Calibration of the ALEPH $dE/dx$

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1 Introduction

Measurement of the normalized ionization energy loss of charged particles in the gas of the ALEPH TPC give us the ability to do particle identification. To do so, first the ionization measurements are converted into the particle’s energy loss per unit length, $dE/dx$. While the former quantity depends on track angles, drift length, etc., the latter quantity, $dE/dx$, should be completely independent of all those parameters. To ensure this requires a profound understanding of the relevant dependencies and an accurate knowledge of their parameters. This procedure we call the calibration of the $dE/dx$.

If the magnitude of the charge of a particle is given, then its energy loss, $dE/dx$, is only a function of its velocity $\beta$:

$$dE/dx = f(\beta \gamma) \quad \text{with} \quad \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}$$

(1)

The function $f$ is given by what we call the Bethe-Bloch curve. Its parameters depend in detail on the design of the ALEPH-TPC and can be calculated theoretically to some extent. However, the accuracy needed for particle identification by $dE/dx$ can only be achieved when $f$ is determined from the experimental data, by fitting the $dE/dx$ information from selected tracks. Thus, in order to achieve the required accurate knowledge of $f$, we depend once again on the $dE/dx$ calibration. Any serious miscalibration can introduce a bias in the particle identification.

Once the energy loss $dE/dx$ is determined and the function $f$ is known, the particle’s velocity is obtained. It momentum $p$ is measured from the curvature of its trajectory inside the TPC. Then its mass $m$ can be calculated easily:

$$m = \frac{p}{\beta \gamma c}$$

(2)

This is the basic principle of particle identification by $dE/dx$. In practice, however, particle identification is realized by calculating weights or $\chi^2$ values for several mass hypotheses. The procedure is described elsewhere in detail [2].
Figure 1: A distribution of individual $dE/dx$ measurements made by the TPC wires for a sample of 45.6 GeV muons close to $\theta = 90^\circ$. The mean and rms width of this distribution are $\bar{x} = 1.72$ and $s = 1.44$, in arbitrary uncalibrated units, while the mean and rms width of the lowest 60% of the distribution are $\bar{x}_t = 0.963$ and $s_t = 0.354$. The 60% point of the distribution is at $x_c = 1.59$.

The calibration procedures and reconstruction algorithms for the ALEPH $dE/dx$ measurements have evolved extensively over the past 5 years. The results still are not perfect, so they probably will evolve some more in the future, depending on how much effort is put into investigation of remaining systematic effects. This note is intended to summarize what has been done and to explain our current understanding of the system and our calibration methods.

2 Truncated Mean Calculation

The individual $dE/dx$ measurements from the thin samples (4 mm of gas at atmospheric pressure) in the ALEPH TPC give a highly skewed distribution with a large Landau tail on the high side. See, for example, figure 1. A simple mean of this distribution is not optimal for distinguishing between particle types. Instead, we use a 60% truncated mean—the mean of the 60% of those samples with lowest $dE/dx$. The value of 60%
is somewhat arbitrary, but studies have shown that there is little advantage for any particular value between about 40% and 70%. The distribution of the 60% truncated mean of sets of 330 measurements taken from this distribution is shown in figure 2. Note that it is nearly perfectly gaussian, in spite of the highly skewed parent distribution.

As is discussed in much more detail in section 11, starting with the 1992 data we also are cutting off the lower 8% of the distribution. From a resolution standpoint this is not desirable, but we have found it to be necessary in order to avoid systematic effects from the threshold of the TPC digitizers.

One additional advantage of the truncated mean is that saturation of the electronics has no effect until more than 40%, in our case, of the samples for a given track are saturated. Such tracks are flagged by JULIA in the last column of the bank TEXS, and in the user interface TIDHYP (called by QDEDX) sectors with such severe saturation are skipped for the track in question.

It is important to distinguish between making a fixed cut on a distribution versus making a percentage cut, as we do for the truncated mean. The latter case is unbiased, which is essential for this purpose, but it also results in a larger variance than the first case. It turns out that the resolution of a truncated mean can be predicted from the variance of the overall distribution, but the calculation is much more involved than for the case of a simple mean, where one simply has, for $n$ uncorrelated measurements,

$$
\sigma_\mu = \sigma_x / \sqrt{n}.
$$

(3)
For the truncated mean, it still is true that the uncertainty of the truncated mean decreases, for sufficiently large \( n \), as \( 1/\sqrt{n} \). However, the formula becomes more involved because, roughly speaking, one must include contributions due to fluctuations of the truncation point.

Let \( F(t) = \int_0^t f(x) \, dx \) be the cumulative probability distribution. Then the truncation endpoint \( x_m \) is on average equal to \( x_c \), where \( F(x_c) = p \) (\( p \) is the fraction of measurements included in the mean and is equal to 0.6 for us). Due to the finite number of measurements, however, \( x_m \) fluctuates both above and below the expected value. In fact, the value of \( x_m \) is not determined by only \( m \) measurements, but rather by all \( n \) of them, since all \( n \) must be ordered before \( x_m \) can be found. Thus the values above the truncation point, as well as those below, play a part in the fluctuations of \( \bar{x}_t \).

The best way to understand how this affects the uncertainty on the truncated mean is to realize that the ordering of the measurements introduces correlations. If all \( n \) values are used in the mean, then obviously the correlations cancel, since the sum of the \( n \) values does not depend on the order in which they are added. But if only the first \( m \) values are included, then the correlations do not cancel and must be included in the expression for the variance of \( \bar{x}_t \). By including properly the correlations, one can show (see the appendix) that the variance of a one-sided truncated mean is given, for uncorrelated measurements, by

\[
\sigma_{\bar{x}_t}^2 = \frac{1}{np} \left[ s_t^2 + (1 - p)(\bar{x}_t - x_c)^2 \right],
\]

where \( s_t^2 = \bar{x}_t^2 - \bar{x}_t^2 \), and \( x_c \) is the truncation point, which can be approximated as \( x_m \).

### 3 Correlations Between TPC Wire Measurements

Equations 3 and 4 apply only to the case where all of the measurements are uncorrelated. That is not the case for the ALEPH TPC, where measurements on adjacent wires have a sizable positive correlation arising from diffusion and delta rays. It is a simple matter to measure the correlation coefficient from the data according to

\[
\rho = \frac{S_{i,i+1}}{\sqrt{S_{ii}S_{i+1,i+1}}}
\]

with

\[
S_{i,i+1} = \sum_{i=1}^{n} (x_i - \bar{x})(x_{i+1} - \bar{x})
\]

and so forth. Here, \( x_i \) is the charge measured on one wire, while \( x_{i+1} \) is that measured on the next adjacent wire. Wires with no pulse were not used, which introduces some bias, since most often such cases simply result from the online threshold. Using tracks of 45.6 GeV muons, we find that \( \rho = 0.249 \) for nearest neighbors when the full distribution is used. On the other hand, the correlation coefficient for next-to-nearest neighbors is -0.003, so only nearest-neighbor correlations need be considered.
To take into account nearest-neighbor correlations, the resolution predictions of equations 3 and 4 need to be multiplied by a factor of $\sqrt{1+2\rho}$. Consider the distribution shown in figure 1 with $x_t = 0.963$, $s_t = -0.354$, and $x_c = 1.59$ for $p = 0.6$. Using the correction factor $\sqrt{1+2\rho}$ with $\rho = 0.249$, equation 4 predicts for $n = 330$ a resolution for the truncated mean of 0.046, compared with the observed resolution of 0.044, as shown in figure 2. The value of $\rho$ used here, however, must be too large, because the truncated mean avoids the tail of the Landau distribution, where one would expect delta rays to make a large contribution to the correlation. In fact, if one measures the correlation only from the lower 60% of the measurements, then the value decreases drastically to $\rho = 0.065$, thus giving a predicted resolution of 0.040. Either way we don’t get exactly the observed resolution, but it is in the ballpark.

