400 MHz Superconducting Cavities in the SPS

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Abstract

For the SPS to work as injector for the LHC it is necessary to install 400 MHz cavities in order to obtain the necessary bunch length prior to extraction of the beam towards LHC. To reduce the RF power requirements for these superconducting cavities the method of non integer harmonic number acceleration is proposed. Bunch compression is then achieved by adiabatically displacing the spectrum of the beam.

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1 Introduction

The transfer of bunches from the SPS injector to LHC requires some form of bunch compression before extraction. Otherwise the matching voltage in the LHC becomes uncomfortably small and the combined effects of injection errors and very heavy transient beam loading in both machines may lead to highly undesired capture losses [1],[2].

Non adiabatic bunch compression by rotation in the 200 MHz buckets cannot be applied in the SPS because the bunches in the head and in the middle of a batch, which experience completely different beam loading effects, could not all be matched to the LHC buckets.

Therefore it has been decided to compress the bunches adiabatically in the SPS prior to ejection, using an additional 400 MHz RF system composed of three single cell superconducting 400 MHz cavities. During acceleration these cavities are made transparent for the high intensity LHC type beam (0.85 A in LHC), while their (beam induced) voltage is raised adiabatically to 2 MV per cavity prior to ejection. To achieve this performance without special means, the power amplifier attached to each cavity (and its RF window) should have a capability in the MW range, far beyond of what can be envisaged.

The solution of the problem is to shift the frequency spectrum of the beam current outside the bandwidth of the 400 MHz cavities (with RF feedback). This is possible, thanks to the large bandwidth of the travelling wave cavities of the SPS, with the technique of non integer harmonic number acceleration already tested in view of heavy ion acceleration.

2 Parameters of the LHC beam in the SPS

The LHC beam in the SPS consists of three PS batches, each containing 81 bunches separated by 25 ns. Between batches 8 bunches are missing leaving enough space to accommodate the rise time of the SPS injection kicker. The total length of the accelerated beam (6.4 $\mu$s) is much smaller than the revolution period of the SPS (23 $\mu$s) leaving ample space for RF manipulations with the SPS travelling wave cavities (filling time 500 ns). The overall beam structure is given in Fig. 1, which shows the envelope of the maximum bunch current at 26 GeV/c for bunches 4.5 ns long. For the ultimate LHC parameters, which determine the design of the hardware, each bunch contains $1.7 \cdot 10^{11}$ protons. Thus the maximum bunch current at injection is 12.1 A (for 4.5 ns bunches of cosine squared shape) and the amplitude of the spectral line at 400 MHz is 26.6 mA (see Fig. 5).

During acceleration the bunch length shrinks and, on the 450 GeV extraction flat top, with the 200 MHz travelling waves alone (maximum available voltage 8 MV), its length is reduced to 2 ns (for a longitudinal emittance of 1eVs per bunch). This leads to a maximum bunch current of 27.3 A (for 2 ns bunches of cosine squared shape). Now the amplitude of the spectral line at 400 MHz is 373 mA, i.e. it is about 14 times larger than at injection.

The 2 ns bunch length is too long for proper capture in the LHC in the presence of heavy transient beam loading and injection errors. Further adiabatic compression is needed at 450 GeV. It is achieved with an additional voltage of 6 MV at 400 MHz. The final bunch length with the two RF systems working in phase will be 1.7 ns [3].
3 400 MHz power requirements with RF feedback

Following present SPS experience the total impedance of the 400 MHz superconducting cavities should not exceed 500 kΩ (166 kΩ/cavity) in order to avoid coupled bunch instabilities during the acceleration ramp. Although this figure has been evaluated for the present high intensity beam for fixed target operation, we propose to keep it also for the LHC type beam, which has a similar average intensity. If, however, coupled bunch instabilities develop, because of the higher peak intensity of the LHC beam, they will be damped by an additional bunch-to-bunch longitudinal feedback.

