} - A_{ave}\right|\right)^2}}$$

$Ip_{ave} := \frac{\sum_{i=0}^{Ifin-1} I_i}{Ifin}$ $Ip_{ave} = 39.8596$ Average current measured

$\alpha p_{ave} := \frac{\sum_{i=0}^{Ifin-1} \alpha_1}{Ifin}$ $\alpha p_{ave} = -0.1649$ Average angle read by inclinometer (mrad)

Average Harmonics:

$C_{ave_1} := A_{ave}$ $I_{ave_1} := Ip_{ave}$ $\alpha_{ave_1} := \alpha p_{ave}$

Average positive harmonic computation at POS3

PAR := READPRN("parPOS3.prn") COUNT := READPRN("counPOS3.prn")

$elem := \mathrm{last}(PAR)$ $Ifin := \frac{elem + 1}{13}$

$i := 0..Ifin - 1$ i= files number, s harmonic number

$I_i := PAR_{(13 \cdot i + 11)}$ $\alpha_i := PAR_{(13 \cdot i + 8)}$

$A_i := \mathrm{Harmonic}\left(\mathrm{Param}(PAR, i), \mathrm{Increm}(COUNT, i), \kappa_a, 0\right)$ Absolute Harmonic on + I

$s := 1..S_l$ Rotation according to gravity (inclinometer reading PAR(7))

$$AS_{s,i} := \left(A_i\right)_s \cdot e^{-j \cdot s \cdot \frac{\alpha_i}{1000}}$$

Normalization respect to specified current Idef (current reading PAR(13))

$$A_{ave} := \frac{I_{def}}{Ifin} \cdot \left[\sum_{i=0}^{(Ifin-1)} \frac{AS^{\langle i \rangle}}{I_i} \right] \qquad A_{stdev} := \sqrt{\frac{1}{Ifin} \cdot \sum_{i=0}^{Ifin-1} \overrightarrow{\left(\left|AS^{\langle i \rangle} - A_{ave}\right|\right)^2}}$$

$Ip_{ave} := \frac{\sum_{i=0}^{Ifin-1} I_i}{Ifin}$ $Ip_{ave} = 39.8944$ Average current measured

$\alpha p_{ave} := \frac{\sum_{i=0}^{Ifin-1} \alpha_1}{Ifin}$ $\alpha p_{ave} = -0.1796$ Average angle read by inclinometer (mrad)

Average Harmonics

$C_{ave_2} := A_{ave}$ $I_{ave_2} := Ip_{ave}$ $\alpha_{ave_2} := \alpha p_{ave}$

Average positive harmonic computation at POS4

$PAR := READPRN("parPOS4.prn")$ $COUNT := READPRN("counPOS4.prn")$

$elem := last(PAR)$ $Ifin := \frac{elem+1}{13}$

$i := 0..Ifin-1$ i= files number, s harmonic number

$I_i := PAR_{(13\cdot i+11)}$ $\alpha_i := PAR_{(13\cdot i+8)}$

$A_i := Harmonic(Param(PAR,i), Increm(COUNT,i), \kappa_a, 0)$ Absolute Harmonic on + I

$s := 1..S_l$ Rotation according to gravity (inclinometer reading PAR(7))

$$AS_{s,i} := (A_i)_s \cdot e^{-j\cdot s\cdot\frac{\alpha_i}{1000}}$$

Normalization respect to specified current Idef (current reading PAR(13))

$$A_{ave} := \frac{I_{def}}{Ifin}\cdot\left[\sum_{i=0}^{(Ifin-1)} \frac{AS^{\langle i\rangle}}{I_i}\right] \qquad A_{stdev} := \overrightarrow{\sqrt{\frac{1}{Ifin}\cdot\sum_{i=0}^{Ifin-1}\left(\left|AS^{\langle i\rangle}-A_{ave}\right|\right)^2}}$$

$$Ip_{ave} := \frac{\sum_{i=0}^{Ifin-1} I_i}{Ifin} \qquad Ip_{ave} = 39.873$$ Average current measured

$$\alpha p_{ave} := \frac{\sum_{i=0}^{Ifin-1}\alpha_1}{Ifin} \qquad \alpha p_{ave} = 0.0918$$ Average angle read by inclinometer (mrad)

Average Harmonics

$C_{ave_3} := A_{ave}$ $I_{ave_3} := Ip_{ave}$ $\alpha_{ave_3} := \alpha p_{ave}$

Average positive harmonic computation at POS5

$PAR := READPRN("parPOS5.prn")$ $COUNT := READPRN("counPOS5.prn")$

$elem := last(PAR)$ $Ifin := \frac{elem+1}{13}$

$i := 0..Ifin-1$ i= files number, s harmonic number

$I_i := PAR_{(13\cdot i+11)}$ $\alpha_i := PAR_{(13\cdot i+8)}$

$A_i := Harmonic(Param(PAR,i), Increm(COUNT,i), \kappa_a, 0)$ Absolute Harmonic on + I

$s := 1..S_l$ Rotation according to gravity (inclinometer reading PAR(7))

$$AS_{s,i} := (A_i)_s \cdot e^{-j\cdot s\cdot\frac{\alpha_i}{1000}}$$

Normalization respect to specified current Idef (current reading PAR(13))

$$A_{ave} := \frac{I_{def}}{Ifin}\cdot\left[\sum_{i=0}^{(Ifin-1)} \frac{AS^{\langle i\rangle}}{I_i}\right] \qquad A_{stdev} := \overrightarrow{\sqrt{\frac{1}{Ifin}\cdot\sum_{i=0}^{Ifin-1}\left(\left|AS^{\langle i\rangle}-A_{ave}\right|\right)^2}}$$

$$Ip_{ave} := \frac{\sum_{i=0}^{Ifin-1} I_i}{Ifin} \qquad Ip_{ave} = 39.8898$$ Average current measured

$$\alpha p_{ave} := \frac{\sum_{i=0}^{Ifin-1}\alpha_1}{Ifin} \qquad \alpha p_{ave} = -0.0548$$ Average angle read by inclinometer (mrad)

Average Harmonics

$C_{ave_4} := A_{ave}$ $I_{ave_4} := Ip_{ave}$ $\alpha_{ave_4} := \alpha p_{ave}$

Analysis starts here...