Anyway, while it is nice to know the relation between the Landau distribution and the error on the truncated mean, no use is made of these equations in the actual analysis. Instead, as is shown at length in section 14, the resolution is measured directly from the truncated mean distributions in data.

4 Calibration Constants Applied in Julia

From a practical standpoint, it is important to understand what calibration constants are applied in Julia, as opposed to those applied after the DST has been produced. The point is that this subset of the constants cannot be altered without redoing the entire processing, starting with raw data tapes. In contrast to other subdetectors, none of the TPC raw data are written to the POT, so most of the reconstruction has to be done correctly the first time.\footnote{Well okay, in practice, it is more like the second or third time.}

Since we never had faith that we would know for sure the relative sector-to-sector gain corrections before the Julia processing, we have always written to the POT (and now to the mini-DST as well) the truncated mean separately for each sector crossed by each track. For similar reasons, the POT and mini-DST always contain information at the level of the raw truncated mean rather than particle-ID information. First, users tend to use this information in different ways for particle identification, and second, it allows most of the calibration corrections discussed in this report to be made after the mini-DST production. These corrections are all applied within the QDedX (or TIDHYP) routines and do not use much CPU time.

Nonetheless, there are a few corrections applied at the hit level that cannot be redone (except in very approximate ways) after the initial processing. These are the logarithmic sample length correction of section 7, the correction for sector edge effects of section 8, the correction for radial dependence of section 9, the nonlinearity correction of section 12 and the drift-length correction of section 13. Note that the average drift length and the average sampling length of each track are saved on the output tape, so a provision has been made to apply some sort of drift-length correction or sampling-length correction at the DST or mini-DST level if necessary (see the bank TC2X). At present,
however, the database parameters are set such that no such corrections are made at the DST or mini-DST level.

It should perhaps be added at this point that the pedestal also cannot be changed after Julia has been run (this is an improvement over pre-1991 data where it could not be changed even at the raw data level, since the pulse length was not saved). Therefore, the pedestal must be correct. In practice, that has only been a problem on isolated runs in which a hardware problem (usually a mistuned gating circuit) has caused a pedestal shift.

Note that the pedestal is an integer, and all of the calculations in the wire reduction are done with integer arithmetic. If a fractional change needs to be made to the pedestal at some time in the future, then Julia must be modified a little bit to make the correction in the routine \textsc{twires} by adding an amount given by the pedestal shift times the pulse length as contained in the bank TSLE, since this pulse length is just the actual number of samples that were added together to give the total charge.

5 \textit{dE/dx} Calibration Using the ALEPH Laser System

In the ALEPH TPC a laser system is installed in order to measure the electrical field homogeneity from the ionization drift velocity. Via a two-photon process, the light of the ALEPH laser can ionize the gas in the TPC. If $L/L_0$ is the normalized intensity of the laser light, then the liberated ionization per unit length is given by:

$$\frac{dE}{dx} \propto \left( \frac{L}{L_0} \right)^2 \quad (7)$$

It is important that the distribution of ionization along a laser track follows a Poisson distribution and not a Landau distribution. Therefore studies of ionization with a laser avoid several problems that arise from the Landau distribution. A few problems with the laser stability (large shot-to-shot variations, long term drift due to laser cooling) can be eliminated when using a large number of shots with frequent measurements of the light intensity in a photodiode.

The laser system of the ALEPH TPC was used to give the first proof that the cutoff of small ionization measurements by the TPD threshold is a serious problem. The intensity of the laser light was changed by changing the laser’s high voltage and was measured with a photo diode. The measured $dE/dx$ as a function of light intensity, $(L/L_0)^2$, is shown in fig. 3. Small $dE/dx$ measurements appear to be shifted artificially when compared with the expectation. The same observation was made for ALEPH data. The effect is due to a cutoff of small ionization measurements in the TPD. This is illustrated in detail in figure 4 for different cut configurations in the TPD. It is seen that releasing the TPD cut clearly helps in measuring small ionization changes.

An example of $dE/dx$ calibration using the ALEPH laser system had been given. We conclude that laser ionization offers a simple and fast way to improve the calibration of $dE/dx$ in a TPC. If needed, it should be used more extensively in the future.
Figure 3: Measured $dE/dx$ as a function of the laser light intensity $(L/L_0)^2$. For large $dE/dx$ values the measurements follow the expected law of proportionality. For small $dE/dx$ values the measurements deviate significantly from the expectation and are stuck at a minimum value of about 0.3. This is a clear signature for a cutoff of small ionization charges.
Figure 4: For three different configurations of the TPD threshold cut, the cutoff effect for small ionization charges is illustrated. The three threshold cuts are defined as follows: 1) A pulse is taken if at least 3 samples are above 10 ADC counts (as in 1991 data). 2) A pulse is taken if at least 3 samples are above 8 ADC counts (as in data taken since 1992). 3) A pulse is taken if at least 2 samples are above 8 ADC counts (not used in ALEPH data taking). The number of good hits per track and the number of tracks with \(dE/dx\) drops significantly towards low ionization. The loss of data is most serious for cut 1) and smallest for cut 3). However, the number of bad hits is slightly increased for cuts 2) and 3).
to understand and to measure subtle effects and to improve particle identification by $dE/dx$.

6 Gain Calibration

Since the TPC operates at atmospheric pressure, the gas gain fluctuates rather frequently as the weather changes from day to day. This pressure dependence obeys a power law

$$G = G_0 \cdot (P/P_0)^{-\alpha},$$

where the exponent $\alpha$ has been measured from our data to be $\alpha = 3.7$ [4]. Up to now, the necessary correction has been handled in the offline calibration simply by breaking the data stream into run blocks, each of which is given a constant normalization factor determined by observing the position of the peak of the $dE/dx$ of minimum ionizing pions. The run blocks have been selected more-or-less by hand by looking at a listing of slow-control pressure readings versus run number. A break is made if the pressure fluctuates significantly, or if there is a large time gap between runs. We also make sure that each run block has sufficient statistics to make a good calibration of the global gain. In recent years this has been no problem, since a typical fill has far more than enough tracks to do the job. No attempt has been made to apply equation 8 by using the measured pressure. The current procedure will not work at LEP-II, however, so eventually the program should be modified to apply a correction based directly on the pressure readings.

The gain can also fluctuate from sector to sector. However, these fluctuations do not vary rapidly with time, except in cases of hardware problems (high voltage changes, or pedestal shifts caused by gating) or in case a sector is changed, of course. Minimum ionizing pions again are used to calibrate the relative sector gains, but the run blocks are much larger than for the global gain.