The necessary impedance reduction is obtained by applying RF feedback (Fig. 2). At the cavity center frequency $f_{c}$ the return path (gain $G_0 = 6 \cdot 10^{-6}$ A/V) is equivalent to a parallel resistance $R = 1/G_0 = 166$ kΩ across the LC circuit which represents the cavity. Then the -3 dB cavity bandwidth (with $G$ purely real) is given by

$$\delta f_{FB} = \pm \frac{1}{2} \frac{f_c R/Q}{R} = \pm \frac{1}{2} G_0 \frac{R}{Q} f_c$$

(1)

that means $\delta f_{FB} = \pm 52$ kHz, with $R/Q = 43.5$ Ω [1] and $f_c = 400$ MHz.

The relation between beam current $i_{b}$ and generator current $i_{g}$ is given by the transfer function (Fig. 2):

$$i_g = -\frac{G Z}{1 + GZ} i_b$$

(2)

with

$$Z = \frac{R}{2 j Q \delta f_{c}}$$

(3)

for a superconducting cavity without losses. $\delta f_{c}$ is the deviation from the resonant frequency of the cavity $f_{c}$.

Note that the unavoidable delay in the return path ($\tau = 500$ ns) makes $G$ complex ($G = G_0 D = G_0 \exp\{ -j (\omega - \omega_c) \tau \}$). The result is that the distance between the -3 dB points becomes ±64 kHz.

However, as $G_0$ is considerably smaller than its maximum value given by loop stability, the loop delay introduces a negligible phase shift in the open loop response (phase margin of 81°, gain margin of 19.6 dB). Consequently, as a first approximation, $G$ can be considered real ($G = G_0$). Eq. 2 becomes in this case

$$i_g \approx -\frac{1}{1 + j \frac{\delta f_{c}}{|\delta f_{FB}|}} i_b$$

(4)

which is the response of a band pass filter (unity gain at the center frequency $f_{c}$, bandwidth $\delta f_{FB} = \pm 52$ kHz, slope 6 dB/octave away from $f_{c}$).

In order to compress the bunches at the flat top before ejection, the additional 400 MHz voltage must be in quadrature with the beam current; above transition this would be obtained with minimum power if the 400 MHz cavity impedance is capacitive at twice the RF frequency. In this case the beam induced voltage in the superconducting cavities would increase the longitudinal focusing. The same argument applies for the negative mass instability, just above transition, which is provoked by the predominantly capacitive impedance of the vacuum chamber at microwave frequencies.
As a consequence the superconducting cavities (working at a fixed frequency) must be tuned below twice the RF frequency at 450 GeV. During acceleration, the RF frequency increases from 200.265 MHz (26 GeV/c) to 200.40 MHz (450 GeV) and necessarily its second harmonic coincides with \( f_c \) during the ramp.

For the ultimate LHC beam current of \( 1.7 \cdot 10^{11} \) protons/bunch, one finds under these circumstances and using the exact Eq. 2, that the maximum amplitude of the generator current is \( i_g = 1.24 \) A. Figs. 3 and 4 show the spectrum of \( i_g \) and the envelope of its time domain evolution when the cavity center frequency \( f_c \) coincides with the second harmonic of the RF frequency (450 GeV, 2 ns bunch length, \( V_{ref} = 0 \)). For all practical purposes \( i_g(t) \) is a 400 MHz amplitude and phase modulated sine wave. Fig. 4 shows additionally the beam current \( i_b(t) \) (batch structure and amplitude of the 400 MHz component for a completely filled ring). As expected, \( i_g(t) \) looks like the output of a band pass filter (Eq. 4) with \( i_b(t) \) as input.

