Hadronic cross section from radiative return*

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Abstract

The impact of final-state radiation (FSR) on the radiative return method for the extraction of the \( e^+e^- \) hadronic cross section is discussed in detail and experimental tests of the model dependence of FSR are proposed for the \( \pi^+\pi^- \) hadronic final state.

HIGHLIGHTS OF THE RADIATIVE RETURN METHOD

Knowledge of the cross section of the electron–positron annihilation into hadrons in the low energy region is crucial for predictions of the hadronic contributions to \( a_\mu \), the anomalous magnetic moment of the muon, and to the running of the electromagnetic coupling (For reviews see e.g. [1-7]; the most recent experimental result for \( a_\mu \) is presented in [8]). Measurements of the electron–positron hadronic cross section were traditionally performed by varying the beam energy of the collider, but the \( \Phi \)- and B-meson factories allow to use the radiative return to explore the whole energy region from threshold up to the energy of the collider. Even if the photon radiation from the initial state reduces the cross section by a factor \( \alpha/\pi \), this is easily compensated by the enormous luminosity of these ‘factories’. A number of experimental results based on the radiative return was already published [9-17] and in these ‘factories’. A number of experimental results based on the radiative return was already published [9-17] and in the near future one can expect much more data covering large variety of hadronic final states.

The radiative return method [18] (see also [19]), relies on the following factorisation property of the cross section

\[
\frac{d\sigma}{dQ^2 d\Omega_\gamma}(e^+e^- \rightarrow \text{hadrons} + \gamma) = H(Q^2, \Omega_\gamma) \sigma(e^+e^- \rightarrow \text{hadrons}, Q^2),
\]

where \( Q^2 \) is the invariant mass of the hadronic system, \( \Omega_\gamma \) denotes the photon polar and azimuthal angles, and the function \( H(Q^2, \Omega_\gamma) \) is given by QED lepton-photon interactions, thus known in principle with any required precision. The formula (1) is valid for a photon emitted from initial state leptons (ISR) and what is more important similar factorisation formula applies for the emission of an arbitrary number of photons or even lepton pairs [20, 21] from initial state leptons. If there is no FSR contribution, one can, by measuring the \( Q^2 \) differential cross section of the process \( e^+e^- \rightarrow \text{hadrons} + \text{photons} + (\text{possibly}) \text{lepton pairs} \) and knowing function \( H(Q^2, \ldots) \), extract the value of \( \sigma(e^+e^- \rightarrow \text{hadrons}) \). The ... in \( H(Q^2, \ldots) \) stand for the phase space variables of photons and/or lepton pairs. In practice that cross section is measured within a given experimental setup, corresponding to an integral over a complicated phase space of photons and/or lepton pairs. As a result the use of Monte Carlo event generators for extraction of the \( \sigma(e^+e^- \rightarrow \text{hadrons}, Q^2) \) become indispensable. Such Monte Carlo programs (EVA [18, 22], PHOKHARA [23, 24, 25], EKHARA[21]), were and are being developed. The analysis presented in this paper is based on the results obtained by means of the program PHOKHARA 3.0 [25] (For further extensive discussions of various aspects of the radiative return method not covered by this article see [26-33], while for related discussion of the scan method look [33, 34]).

Further complication arises as the photons and/or lepton pairs are emitted also from final state charged hadrons and, as that emission is model dependent, it has to be tested experimentally. That problem is discussed extensively in the next section.

FSR AT LO AND NLO

At the leading order (LO) FSR contributions to the process \( e^+e^- \rightarrow \pi^+\pi^-\gamma \) can be easily controlled by suitable cuts at \( \Phi \)-factories as shown in Fig.1a and are completely negligible at B–factories (see Fig.1b). The remaining FSR contribution at \( \Phi \)-factory, which is less than 1%, can be subtracted from the data, relying on a MC generator, and the procedure of the hadronic cross section extraction described in the previous section can be used after that subtraction. The FSR contribution is however model dependent and one needs an independent experimental check on the accuracy of the model used. A simple observation that the interference of ISR, which leads to a C-odd (C stands for charge conjugation) configuration of \( \pi^+\pi^- \) pair, with the FSR amplitude, corresponding to C-even \( \pi^+\pi^- \) configuration, vanishes if a charge symmetric event selection is used, but it gives rise to charge asymmetries and charge induced forward–backward asymmetries, is crucial for that tests. By relaxing the cuts and measuring various charge asymmetric distributions, extensive tests of FSR models are possible, and as the actual contribution of FSR to the radiative return cross section, is of the order of 1%, a modest 10% accuracy of the model will lead to an error of 0.1%, sufficient for any high precision measurement. Some of the tests of the model used in EVA and PHOKHARA for

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shown that the charge asymmetry in [18], were already done by KLOE [9], where it was noted that (point-like pions and scalar QED (sQED)), proposed in [18], were already done by KLOE [9], where it was noted that 10.52 GeV (b).

Figure 1: Relative contribution of FSR with respect to ISR to the inclusive photon spectrum at $\sqrt{s} = 1.02$ GeV (without and with cuts (multiplied by a factor 10)) (a) and $\sqrt{s} = 10.52$ GeV (b).