Offline global gain adjustments appear in no less than four places, while the sector-to-sector gain adjustments appear in two places. First, in both the TC1X and TC2X banks there are global and sector gain adjustments. The TC1X constants are used in JULIA and generally are not changed more than once per year. They have turned out not to be important, since JULIA itself does not make serious use of the $dE/dx$. There also is a global constant in TC5X that affects the overall $dE/dx$ normalization in JULIA but should not be changed. The TC2X constants are the most important and are not applied until the user routine TDHY (or QDHY) is called at analysis time. (Note: recently two new banks TC9X and TCSX have been defined to take the place of TC2X. They may be accessed by means of the routine TDXC.) There also is another global gain adjustment in TC3X which is used to fine tune the calibration at a late stage. It also is applied in the user routine, so to get the latest calibration for analysis it is only necessary to have the latest version of the data base.
7 Logarithmic Sample Length Correction

In principle, we would like to define the $dE/dx$ to be the truncated mean of the distribution $Q/\Delta x$, where $Q$ is the calibrated pulse height and $\Delta x$ is the sample length—the length of track which projects onto a single wire. However, such a definition of the $dE/dx$ is found not to be independent of $\Delta x$ for a given particle type of a given momentum. This effect is understood to be due simply to the shape of the Landau distribution.

The basic idea is that as the sampling thickness increases, the distribution changes shape toward one with less tail on the high side. As a result, the truncated mean does not simply scale with $\Delta x$ (nor does the most probable value of the distribution). This is a well known effect and arises simply from statistics. As the sampling thickness increases, the number of clusters of ionization per sample increases. In the limit that the number of clusters becomes large, the fluctuations in the total energy deposit per sample are expected to become small and gaussian distributed. In practice, the gaussian limit predicted by the central limit theorem is never reached for sample thickness relevant to us, but the tail does become significantly smaller and the peak moves upwards as the samples become thicker.

This effect can be seen in other ways, as well. For example, it results in the measured $dE/dx$ of two overlapping tracks being more than twice the $dE/dx$ of a single track. This has been verified by looking at overlapping electrons from photon conversions. The effect can be most easily seen, however, by playing with a Landau random number generator. For this purpose, we used the CERNLIB routine RANLAN. This routine does not always return positive values; experimentally we find that the minimum value is $-3.58725$, so we always add minus this number to the value returned to give the distributions used in the analysis. Figure 5 shows a histogram of the resulting distribution overplotted with the distribution of the average of pairs of random numbers from the same generator. One can clearly see the upward shift of the peak and the decreasing importance of the tail when the pairs of numbers are added. We calculate that the truncated mean of the distribution increases from 3.70 to 4.39. We can take this further by adding triplets of random numbers, quadruplets, and so forth. When we do so and plot the truncated mean divided by $N_t$, where $N_t$ is the number of random numbers added together for each trial, versus the logarithm of $N_t$, then we get an approximately linear relation, as shown in figure 6. By fitting the points in figure 6, we derive a new definition of the $dE/dx$ that is approximately invariant with the size of $\delta x$:

$$I = \frac{Q}{\delta x} \cdot \frac{1}{1 + C \cdot \ln \delta x / x_0}$$

(9)

where $x_0$ is the size of the minimum sampling length (4 mm in the TPC), $Q$ is the measured charge, and $I$ is the $dE/dx$ to be passed to the truncated-mean algorithm.

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Strictly speaking, the central limit theorem can only be applied if the underlying "Landau" distribution is integrable. But that is only an academic problem, since the true physical distribution must be integrable—the tail cannot really extend to infinity, and in practice it generally gets truncated by the dynamic range of the apparatus.
Figure 5: Solid curve is a histogram of Landau random numbers generated by the CERNLIB routine RANLAN. A constant equal to 3.58725 has been added to every number. The dotted curve is a histogram of the average of pairs of such random numbers.
$\chi^2 = 70.4$ for 8 - 2 d.o.f., C.L. = $3.34E-10$

Function 1: Polynomial of Order 1
NORM: 3.9266, ±3.6380E-03, -0.0000E+00, +0.0000E+00
POLY01: -9.6664, ±2.3737E-03, -0.0000E+00, +0.0000E+00
OFFSET: 0.0000E+00, ±0.0000E+00, -0.0000E+00, +0.0000E+00

Figure 6: Truncated means calculated from sets of “measurements” generated from Landau random numbers. $N$ is the number of random numbers that have been averaged to yield each “measurement.”
Figure 7: Truncated means calculated from sets of $dE/dx$ “measurements” which were generated by averaging in each case $N$ ionization samples from tracks of minimum ionizing pions.

The experiment with the random number generator yields $C_p = 0.246$, but as shown in the following, slightly different numbers follow from data and from the TPC Monte Carlo.

The parameter $C_p$ can also be measured from individual hits on tracks by the following method, which is equivalent to what is done with the random number generator above. We start with a set of minimum ionizing pions, selected by taking all tracks in the momentum interval from 300 to 600 MeV which have $dE/dx$ well below that expected for an electron. Only tracks within $10^\circ$ of the $z = 0$ plane are used. Lumping together all wire hits from all tracks, a truncated mean charge is calculate, as is the mean value of the sampling length. The procedure then is repeated after adding together adjacent pairs of hits to produce a new set of hits of twice the sampling length. Then triplets of hits are added together, and so forth, giving in the end the plot shown in figure 7. The slope of this plot gives a value of 0.2 for $C_p$. However, the value that we have used in our data processing from the beginning of LEP operation is $C_p = 0.254$, which was derived in the early days from the TPCSIM Monte Carlo. In any case, varying this correction from 0.2 to 0.3 does not make a big difference to the quality of the calibration.
8 Correction for Sector Edge Effects

The gas gain of the TPC proportional chambers is sufficiently uniform across a given sector, with only insignificant random fluctuations, that no corrections need to be made except at the very edges of the sectors. The first and last sense wires in each sector are ignored by the reconstruction, since they tend to get saturated by gating noise anyway. Also, all hits within 0.5 cm of the sector edges to which wires are attached are also ignored by the program. Still, there is a significant loss in gain even within the fiducial region defined by this 0.5 cm cut. The sectors were built with field correction strips located along the sector edges, and studies were made which showed that by applying the appropriate voltage to those strips the gain could be made constant to well within 0.5 cm of the edge [10]. However, since LEP data taking began, we have never run the TPC at those voltages. Instead, the voltages have been deliberately set to reduce the gain at the sector edge in order to avoid electrical discharges in this sensitive region of the chambers.

For tracks with appreciable curvature this does not cause much of a problem, since only a short length of the track can be in the area of reduced gain. However, the problem shows up clearly for straight $\mu$-pair or Bhabha tracks if the $dE/dx$ is plotted as a function of the angle from the sector center. That is simply because the straight radial tracks can follow a sector edge for a long distance and therefore be severely affected by the reduced gain. We have measured this effect by plotting the truncated mean $dE/dx$ as a function of the distance of a hit from the sector edge. The results, as can be seen in figure 8 for K sectors (W and M sectors have also been measured and look similar), have been fit to an exponential function of the form

$$Q = 1 - c_1 e^{-\delta y / c_2},$$  \hspace{1cm} (10)

where $\delta y$ is the distance from the sector edge, and $c_1$ and $c_2$ are fit to the data. This correction is applied to the individual wire hits in the routine TFTWTL of the offline reconstruction program. It cannot be adjusted at the level of the POT or DST.

9 Correction for Radial Dependence

Not all wires within a given sector show exactly the same response or gain. One reason is that the gas gain can vary slightly with radius. More important, however, is the fact that the electrical characteristics of the wires as seen by the preamps varies due to changing lengths of sense wires and the busses that lead to the preamps. The wire channels are calibrated by the online calibration system to remove such effects, but that system has its own systematic dependences on the signal busses and, most obviously, on the lengths of the wires, since it relies on capacitive coupling between the field wires and sense wires. The latter effect has been largely taken out in the calibration program, but there may still be left a residual slope with wire number.