The klystron connected to the superconducting cavity has to deliver the power necessary to produce a current of 1.24 A for a gap voltage of \( V_{cau} = 2 \) MV (during bunch compression). As the cavity voltage has to be in quadrature to the beam current its impedance is nearly pure reactive, such that practically all the incident power is reflected, leading to a cavity voltage amplitude which is about twice that of the incident wave. Therefore the klystron power is given by

\[
\hat{P} = \frac{1}{4} V_{cau} i_g = 620 \text{ kW} \quad (5)
\]

exceeding by far the RF window power limit of about 220 kW.

4 Frequency modulation in the SPS

To reduce the power demand one can shift the frequency spectrum of the beam current above \( f_c \). This is possible because the LHC beam only fills about 28% of the SPS circumference: the instantaneous RF frequency during the passage of the batch (the batch frequency) does not need to be an exact multiple of the revolution frequency \( f_0 \). Rephasing can take place in the long gap, thanks to the rapid filling time (about 1 to 2 \( \mu s \)) of the travelling wave system. This technique will be used for lead ion acceleration; it has already been tested successfully with protons [4].

The center frequency of the superconducting cavity is very close to a harmonic of the revolution frequency at 450 GeV, \( f_c = 400.79 \) MHz.

At 26 GeV/c the peak of the spectrum of the injected LHC beam falls below \( f_c \) (see Fig. 5). It is centered at 400.53 MHz.

As already mentioned, the envelope of the beam spectrum can be displaced by increasing the batch frequency \( f_{batch} \) from its original value of 200.255 MHz to 200.560 MHz. The result is that now the peak of the beam spectrum lies above \( f_c \) (see Fig. 6). The spectrum is now centered at 401.12 MHz. The batch frequency change of 295 kHz at 200 MHz corresponds to twice that value at 400 MHz. In both spectra the spectral lines are at multiples of the revolution frequency corresponding to 26 GeV/c.

During acceleration the batch frequency will be kept constant. Therefore the beam spectrum at 450 GeV is essentially the same as at 26 GeV/c (see Fig. 7). The spectrum is still centered at 401.12 MHz. The only difference is, that now the spectral lines are at
multiples of the revolution frequency corresponding to 450 GeV and that the magnitude of the spectral components is higher than at 26 GeV/c because the bunch length is reduced from 4.5 ns to 2 ns.

Therefore the proposed scenario for acceleration of the LHC beam in the SPS is the following:

- Capture the bunches (4.5 ns long) from the PS at the 26 GeV/c flat bottom with the 200 MHz travelling wave cavities (harmonic number $h = 4620$, $f_{RF} = 200.265$ MHz, $f_{batch} = 200.265$ MHz).

- Increase the batch frequency from its original value of 200.265 MHz to 200.560 MHz and thus displacing the envelope of the beam spectrum above $f_c$.

At 26 GeV/c the bunches are 4.5 ns long and therefore their 400 MHz component is still relatively small. The maximum 400 MHz generator power is needed during this operation when the beam spectrum coincides with the cavity frequency $f_c$. One finds under these circumstances $i_g = 89$ mA, $\dot{P} = 44.5$ kW.

The maximum batch frequency is determined by the bandwidth of the travelling wave cavities and their power amplifiers. We take as maximum batch frequency 200.4 MHz + 0.16 MHz. The value of 200.4 MHz is about $4620 \cdot f_0$ at 450 GeV. This maximum batch frequency displaces the center of the spectrum of the beam by about 330 kHz with respect to the cavity frequency $f_c$. However, shifting the batch frequency from 200.4 MHz to 200.56 MHz leads to a reduction of the available voltage of the 4 sections long travelling wave cavities by 4.2% due to the transit time effect [5]. This is still considered acceptable.

- Acceleration to 450 GeV with the travelling wave cavities at constant batch frequency.

In order to minimize the generator current, the bunch length should not decrease to values smaller than 2 ns. During acceleration the 400 MHz generator current will be highest at the final energy, because the bunch length will be minimal at that moment.

For the flat top energy of 450 GeV the spectrum of the generator current is shown in Fig. 8 and its time domain representation in Fig. 9 ($V_{ref} = 0$). Thus we obtain $i_g = 416$ mA and $\dot{P} = 206$ kW, for the batch frequency of 200.56 MHz.