$\left|\left(C_{ave_0}\right)_2\right| = 0.01549$ $\left|\left(C_{ave_1}\right)_2\right| = 0.03467$ $\left|\left(C_{ave_2}\right)_2\right| = 0.03449$

$\left|\left(C_{ave_3}\right)_2\right| = 0.03459$ $\left|\left(C_{ave_4}\right)_2\right| = 0.02507$ Magnitude of C2

$$Ccentre := \frac{\left|\left(C_{ave_1}\right)_2\right| + \left|\left(C_{ave_2}\right)_2\right| + \left|\left(C_{ave_3}\right)_2\right|}{3} \qquad Ccentre = 0.03458$$

Magnetic lenght

$$Lm := \frac{\sum_{i=0}^{4}\left|\left(C_{ave_i}\right)_2\right|\cdot 0.74805}{Ccentre} \qquad Lm = 3.12145$$

Integral harmonics are obtained by summing the harmonics at the five **positions** and averaged them with their gradient C2 respect to the gradient at central position (Cs).

$$C_S := \frac{\overrightarrow{\sum_{i=0} C_{ave_i}\cdot\left|\left(C_{ave_i}\right)_2\right|}}{5\cdot Ccentre}$$

We use the average of the harmonics at the inner positions to calculate the magnetic centre (C_{Ax})

$$C_{Ax} := \frac{\sum_{i=1}^{3} C_{ave_i}\cdot\left|\left(C_{ave_i}\right)_2\right|}{3\cdot Ccentre}$$

Field module $\left|C_{Ax_2}\right| = 0.03459$

Center localization for the integral harmonics

$R_{ref} := 0.017$ $m := 10$ $B_{Ax} := Re(C_{Ax})$ $A_{Ax} := Im(C_{Ax})$

$$\Delta x := \frac{R_{ref}}{m-1}\cdot\frac{B_{Ax_m}\cdot B_{Ax_{m-1}} + A_{Ax_m}\cdot A_{Ax_{m-1}}}{\left(B_{Ax_m}\right)^2 + \left(A_{Ax_m}\right)^2} \qquad \Delta x = -2.25848\times 10^{-4}$$

$$\Delta y := \frac{R_{ref}}{m-1}\cdot\frac{B_{Ax_m}\cdot A_{Ax_{m-1}} - A_{Ax_m}\cdot B_{Ax_{m-1}}}{\left(B_{Ax_m}\right)^2 + \left(A_{Ax_m}\right)^2} \qquad \Delta y = 3.35516\times 10^{-4}$$

$$\Delta z := \Delta x + i\cdot\Delta y$$

$$\Delta z = -2.25848\times 10^{-4} + 3.35516i\times 10^{-4}$$

$r := 1..S_l$ Computation of the harmonics with respect to the new center for integral harmonics

$$C_{nave_r} := \sum_{k=r}^{S_l}\frac{(k-1)!}{(r-1)!\cdot(k-r)!}\cdot C_{S_k}\cdot\left(\frac{-\Delta z}{R_{ref}}\right)^{k-r}$$

Main field module $m := 2$

$G_m := \left|C_{nave_m}\right|$ $g_m := \frac{G_m}{R_{ref}^{m-1}}$ $G_m = 0.02577$

$B := Re(C_{nave})$ $A := Im(C_{nave})$

Main field Phase

$main_{(m)} := B_m + i \cdot A_m$

$\varphi_m := arg(main_m)$ $arg(main_m) = -3.08618$ $\varphi_m = -176.82529 deg$

$\varphi_m := if\left(\varphi_m < -\frac{\pi}{2}, \varphi_m + \pi, \varphi_m\right)$ $\varphi_m := if\left(\varphi_m > \frac{\pi}{2}, \varphi_m - \pi, \varphi_m\right)$

$\varphi_m = 0.05541$ $\varphi_m = 3.17471 deg$

Main Field direction $\beta_m := \frac{\varphi_m}{m}$ $\beta_m = 0.0277$

Rotation of harmonic coefficients with respect to the main field direction for the integral harmonics

$s := 1 .. S_l$

$sign := \frac{Re(C_{nave_2})}{\left|Re(C_{nave_2})\right|}$ $sign = -1$

$C_{rot_s} := C_{nave_s} \cdot e^{-i \cdot s \cdot \beta_m}$

Normalization

$c_{rot} := 10^4 \cdot \frac{C_{rot}}{sign \left|C_{rot_2}\right|}$

$c_{rot} =$

	0
0	0
1	90.47496-219.87901i
2	$10 \cdot 10^3$ +0.01387i
3	-0.77107+4.69221i
4	2.19555-0.54809i
5	3.12593+0.06076i
6	0.73593+0.25767i
7	-0.76879-0.51284i
8	0.17431+0.24002i
9	$-7.59757 \cdot 10^{-3}$ -0.05238i
10	0.77591+0.05205i
11	-0.11395+0.09619i
12	-0.02959+0.07891i
13	0.02716+0.01612i
14	-0.12101+0.12274i
15	$-1.35651 \cdot 10^{-3}$ +0.02939i