Random variations from one wire to the next have no effect on our resolution until those variations are similar in size to the huge Landau fluctuations in the individual wire
Figure 8: The truncated mean $dE/dx$ as a function of the distance from the sector edge in K sectors, as measured from wire hits in dilepton events.
Figure 9: The dependence of the truncated mean on wire number in the inner (K) sectors, as measured from dilepton tracks with $\theta$ close to 90°.

measurements. All we are concerned about then is a systematic dependence on radius. Even that would only have second order effects, since to first order all tracks cross the same wires. A possible second order effect is that tracks in jets, which typically miss the inner wires due to high track overlap, may have slightly different response compared with isolated tracks.

We have investigated this by measuring the truncated-mean $dE/dx$ on each wire by using hits from a large set of dilepton events. Only tracks with polar angle between 80° and 100° are used, in order to avoid correlations with angle and drift distance. Minimum ionizing pions are not used, since their angle with the wires is strongly correlated with radius, and there are other systematic effects related to that angle. Muons and electrons are combined by plotting the truncated mean of the $dE/dx$ divided by the expected $dE/dx$. This has not been done separately for each of the 36 sectors but only each of the three sector types (K, M, W). The results are shown in figures 9 and 10. In these plots we see evidence for a dependence on wire number that has a period of 16, just the number of channels connected to each preamp. No attempt has been made to remove this effect since it is of a short enough period that it cannot affect the $dE/dx$ results in the end. But we also see small slopes of about 5% per meter in the outer sectors and a distinct bow in the inner sectors. The bow probably is related to the shape of the $K$ sectors, which first get more broad with radius and then become more narrow toward the outer radius. It has been fit to a quadratic curve, while the outer sectors have been fit to a linear function. These fits are used to correct the data in the offline

3Actually, this dependence is seen much more clearly when higher statistics are obtained by suppressing the angular cuts.
reconstruction in the routine TFTWTD. No effects from the correction are visible in the average resolution, whether measured from dileptons or from pions in jets, so it is not a major issue.

10 Threshold Effects

The TPC wire thresholds have a somewhat long and confused history. The algorithm used in the TPD has always required at least three consecutive FADC samples greater than or equal to the threshold value in order to form a valid hit. During the 1989 run the threshold value was allowed to vary between sectors, depending on the noise level in the sector, but in 1990 it was fixed to a consistent value of 8 counts above pedestal for all sectors.\footnote{The pedestal has always been a constant 4 ADC counts.} This value was eventually realized to be unnecessarily high and had been chosen more-or-less by looking at the online display. Therefore, we lowered it to 6 counts above threshold for the 1991 run. It was only realized some time later that this threshold still was high enough to be responsible for most of the systematic problems that were plaguing the \( dE/dx \) reconstruction. Therefore, before the 1992 run, tests were made to determine whether it could be lowered further still. It was found that it could be lowered to 4 counts above threshold, still keeping the 3-sample requirement, without too seriously impacting the data volume. It should be clearly understood that how low the threshold can be placed depends only on the data rate that can be sustained
in the online system and onto the raw data tape. The reconstruction always prefers a
threshold as low as possible, since noise hits cause no problem other than to increase
slightly the CPU time needed for reconstruction. Also, the threshold has almost no
impact on the size of the POT, DST, or MINI files, since wire hits are not stored on
any of those.

A non-zero threshold results in a nonlinear dependence of the measured truncated-
mean pulse height on the deposited charge. That is because, for example, the Landau
distribution from a minimum-ionizing particle is substantially truncated on the low
side by the threshold, resulting in an artificially high mean, while that of an electron
is only slightly affected by the threshold. As a result, the ratio of the electron versus
MIP truncated means appears to be too small. The most serious consequence of this
problem is that, since the pulse heights increase as the sampling length increases due to
changing track angles, the effect of the threshold on the MIPs also varies with sampling
length, resulting in an electron-to-MIP ratio that varies with the track direction. The
particle identification works on the principle that the measured $dE/dx$ depends only
on the velocity (and magnitude of the charge) of the particle. The threshold effect,
however, prevents us from deriving a unique relation between velocity and mean $dE/dx$
by introducing systematic effects that depend on the direction of the particle and the
drift length of its ionization.

This effect is most clearly seen by the following method. We select minimum-ionizing
pions by taking all tracks in the momentum interval from 400 to 500 MeV which have
$dE/dx$ well below that expected for an electron. We select only tracks near $\theta = 90^\circ$ in
order to avoid dependences on drift length and dip angle. There still is a large variation
in sampling length due to the large curvature of these tracks, as well as the up to $\pm 30^\circ$
variation in angle between the radial direction and the wire direction due to the size of
the sectors (especially the inner sectors). To analyze the dependence of the truncated
mean on sampling length, all hits from all tracks are thrown together and then sorted
into bins of sampling length. The truncated mean is taken for each bin and then divided
by the logarithmic correction discussed in section 7. This result should be expected to
depend linearly on the sampling length and extrapolate to zero at zero sampling length.
In fact, as seen in figure 11 for 1991 data, the plot does not extrapolate to zero. That
is understood to be largely due to the threshold, which truncates the lower side of the
Landau distribution most heavily for those tracks with small sampling length, giving
them an artificially high mean.

The situation improved substantially for 1992 data, because of the reduced thresh-
old. A measure of the problem can be taken to be the parameter $C_w$, which we define
to be the intercept of the plot in figure 11 divided by 0.4. For 1991 we had $C_w = 0.31$,
while for 1992 it was $C_w = 0.13$. The effect still remains significant, however. In ret-
rospect, the problem should have been anticipated much earlier, but the significance of
the threshold was not appreciated, partly because the TPC simulation had been mis-
takenly set up to require just one sample above threshold and therefore did not show
any detrimental effects, leading us to believe for some time that the problem in data
was something more mysterious. With the simulation set up properly, the threshold
effect should be, and is, easily simulated.
Figure 11: The truncated-mean charge in 1991 data, after the logarithmic correction discussed in section 7, as a function of sample length $\delta x$ for minimum ionizing pions with $80^\circ < \theta < 100^\circ$. The fact that the points do not extrapolate to the origin is due primarily to the TPD threshold.
11 Modification to Julia to Correct the Threshold Problem

For data taken starting in 1992, the reconstruction program was heavily modified in order to introduce a new algorithm that is not too sensitive to the threshold. The idea was to try to determine for each track how many wires did not fire simply because the signal was below the TPD threshold. Since the old program only considered valid those wire hits that matched with track extrapolations, major changes in the code were required. The basic idea is simply to count the number of wires crossed by a given track that have no hit at the appropriate time (a below-threshold “hit”), but a number of issues had to be addressed in order to make the new code sufficiently robust. There are a number of reasons why a wire hit might be missing at the time expected for a given track. First, the wire might be dead, so the new program is careful not to count those dead wires flagged by the calibration system in the TKAP bank. Also, if a single wire with no data in the hit list is crossed by more than three tracks, then that wire is assumed to be dead even if not flagged in TKAP (however, this information is not remembered for the following events). Second, the track extrapolation may not be valid, perhaps because the track really begins or ends in the middle of the TPC due to a particle decay. Those problems are avoided by deleting all long runs of below-threshold hits (greater than 4 below-threshold hits in a row) and by deleting all below-threshold hits at the beginning or end of any track. Finally, two or more tracks may merge to form a single pulse. The time estimate for such a pulse may match only one of the tracks, leaving the others to be candidates for below-threshold hits. Fortunately, near the beginning of the 1991 run a new bank, called TRLE, was added to the TPP output. This bank saves the length of each pulse for those pulses reduced in the TPP and thus allows the program always to check whether a track falls within a pulse. Therefore, if the track extrapolation falls within the extent of such a pulse, even if the track does not match the time of the leading edge of the pulse, the wire is not assigned to be a below-threshold hit (however, as discussed in section 18, there were some problems with this cut in the 1992 processing that reduced its effectiveness).