## 5 Notch filter in the feedback path

The spectrum of the generator current $i_g$ (Fig. 8) shows a high peak close to $2f_{batch}$. To reduce $i_g$, one can envisage a notch filter inserted in the return path of the RF feedback. The zero of the filter should coincide with this peak at $9247f_0$. The maximum of the notch filter transfer function should be at $f_c$. Such a filter can be made using a delay (Fig. 10),

$$\tau_{notch} = \frac{1}{2(9247f_0 - f_c)}$$  \hspace{1cm} (6)

In our case $\tau_{notch} = 1.64\mu$s.
The transfer function of the filter is given by

\[ H(j\omega) = \frac{1}{2} \left( 1 + e^{-j(\omega - \omega_0)r_{\text{notch}}} \right) \tag{7} \]

Analysis of the open loop response shows that with this notch filter, there is still a phase margin of 65° and a gain margin of 16.2 dB in the RF feedback loop.

The new transfer function

\[ i_g = \frac{-GZH}{1 + GZH} i_b \tag{8} \]

applied to the displaced beam spectrum, leads to a spectrum of \( i_g \) as shown in Fig. 11 (\( V_{\text{ref}} = 0 \)). Its time domain representation (Fig. 12) shows a further decrease of the peak current to be delivered by the power amplifier (now \( i_g = 258 \) mA). The generator peak power for the ultimate LHC parameters is then \( \dot{P} = 129 \) kW, which can be obtained with the proposed LHC klystrons (which deliver a peak power of 300 kW).

6 Bunch compression

6.1 Principle

At 450 GeV the center frequency of the superconducting cavity is given by

\[ f_c = 2 \cdot 4620 \cdot f_0 + \Delta f_{\text{DT}}. \tag{9} \]

The cavity detuning of \( \Delta f_{\text{DT}} = -0.035 f_0 = -1.5 \) kHz, is such that one obtains a maximum cavity voltage of \( V_{\text{cav}} = 2.1 \) MV with nominal beam characteristics (2 ns bunches).

In the following, the displacement of the beam spectrum will not be described by the batch frequency \( f_{\text{batch}} \) but by a parameter \( \Delta f_c \) representing the distance of the center of the beam spectrum, which is at \( 2 \cdot f_{\text{batch}} \), with respect to the harmonic 400 MHz frequency of \( 2 \cdot 4620 f_0 \): \( \Delta f_c = 2(f_{\text{batch}} - 4620 f_0) \). So the maximum value of the frequency displacement \( \Delta f_c \) is 330 kHz (see Chapter 4).

Bunch compression is obtained by adiabatically reducing \( \Delta f_c \) from the maximum value of 330 kHz to zero. While reducing \( \Delta f_c \) the generator current \( i_g \) will be kept zero by applying an appropriate voltage \( V_{\text{ref}} \) to the RF feedback loop (Fig. 2). During this process the cavity voltage will increase like shown in Fig. 13.

The cavity voltage will not be exactly the same for all bunches at the end of the compression process as there are residual amplitude and phase modulations. However, they are small enough to ensure nearly equal compression for all bunches. Figs. 14 and 15 show these amplitude and phase modulations. Here the phase \( \Phi_{\text{rel}} \) is the relative phase of \( V_{\text{cav}} \) with respect to the harmonic frequency \( 2 \cdot 4620 \cdot f_0 \).