FSR (point-like pions and scalar QED (sQED)), proposed in [18], were already done by KLOE [9], where it was shown that the charge asymmetry

$$A(\theta) = \frac{N^{\pi^+}(\theta) - N^{\pi^-}(\theta)}{N^{\pi^+}(\theta) + N^{\pi^-}(\theta)}$$

agrees well with the EVA MC [18]. However additional tests are needed to assure the accuracy of the model at the required level and comparisons of various charge asymmetric distributions between experimental data and MC are indispensable, especially when a measurement will use configurations with higher FSR contribution, necessary to cover the region of low $Q^2$. If only pions four-momenta are measured, as done in the KLOE experiment at the moment (see [15, 16]), one arrives at distributions as shown in Fig.2. With 500 pb$^{-1}$, collected till now by KLOE, the 0.1 nb/bin in the plot corresponds to 2000 events per bin. Thus that kind of measurement is feasible and tests can be done with the required precision, provided systematic errors are small enough. The nontrivial $Q^2$ and polar angle dependence of $\Phi$–distributions (definition of angle $\Phi$ is shown in Fig. 3) provides profound cross checks of the tested model.

At the next-to-leading order (NLO) the relevant diagrams contributing to the studied process are shown schematically in Fig.4. They were implemented in the PHOKHARA event generator version 3.0 [25]. From the analysis of the corresponding corrections to the $e^+e^- \rightarrow \pi^+\pi^-$ process [35] in the framework of sQED, this contribution is expected to be of the order of 1%. However, the emission of the initial photon reduces the invariant mass of the $\pi^+\pi^-$ (or $\pi^+\pi^-\gamma$) system to the $\rho$ mass with high probability due to the peak of the pion form factor at the $\rho$ mass. As a result, this contribution is strongly enhanced in the region of invariant mass of the $\pi^+\pi^-$ system below the $\rho$ resonance, as shown in Fig.5a for KLOE energy and in Fig.6a for B–factory energy. Suitably chosen cuts can be applied to suppress these NLO FSR contributions. In Fig.5b one can see that the standard KLOE cuts [16], which consist of the cuts on pion angles, the missing momentum angle and the track mass ($M_{tr}$), keep the NLO FSR contribution below 2% with respect to the ISR cross section in the whole interesting region of the two–pion invariant...
mass. Similarly at B–factories, applying the track mass cut only for events with $Q^2 < m^2$, the NLO FSR contribution is kept at a negligible level (Fig.6b). The radiative corrections to $e^+e^-$ vertex with photon emitted from final state, which are not included in version 3.0 and are included in PHOKHARA 4.0 [36], might be as big as 2%, if no cuts are applied, but are well below 0.1% for the standard KLOE cuts.

Again, as in the case of LO FSR contributions, the main problem consists in the model dependence of FSR. Till now only few tests were performed to verify the model for FSR. However, if one aims at a measurement of the accuracy below 1% such tests become indispensable. In the present KLOE experimental setup, where only four momenta of the pions are measured, a possibility to test the hard part of the NLO FSR contribution is to look at the dependence of the cross section on the missing invariant mass. Completely different effect of the cut on missing invariant mass on ISR (ISRNLO) and FSR at NLO, as shown in Fig.7, provides a powerful tool for testing the hard part of the IFSNLO contributions. A measurement of this few percent effect, depending on the two–pion invariant mass $Q^2$ is within reach of the KLOE experiment and a detailed discussion of possible tests can be found in [25, 31].

Figure 3: Angle $\Phi$ defined as the angle between normal to the production plain and the direction of the initial positron.

Figure 4: NLO contributions to the reaction $e^+e^- \rightarrow \pi^+\pi^-\gamma$ from real (both soft and hard) FSR emission (a) and virtual corrections to the $\pi^+\pi^-\gamma^*$ vertex (b).

Figure 5: Comparison of the $Q^2$ differential cross sections for $\sqrt{s} = 1.02$ GeV: IFSNLO contains the complete NLO contribution, while IFSLO has FSR at LO only. The pion and photon(s) angles are not restricted in (a). In (b) cuts are imposed on the missing momentum direction and the track mass (see text for description).

CONCLUSIONS

Huge event rates for $R(Q^2)$ measurements via radiative return method are, and will be available in near future at $\Phi$– and $\Phi$– factories. They cover large range of $Q^2$ allowing for significant reduction of theory errors on muon anomalous magnetic moment and the running electromagnetic coupling. Experimental studies of the model dependence of the FSR contributions will allow for reduction of the error coming from that contribution to a negligible level.

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Figure 6: Comparison of the $Q^2$ differential cross sections for $\sqrt{s} = 10.52$ GeV: IFSNLO contains the complete NLO contribution, while IFJLO has FSR at LO only. The pion and photon(s) angles are not restricted in (a). In (b) cuts are imposed on the track mass for $Q^2 < m_p^2$.


Figure 7: Dependence of the relative IFSNLO contribution on the cut on missing invariant mass $M^2$.