Once the below-threshold hits are counted for each track, the question remains as to what to do about them. It is not optimal to assign them a pulse height of zero. Instead, TPCSIM was used to estimate a reasonable average pulse height for such below-threshold hits by making a histogram of the pulse heights of all rejected hits from a set of generated minimum ionizing tracks. This value was set to 7.0 ADC counts. However, that must have resulted from an error, since a repeat of the same procedure (after the 1992 reprocessing) indicated that for MIPs the appropriate value is 12.5 ADC counts. In any case, a constant value obviously cannot be correct in general, so it was decided to modify the truncated mean to be a 2-sided truncated mean. The upper 40% of all hits still are truncated, but in addition, the lower 8% of all hits also are truncated. Since this lower truncation is done as a percentage, as opposed to the constant cut that had been imposed by the threshold, then it is not biased, as long as the number of below-threshold hits is counted correctly. The 8% value was chosen because it was found that for minimum ionizing pions the average number of below-threshold hits per
Figure 12: Raw ADC distribution of wire hits from pion tracks with $0.3 < p < 0.6 \text{ GeV}$, $75^\circ \leq |\theta| \leq 85^\circ$, and sample length $\delta x < 0.47 \text{ cm}$. The bin at zero pulse height counts the number of wires crossed by the tracks which produced no signal above threshold. It represents 8.1% of the overall distribution.

track is 8.1% (see the pulse-height distribution in figure 12). Therefore, not too many of the below-threshold hits ever actually enter into the mean.\textsuperscript{5} The downside, of course, is that such a truncation increases the statistical error on the truncated mean. For example, we end up throwing away 8% of the good measurements for electron tracks in order to treat them exactly as we treat the MIP tracks, for which on average 8% of the measurements were thrown away online by the TPD. In the ideal world, one would prefer to design the TPC and operate it at a gain such that the online threshold is negligible.

Note that in the 1991 data 16% of the hits from a minimum ionizing pion fall below the threshold with an average of 19 ADC counts, so the problem is worse by a factor of two, as already indicated in the previous section. One would expect that the same algorithm could nonetheless be applied, with the truncation point moved up to 16%. That was tried, but for unknown reasons the results, in terms of systematics, resolution, and particle separation, were found to be unacceptable. Only a couple of weeks were

\textsuperscript{5}The 8.1% is an average over tracks at small dip angle. Since the actual number fluctuates, some below-threshold hits do make it into the truncated mean for minimum ionizing particles and, without doubt, must cause some residual systematic effects.
available at that time before the beginning of reprocessing of the 1991 data, so we gave up and decided to leave the 1991 data as they were.

12 Correction for Nonlinearity with Sample Length

An ad-hoc correction was applied to the 1991 data that was based essentially on figure 11. The idea is to add just enough to the measured charge as a function of increasing sampling length in order to make the result extrapolate to zero. The formula is

\[ Q_{\text{corr}} = G \cdot \left[ \frac{Q}{\Delta x(1 + C_p \ln(\Delta x/0.4)} + \left(1 - \frac{0.4}{\Delta x} \right) C_w \right] \]  

(11)

where \( G \) is a constant normalization factor, \( C_p \) is the correction discussed in section 7, \( C_w = 0.31 \) for 1991, and \( \Delta x \) is the sampling length measured in centimeters. This correction is somewhat successful at correcting for the systematic effects caused by the threshold but clearly doesn’t really address the problem properly. One reason is that the threshold affects the sampling length dependence differently depending on whether the track angle with respect to the sense wires is in the \( r\phi \) plane, due to curvature, or in the \( rz \) plane, due simply to the track polar angle. As the polar angle and drift lengths change, the pulses become more or less elongated. For a given charge, longer pulses have smaller ADC values and therefore are more sensitive to the threshold. One cannot correct such complicated dependences by a simple function of \( \Delta x \).

For the 1992 data, using the new algorithm that includes below-threshold hits, the parameter \( C_w \) is measured by the procedure of section 10 to be \(-0.05\) (notice the sign). Why it doesn’t turn out to be exactly zero is not understood. We decided to continue making the correction of equation 11 with this new value of \( C_w \), except that in the equation \( \Delta x \) is replaced by \( \Delta x_p \), the projection of the sampling length on the \( r\phi \) plane. As a result, this correction now has a significant effect only on highly curved (low momentum) tracks and is not of great importance.

If one now takes all hits from Bhabha tracks in 1992 data going through \( \pm 10^\circ \) of the center of a sector and sorts them into bins of sample length, the truncated mean, with the logarithmic correction of section 7, is nicely linear and extrapolates perfectly to zero, as seen in figure 13. The same is seen to be true for hits from minimum-ionizing pions as a function of dip angle (again taking only track segments within \( \pm 10^\circ \) of being perpendicular to the sense wire in the \( r\phi \) plane). That is why the ad-hoc correction of equation 11 is now applied only to \( \Delta x_p \) rather than to \( \Delta x \).

13 Drift Length Correction

In principle, a correction which increases linearly with drift length should be applied to the detected charge in order to correct for loss of ionization by attachment to impurities in the gas. This loss has been measured using a radioactive source and a special chamber [8] to be about 1.3% per meter. In practice, such a small effect is overwhelmed
Figure 13: The truncated-mean charge, after the logarithmic correction discussed in section 7, as a function of sample length for electrons in Bhabha events from 1992 data. The new algorithm, which takes into account below-threshold hits, has been used here.
by systematics arising from the interplay of the threshold (among other things?) with the track dip angle, which always is correlated with drift length for tracks that come from the origin. The broadening of pulses by diffusion also introduces complications into the drift-length dependence. Thus, while it is important to make a correction for drift length, the correction does not have a simple interpretation and cannot be directly related to the predicted 1.3% per meter loss.

We evaluate the correction by binning hits from many tracks, all of the same type and momentum (μ-pairs, Bhabhas, or minimum ionizing pions), in bins of drift length. For each bin the truncated mean ionization is calculated after correcting for angular effects according to sections 12 and 7. The slope used for correcting 1991 data is 2% per meter, while for 1992 data it is 0.3% per meter. It should be kept in mind, however, that the difference between these numbers probably has little to do with possible differences in absorption between the two running periods.