The frequency spectrum of the beam will change during the compression process due to a change of \( \Delta f_c \), a change of bunch length and due to a phase modulation produced by the phase modulated voltage of the superconducting cavities. As a first approximation the effect of the phase modulation will be neglected, i.e. the assumption is, that the voltage of the 200 MHz travelling wave cavities determines entirely the azimuthal position of the bunches within the beam.
6.2 The reference voltage for minimal generator power

During flat bottom and during acceleration it is sufficient to set the reference voltage \( V_{ref} = 0 \) in order to keep the maximum generator current \( i_g \) low, as it was explained before. As the revolution frequency stays constant at flat top it is theoretically feasible to reduce \( i_g \) to zero (no demand of generator power) by application of an appropriate \( V_{ref} \neq 0 \). This reference voltage \( V_{ref} \) has to be calculated for each step of the compression process as a function of the actual beam spectrum, which is mainly determined by the frequency displacement \( \Delta f_c \), the bunch length and the bunch current.

However, \( i_g \) will not be exactly zero because the parameters for calculating \( V_{ref} \) will not be exactly known. Therefore it is necessary to ensure that these errors do not lead to an \( i_g \) greater than the design value given in Chapter 5 (\( i_g = 258 \) mA).

From Fig. 2 (RF feedback schematics) we obtain the reference voltage \( V_{ref} \) for the ideal case of \( i_g = 0 \) as

\[
V_{ref} = ZHDi_b. \tag{10}
\]

For the bunch length of \( t_0 = 2\)ns and no notch filter, Fig. 16/17, Fig. 18/19 and Fig. 20/21 show the amplitude and phase of \( V_{ref} \) for \( \Delta f_c = 0 \), 100 kHz and 300 kHz. With a notch filter in the feedback loop, \( V_{ref} \) amplitude and phase are shown in Fig. 22/23, Fig. 24/25 and Fig. 26/27 also for \( \Delta f_c = 0 \), 100 kHz and 300 kHz. The phase \( \Phi_{ref} \) is the relative phase of \( V_{ref} \) with respect to the harmonic frequency \( 2 \cdot 4620 \cdot f_0 \). The amplitude of the cavity voltage as a function of time, \( V_{cav}(t) \), has the same shape as \( V_{ref}(t) \) (for no notch filter in feedback path, \( H = 1 \)), except for the displacement in time due to the loop delay.

The development of min./max. of \( V_{ref} \) as a function of the frequency displacement \( \Delta f_c \) is exactly the same as for \( V_{cav} \) (see Fig. 13) in case of \( H = 1 \), and it is practically the same if a notch filter is present.

This reference voltage \( V_{ref} \) could be generated by fast programmable function generators driving RF amplitude and phase modulators.

6.3 Error analysis of \( V_{ref} \)

6.3.1 Introduction

The reference voltage \( V_{ref} \) is a function of \( i_b \) (bunch intensity, bunch length, bunch spacing, etc). The generator current will be zero only for the reference voltage \( V_{ref} \) which matches exactly with the beam parameters. However, not all parameters of \( i_b \) are exactly known, thus leading to \( i_g \neq 0 \).

In the following the effect of a mismatch of the parameters bunch length \( t_0 \), frequency displacement \( \Delta f_c \) and individual batch current will be examined. The reference voltage \( V_{ref} \) will be calculated for an ideal set of beam parameters. Using the actual beam parameters the generator current \( i_g \) is then given by

\[
i_g = \frac{G_0}{1 + ZDH} \left( V_{ref} - ZDHi_b \right). \tag{11}\]

There are parameters which are supposed not to vary significantly from pulse to pulse but may be different than expected at a certain time within the cycle. It seems that these errors
can be trimmed out (like errors on bunch length and frequency displacement). Parameters which might vary from pulse to pulse are the individual batch currents.

6.3.2 Influence of Unequal Batch Currents on $i_g$

The reference voltage $V_{ref}$ is based on the average batch current (which is in principle known at the time of bunch compression). If the individual batch currents are not identical, then $i_g$ will not be zero (Eq. 11).

Examination of several cases shows that a deviation of an individual batch current from the average batch current leads to the highest $i_g$ values at $\Delta f_c = 0$ for a RF feedback loop without notch filter ($H = 1$). If there is a notch filter, the frequency dependence is less pronounced. Fig. 28 and Fig. 29 show this frequency dependence for the case of relative batch amplitudes 0.8, 1.0, 1.0 (1.0 being the nominal batch intensity).

If there is a deviation from the nominal batch current of equal size, the effect on $i_g$ is worst if the deviation happens to be on the first batch, less pronounced if it happens to be on the second batch and even less in case of the third batch. The following Table shows the corresponding values for a typical example.

<table>
<thead>
<tr>
<th>rel. batch amplitudes</th>
<th>$i_g$ [mA]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8 1.0 1.0</td>
<td>103</td>
</tr>
<tr>
<td>1.0 0.8 1.0</td>
<td>81</td>
</tr>
<tr>
<td>1.0 1.0 0.8</td>
<td>74</td>
</tr>
</tbody>
</table>

Several cases of amplitude variations were examined: the individual batch intensity may be up to 20% smaller than the nominal one and $i_g$ will still be less than 100 mA. An example is given in Fig. 30 (2 ns bunches, $\Delta f_c = 0$, without notch filter, relative batch amplitudes 0.8, 1.0, 1.0).

6.3.3 Influence of Bunch Length on $i_g$

The reference voltage $V_{ref}$ depends on the bunch length (Eq. 10). If the actual bunch length is different than the one used for calculating $V_{ref}$ this will lead to $i_g \neq 0$. From the inspection of several cases it follows that the effect of a bunch length variation does not depend significantly if a notch filter is used or not within the RF feedback loop.

Two extreme examples are used to demonstrate the influence of bunch length error on the maximum generator current $i_g$. $V_{ref}$ was calculated for $t_0 = 2$ ns, $\Delta f_c = 0$. For an actual bunch length of $t_0 = 1.7$ ns and $t_0 = 2.5$ ns one obtains $i_g = 175$ mA and $i_g = 305$ mA, respectively. Fig. 31 shows $i_g$ versus time for the second case ($t_0 = 2.5$ ns) and with notch filter. Without notch filter, $i_g(t)$ has about the same maximum amplitude, but the shape of $i_g(t)$ is like in Fig. 4. For $t_0 = 1.7$ ns the envelope of the generator current $i_g(t)$ has the same form like for $t_0 = 2.5$ ns but scaled down to the maximum amplitude of about 175 mA.

These peak currents seem to be high but it is expected that the actual bunch length at a certain time within the cycle will be relatively constant, allowing to trim out corresponding errors.

Another way to reduce these high $i_g$ values is the use of a real time bunch length measurement and a permanent reevaluation of $V_{ref}$ as a function of bunch length. However, a simple
scaling of $V_{ref}$ with the Fourier coefficients of a train of cosine squared pulses, $C_{10}/C_0$ [7], which are a function of the bunch length $t_0$, reduces $i_g$ in the two aforementioned cases to about 1 mA.

6.3.4 Influence of $\Delta f_c$ errors on $i_g$

The reference voltage $V_{ref}$ is also a function of the frequency displacement $\Delta f_c$. If the actual frequency displacement is different than it was assumed for the $V_{ref}$ calculation, this will lead to $i_g \neq 0$.

Inspection of several cases shows that for otherwise nominal beam parameters, $i_g$ will be smaller than 100 mA for $\Delta f_c$ errors smaller than 3 kHz. Fig. 32 shows the envelope of $i_g(t)$ for a typical case (2 ns bunches) with $\Delta f_c = 0$ nominally but actually being 3 kHz larger (no notch filter). With notch filter $i_g$ is about 10% smaller.

6.3.5 Influence of combination of errors on $i_g$

It is difficult to estimate the effect of a combination of batch intensity variations, bunch length variations and errors in the frequency displacement without actually calculating $i_g$ for each of these combinations. Taking a number of examples it was seen that these errors compensate or enhance each other depending on the exact circumstances.