## 14 The Resolution Function

The $dE/dx$ statistical resolution depends most strongly on the number of wire hits or samples used to calculate the truncated mean. However, it also depends significantly on the number of ionization clusters which contribute to each wire measurement. The number of such clusters depends both on the length of track, $\delta x$, which projects onto a single wire and on the ionization density, $dE/dx$, itself. Let $I$ represent the measured $dE/dx$. Then the resolution, $\sigma_I$, may be parameterized as

$$\sigma_I/I = \sigma_0 N^{p_1} \Delta^{p_2} (I/I_0)^{p_3}$$

where $\Delta \equiv \delta x/4$ mm, and $I_0$ is the $dE/dx$ at minimum ionizing (we normally define $I_0 = 1$). In the limit of large statistics one would expect all of the three exponents $p_i$ to be $-0.5$. However, while $N$ can be quite large in the ALEPH TPC, the number of clusters contributing to a pulse on a single given wire is generally quite small, due to our thin samples (≈ 4 mm of gas at atmospheric pressure). The thin sampling length, together with the highly skewed Landau shape of the distribution of ionization, can lead to exponents $p_2$ and $p_3$ which are quite different from $-0.5$. Therefore, we have tried to measure all of the exponents $p_i$ in various ways and with various data samples.

### 14.1 The $N$ Dependence of the Resolution

From basic statistics, the central limit theorem guarantees that for a sufficiently large number of samples, $N$, the resolution on the truncated mean must improve as $N^{-0.5}$, regardless of the fact that the distribution of ionization from which the truncated mean is made is highly non-Gaussian. All that is theoretically required is that the size of the Landau tail not be infinite, which the finite dynamic range of our electronics guarantees. The result should also hold even if there is a small nearest-neighbor correlation, as is indeed the case in the TPC wire data.
In practice, one might see a deviation from an exponent of \(-0.5\) for one of two reasons:

- Calibration problems or systematic errors may introduce an effective constant term such that the resolution does not go asymptotically to zero as \(N\) goes to infinity.

- Maybe \(N\) isn’t large enough that the central limit theorem can be applied.

Regarding the first possibility, note that the resolution generally is measured by summing over many tracks accumulated over a long period of time, with the result that the measurements can be influenced by variations of pressure, gas composition, temperature, \textit{etc.} The second possibility is easily tested using a Landau random number generator. To do this, we use a generator that produces a most probable value of about 3.5. To simulate the dynamic range of the \textsc{Aleph} electronics, we truncate the distribution at 30, such that all “measurements” greater than 30 are set to 30. A 60% truncated mean is then taken, and the rms width of the distribution of the truncated mean is studied as a function of \(N\), for \(N = 10, 20, 30 \ldots\). The resulting plot is shown in figure 14. It fits perfectly to a power law with an exponent of \(-.5\). In general, for \(N > 50\), the distribution of the truncated mean looks almost perfectly gaussian, so the central limit theorem applies very well in the region of interest to us.

In most reports of measurements of this exponent, a value somewhat less negative than \(-0.5\) is obtained. A widely quoted reference, Allison and Cobb \cite{11}, gives a value of \(p_1 = -0.46\) from a fit to calculations of a \(dE/dx\) model. More recently, \textsc{Opal} finds an exponent of \(-0.43\) \cite{5} from their data. They explain that “the exponent differs from \(-0.5\) (as expected for a Gaussian distribution) since the truncated mean distribution is, for the given number of samples, close to, but not exactly a Gaussian distribution.” Judging from the test done with Landau random numbers, this explanation must refer mainly to the region of very small \(N\) (their plot goes all the way down to \(N < 5\)). If they were to fit only the region of large \(N\) (say, \(N > 30\)), then we expect that they should find an exponent closer to \(-0.5\). If not, then the deviation from the expected value must be from systematic effects, not statistics.

We have attempted to measure the exponent \(p_1\) from our data by two methods. In the first method, which we believe is identical to the method used in Ref. 5, we choose a bin in ln\(\Delta\) (from 0.1 to 0.3) and collect all hits from a set of tracks all of the same velocity. This single set of hits is then divided into groups of 10 hits, from which truncated means are calculated, then into groups of 20 hits, 30 hits, and so forth, up to \(N = 340\). For each value of \(N\) a histogram of the truncated means is made and fit to a gaussian. Figure 15 shows a plot of the resulting \(\sigma\) as a function of \(N\) for Bhabha electrons, while figure 16 shows the same thing for minimum ionizing pions. The plot fits very well to the power law form, with an exponent \(p_1 = -0.485 \pm 0.006\) for the electrons and \(p_1 = -0.498 \pm 0.006\) for the pions, where only the region \(N \geq 50\) has been included in the fit. If the full range down to \(N = 10\) is included in the fit, then the confidence level of the \(\chi^2\) fit falls (to 0.8\% in the pion case) and the exponent changes to about \(-0.47\) in both cases. This is not surprising, since for \(N < 50\) we find that the distribution of the truncated mean fits very poorly to a gaussian curve.
\[ \chi^2 = 37.5 \text{ for } 35 - 2 \text{ d.o.f.} \quad \text{C.L.} = 27.0\% \]

Function | Power Law | Parabolic | Minos
---------|-----------|-----------|------
Norm     | 2.4108    | ±5.378E-02 | ±0.000E+00 + 0.000E+00
Power    | -0.49409  | ±4.455E-03 | ±0.000E+00 + 0.000E+00
Slope    | 1.0000    | ±0.000E+00 | ±0.000E+00 + 0.000E+00

Figure 14: The truncated mean as a function of \( N \), the number of "measurements," as calculated from a Landau random number generator. The smooth curve is a fit to a power law, which gives an exponent of \(-0.5\), as expected.
Figure 15: The $dE/dx$ resolution versus number of samples for high-momentum electrons, calculated by combining hits from all tracks. The fitted exponent is $-0.485$. 
Figure 16: The $dE/dx$ resolution versus number of samples for minimum-ionizing pions, calculated by combining hits from all tracks. The fitted exponent is $-0.498$. 

\[ \chi^2 = 29.7 \text{ for } 30 - 2 \text{ d.o.f.} \] 

C.L. = 38.0%
Figure 17: The $dE/dx$ resolution versus number of samples for minimum ionizing pions, calculated by using tracks which actually have the given number of samples. The fitted exponent is $-0.49$

The second method of measuring $p_1$ is probably less rigorous but is more similar to the method by which data actually are analyzed. Minimum ionizing pions are selected by taking all tracks in the range $0.3 \leq p \leq 0.6$ GeV with $dE/dx$ well below that expected for an electron. For each track, a histogram is made of the quantity

$$\frac{I - I_e}{I_e^{-p_2} \Delta^{-p_2}},$$

with $p_2 = -0.4$ (see below), for each bin of $N$, where $N$ is the actual number of wire hits for the track in question and $I_e$ is the expected $dE/dx$ for a pion of the measured momentum. The resulting plot of $\sigma$ versus $N$ is shown in figure 17 and fits a power law reasonably well with $p_1 = -0.49 \pm 0.01$.

This second method can also be applied to electrons in hadronic jets, although with a coarser binning in $N$. In this case, one has to be careful to subtract the background, since the electron sample is not nearly as pure as the pion sample. For all electron candidates in a given bin of $N$ we make a histogram of the $dE/dx$ minus the expected
Figure 18: The $dE/dx$ resolution versus number of samples for electrons in hadronic jets (square points), calculated by using tracks which actually have the given number of samples. The solid circular points are identical to those in figure 17, only scaled by $(1.65)^{-0.4}$.

The $dE/dx$ divided by $\Delta p^2$. We then fit this histogram to the sum of a gaussian curve plus a background histogram with floating normalization. The background shape is easily obtained from the equivalent $dE/dx$ distribution of non-electrons, just as was done in our first heavy-flavor publication [9]. The results are given as a function of $N$ by the square data points in figure 18. The exponent comes out to be reasonably consistent with the expected value of $p_1 = -0.5$.

$^6$The small error on the fitted value of $-0.560 \pm 0.002$ cannot be taken seriously, since the individual points have large systematic errors which have not been taken into account, as is evident from the terrible $\chi^2$ of the fit.
14.2 The Sample-Length Dependence

The exponent $p_2$ expresses the fact that for a set of particles of equal velocity, those with small polar angles, or those which have curved away from the radial direction, produce more ionization per wire and therefore should be expected to have better resolution than radial tracks at $\theta = 90^\circ$. This is an important effect for the ALEPH TPC, which has a high magnetic field and a very large angular acceptance. As discussed above, we are in a statistical regime where the asymptotic limit $p_2 = -0.5$ generally doesn't hold. Allison and Cobb [11] give an empirical exponent of $-0.32$ for the dependence on sampling length, but there is no reason to believe that all experiments should find the same value.

Unfortunately, $p_2$ has proven to be a difficult number to nail down precisely. For guidance, we turn to the Landau random number generator and simulate increasing sampling length by adding together sets of random numbers, first in pairs, then in triplets, etc., just as was done in section 7 to investigate the dependence of the truncated mean on sampling length. At first sight, it may appear that this procedure is exactly the same as investigating the dependence of the resolution on $N$, since a so-called doubling, for example, of sampling length here is achieved essentially by doubling $N$. The difference is that when we add pairs of samples in this procedure, we are effectively taking a straight mean. That mean then is used as input, along with 329 other such pairs, to a truncated mean. Therefore, we expect that adding pairs of samples to give a total of 330 pairs will not give as much an improvement in resolution as simply doubling the number of samples to 660—we lose some of the benefit of a truncated mean in the former case. In fact, as shown in figure 19 for the random number generator, the resolution only improves by an exponent of about $-0.3$ as the sampling length increases.

We should expect a similar exponent for the dependence of the actual TPC resolution on sampling length. The random number game makes it clear why the exponent is not $-0.5$. Something equivalent to what was done with the random numbers would be, for example, to reduce the number of TPC wires by 1/2 while maintaining the same overall length of gas. It is probably intuitively clear that doing such a thing would be detrimental to the resolution—increase the number of samples allows one to isolate better the Landau tail, the bad effects of which can then be avoided more effectively by taking a truncated mean. However, going back to the random number generator, we note that as $N_s$, the number of random numbers added together for each “sample,” gets very large ($N_s > 10$), the exponent approaches more closely the value of $-0.5$, as seen in figure 20. This is because the distribution of a sum of $n$ random numbers becomes gaussian as $n$ gets large (the central limit theorem)—the Landau tail essentially goes away. For the actual TPC, this corresponds to the well-known fact that as the samples get very thick, the ionization distribution becomes less skewed and more gaussian. Thus the situation is complicated—for thin samples the dependence of the resolution on $\Delta$ cannot really be described by a single exponent. If we do describe it by an exponent, then the value that fits best will depend on details such as the sampling length itself.

\footnote{On the other hand, adding more wires in the same gas volume also would not bring much improvement in the case of our TPC, because the correlation between measurements would increase dramatically due to diffusion.}
Figure 19: From a Landau random number generator, the resolution on the truncated mean as a function of the number of random numbers averaged together to form each "measurement."
Figure 20: From a Landau random number generator, the resolution on the truncated mean as a function of the number of random numbers averaged together to form each “measurement.” This is identical to figure 19 except that the range in $N$ has been increased.
Figure 21: From minimum ionizing pions, the resolution on the truncated mean \( dE/dx \) as a function of the effective sample length, where increasingly large sample lengths have been obtained by adding together sets of adjacent wire hits.

and can be expected to vary from one experiment to another.

We can play the same game with data that we played with the random number generator. We take a set of minimum ionizing pions within a limited range of sampling length and then add the hits in pairs, in triplets, etc., just as was done for the random number generator. This generates the plot shown in figure 21, which exhibits an exponent of \( p_2 = -0.39 \), somewhat steeper than for the random number generator.

We also have tried to measure \( p_2 \) more directly from data. We begin by taking all hits from a set of tracks, all of the same velocity (such as \( \mu \)-pairs or MIPs), and sorting them into bins of sample length. The hits in each bin are divided into groups of 330, and a truncated mean is calculated for each group and entered into a histogram. The resulting gaussian distributions are fit to determine the \( \sigma \), and a plot is made of \( \ln \sigma \) versus \( \ln \Delta \), as shown in figure 14.2 for \( \mu \) pairs, giving \( p_2 = -0.48 \), and in figure 23 for bhabhas, giving \( p_2 = -0.27 \). We do not understand why these different measurements are not consistent. However, it must be kept in mind that this method could easily be biased by other effects associated with the track dip angle, since for these tracks the sampling length is almost completely correlated with the dip angle. Therefore, we
Figure 22: The $dE/dx$ resolution as a function of sample length for high-momentum muons. The fitted exponent is $-0.48$.

Figure 23: The $dE/dx$ resolution as a function of sample length for high-momentum electrons. The fitted exponent is $-0.27$. 

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do not consider these measurements to be reliable. We have chosen $p_2 = -0.4$ to be the best value to use in data analysis, but it would be more satisfactory to understand better the sample-length dependence of the resolution for the muons pairs and Bhabha electrons.

14.3 The Dependence on the $dE/dx$ Itself

Comparing figure 18 with figure 17, we see that the relative resolution ($\sigma_I/I$) for electrons in jets appears to be substantially better than that for minimum-ionizing pions, by a factor of almost $\sqrt{1.65}$. In fact, the solid round points in figure 18 are just the pion points from figure 17 multiplied by $1.6^{0.4}$, and they agree rather well with the electron measurement. This should not come as a surprise, since the electrons produce more ionization clusters by a factor of about 1.65 and therefore should have a smaller statistical error. The exponent, $p_3$ in this case, can reasonably be expected to be the same as the exponent $p_2$. We don’t expect $p_3 = -0.5$ for exactly the same reason that $p_2 \neq -0.5$. In fact, $p_3 = -0.4$ describes these data well, although we have not found it to be possible to nail down this exponent with great precision.

It is interesting to check what resolution is obtained from bhabhas and $\mu$-pairs, since they represent ideal, straight, isolated tracks. Figures 14.3 and 25 show the truncated mean distributions for 45.6 GeV electrons and muons. All of the tracks are required to have $N > 280$, and the average is $\bar{N} = 316$. We find that both the muons and bhabha electrons have a resolution of 4.8%. The calibration from electrons in jets predicts that bhabha electrons with $N = 316$ should have a resolution of about 5.0%, which is fairly consistent with the observed resolution. We might expect that the straight, isolated tracks should have a slightly better resolution than the curved tracks in jets, due to track confusion in jets and remaining systematic problems with the large angles produced by curving tracks.