References

A Figures

Figure 1: Envelope of max. bunch current (26 GeV/c, 4.5 ns bunches)

Figure 2: RF feedback block diagram
Figure 3: Spectrum of $i_g$ ($f_c = 2f_{RF}$, 450 GeV, 2 ns bunches)

Figure 4: Envelope of $i_g$ and RF component of $i_b$ (450 GeV, 2 ns bunches)
Figure 5: Spectrum of $i_b$ for $f_{\text{batch}} = 200.265$ MHz, (26 GeV/c, 4.5 ns bunches)

Figure 6: Spectrum of $i_b$ for $f_{\text{batch}} = 200.56$ MHz, (26 GeV/c, 4.5 ns bunches)
Figure 7: Spectrum of $i_b$ for $f_{\text{batch}} = 200.56$ MHz, (450 GeV, 2 ns bunches)

Figure 8: Spectrum of $i_g$ ($f_{\text{batch}} = 200.56$ MHz, 450 GeV, 2 ns bunches)
Figure 9: Envelope of $i_p$ and RF component of $i_b$,
$(f_{batch} = 200.56 \text{ MHz, } 450 \text{ GeV, } 2 \text{ ns bunches})$

Figure 10: Notch filter
Figure 11: Spectrum of $i_g$ with notch filter ($f_{batch} = 200.56$ MHz, 450 GeV, 2 ns bunches)

Figure 12: Envelope of $i_g$ and RF component of $i_b$ with notch filter, ($f_{batch} = 200.56$ MHz, 450 GeV, 2 ns bunches)
Figure 13: Amplitude of min./max. $V_{CAV}$ vs. $\Delta f_c$
Figure 14: Amplitude of $V_{cau}$ vs. time, $\Delta f_c = 0$, (for $i_g = 0$)

Figure 15: Phase of $V_{cau}$ vs. time, $\Delta f_c = 0$, (for $i_g = 0$)
Figure 16: Amplitude of $V_{ref}$ vs. time, $\Delta f_c = 0$, without notch filter

Figure 17: Phase of $V_{ref}$ vs. time, $\Delta f_c = 0$, without notch filter
Figure 18: Amplitude of $V_{ref}$ vs. time, $\Delta f_c = 100$ kHz, without notch filter

Figure 19: Phase of $V_{ref}$ vs. time, $\Delta f_c = 100$ kHz, without notch filter
Figure 20: Amplitude of $V_{ref}$ vs. time, $\Delta f_c = 300 \text{ kHz}$, without notch filter

Figure 21: Phase of $V_{ref}$ vs. time, $\Delta f_c = 300 \text{ kHz}$, without notch filter
Figure 22: Amplitude of $V_{ref}$ vs. time, $\Delta f_c = 0$, with notch filter

Figure 23: Phase of $V_{ref}$ vs. time, $\Delta f_c = 0$, with notch filter

21
Figure 24: Amplitude of $V_{ref}$ vs. time, $\Delta f_c = 100$ kHz, with notch filter

Figure 25: Phase of $V_{ref}$ vs. time, $\Delta f_c = 100$ kHz, with notch filter
Figure 26: Amplitude of $V_{ref}$ vs. time, $\Delta f_c = 300$ kHz, with notch filter

Figure 27: Phase of $V_{ref}$ vs. time, $\Delta f_c = 300$ kHz, with notch filter
Figure 28: Max. generator current $i_g$ vs. $\Delta f_c$, effect of batch amplitude error (without notch filter)

Figure 29: Max. generator current $i_g$ vs. $\Delta f_c$, effect of batch amplitude error (with notch filter)
Figure 30: Envelope of $i_d$ and of RF component of $i_b$ versus time, effect of batch amplitude error (without notch filter)

Figure 31: Envelope of $i_d$ and of RF component of $i_b$ versus time, effect of bunch length error (with notch filter)
Figure 32: Envelope of $i_g$ and of RF component of $i_h$ versus time, effect of $\Delta f_c$ error (without notch filter)