Roughly the same effects are seen by OPAL [5], but they ascribe all of the difference in resolution between MIPs and electrons to systematic effects associated with multi-track confusion in jets. They find a resolution of 3.1% for dimuons and 3.8% for minimum ionizing pions.\footnote{In terms of raw resolution OPAL does considerably better than ALEPH, because of their 4 atmospheres of pressure in the jet chamber.} They explain that “the fact that the resolution for tracks in dense environments is slightly worse than for isolated tracks can again be attributed to the insufficient multiple hit correction.” Note that they do not give a resolution figure for electrons in jets. We suspect that their interpretation is incorrect and wonder whether if they did measure electrons in jets they would find a resolution similar to that of the dimuons. In other words, the dense track environment may not have much to do with it—after all, there is a purely statistical effect (namely the exponent $p_3$) of roughly the same magnitude which must contribute but is not mentioned in that publication. In fact, if one takes the mean $dE/dx$ for their muons divided by that of their MIPs, one finds a ratio of about 1.45. Assuming then an exponent $p_3 = -0.4$, one predicts that if the dimuon resolution is 3.1%, then the MIP resolution should be about $3.1\% \cdot (1.45)^{0.4} = 3.6\%$, which explains most of the difference that they see. This
Figure 24: The $dE/dx$ distribution for bhabha electrons. Each track is required to have greater than 280 measurements.
Figure 25: The $dE/dx$ distribution for 45.6 GeV muons. Each track is required to have greater than 280 measurements.
is in fact important, because if we are correct, then they are predicting too poor a resolution for electrons that they identify in hadronic jets by \( dE/dx \), which could have implications for their efficiency and background estimates.

Until 1992 we also assumed \( p_3 = 0 \), since we were seeing the same resolution for dileptons as for MIPs. What changed in 1992, of course, is that we began correcting for the threshold effects. In fact, the threshold had been producing an artificially good resolution for the MIPs. That is easy to understand: by chopping off the low part of the Landau distribution, the threshold was not only increasing the truncated mean but also was decreasing the rms of the truncated mean. The ratio of the two therefore gave a much-to-low estimate of the resolution. With this effect removed by the new algorithm, we should be seeing something much closer to the truth, and in fact the data above (for both ALEPH and OPAL) are quite consistent with \( p_3 \approx -0.4 \). It is hard to nail down this exponent precisely from the data, but it clearly cannot be zero.

In summary, we have concluded that the best values for the exponents describing the \( dE/dx \) resolution are \( p_1 = -0.5 \), \( p_2 = -0.4 \), and \( p_3 = -0.4 \). The best value for \( \sigma_0 \) in equation 12 is 1.186, as obtained from a fit to the plot of resolution versus number of samples. From this formula, a sort of best-case resolution would be 4.5% for a Bhabha track at \( \theta = 45^\circ \). The more optimistic resolutions that we claimed in reference 3 are, in short, simply incorrect and resulted from not taking into account the effect of the threshold.

### 15 The Bethe-Bloch Curve

For a given medium, the expected \( dE/dx \) depends only on the incident particle velocity \( \beta \) and its charge. The parameterization in terms of \( \beta \) is taken to be the usual Bethe-Bloch form, as can be found in the Review of Particle Properties:

\[
f(\beta \gamma) = \frac{Q^2 \chi}{\beta^p} [K + 2x - \beta^p - \delta] ,
\]

where \( Q \) is the charge, \( x \equiv \ln \beta \gamma \), and \( \chi, K, \) and \( p \) are constants to be fit. The constant \( p \) should be exactly 2, theoretically. We don’t fix it to that value because detector effects, such as saturation, can cause it to appear to be different from 2. Note that this form does not apply to very low values of \( \beta \), and in fact the prediction goes negative as \( \beta \) goes to zero. The saturation, or transition from the relativistic rise to the plateau, is described by the function \( \delta \). We use an arbitrary, nonanalytic form for \( \delta \), which is however continuous and smooth:

\[
\delta = \begin{cases} 
0 & x < x_0 \\
2(x - x_0) + \sum_{i=2}^5 a_i (x - x_1)^i & x_0 < x < x_1 \\
2(x - x_0) & x > x_1
\end{cases}
\]

Requiring continuity and smoothness at \( x = x_0 \) fixes \( x_0 \) and \( a_2 \), leaving \( x_0, x_1, a_3, a_4 \) and \( a_5 \) as free parameters, for a total of 8 free parameters in the fit. This is not an ideal parameterization, since many of the parameters are highly correlated in the fit. However, in practice it works fine.
For the most part, tracks in hadronic events are used to fit the free constants in equation 14. The only exception is that dimuons and bhabha tracks are used to provide two points at very high velocity. Tracks from tau decays also are useful for this purpose, but so far they have only been used as a cross check. Our procedure is first to derive from the data a plot of mean $dE/dx$ as a function of $\beta \gamma$ and then fit that plot to equation 14.

We begin by dividing the data into small momentum bins, with bin width generally increasing with momentum. The dimuons and bhabha tracks easily give two isolated points. For the other momentum bins we always have a mixture of all particle types found in hadronic $Z$ decays. Each momentum bin gives a histogram of the truncated-mean $dE/dx$, which appears as a superposition of gaussian peaks, one for each particle type. We fit each histogram to a sum of four gaussians, for electrons, pions, kaons and protons. Only tracks with at least 200 $dE/dx$ measurements are included in the histograms, so there is not a large variation in resolution. Therefore, the widths of the gaussian peaks are taken from our resolution parameterization, using the mean values of $N$ and $\Delta$ for the tracks within the momentum bin. We also add in quadrature 0.85 times half the momentum bin width times the derivative of the Bethe-Bloch curve with respect to momentum, on order to take into account the variation in expected $dE/dx$ within a single momentum bin. This contribution, however, is important only in the $1/\beta^2$ region of the curve.

The amplitudes of the gaussian peaks always are left free in the fit, and the position of the pion peak also is left free. The positions of the electron, kaon, and proton peaks, however, are left free only in those cases where they are well separated from the other particle types, in particular the pions. Otherwise, they are fixed to the prediction of the previous iteration of the fit to equation 14. Figures 26 and 27 show the $dE/dx$ distributions for several example momentum bins, including the fit to the contributions from the various particles.

The results of the fits to the various momentum bins, including the points from $\mu$-pairs and bhabha electrons, are shown in figure 28, overplotted by the final fitted curve. Figure 29 shows the same fitted curves, with the data plotted as a scatterplot. The data can also be viewed all on one curve when plotted as a function of $\beta \gamma$, as shown in figure 30 (the low momentum electrons are not shown in this plot and were not included in the fit). This figure also shows the residuals. Clearly, from the point of view of the statistical errors on the points, the fit is very poor. The wiggles in the data are from residual systematic effects arising from track angles, drift lengths, saturation effects, multi-track effects, and so forth. There is no point in trying to modify the parameterization to follow those wiggles. In the momentum region of interest, the largest deviations are around 1%, while the optimum resolution is about 5%, so while the result is not perfect, it is acceptable. It is interesting, however, to look at the low momentum electrons in figure 28. They give a slightly lower $dE/dx$ than do the high momentum bhabha electrons. This could be a remaining effect from track curvature (the discrepancy used to be far worse), or it could be partially due to a plateau which is not absolutely flat (some models predict a slightly rising “plateau”). Whatever the case, we choose to define the relativistic rise to be the ratio of electron to pion $dE/dx$.
Figure 26: The measured $dE/dx$ in hadronic events for two narrow momentum bins at low momentum. The histograms have been fit to a sum of four gaussians, to account for pions, electrons, kaons, and protons. Only the amplitudes of the four gaussians and the positions of the pions and proton peaks have been allowed to vary freely in the fit.
Figure 27: The measured $dE/dx$ in hadronic events for two narrow momentum bins at high momentum. The histograms have been fit to a sum of four gaussians, to account for pions, electrons, kaons, and protons. Only the amplitudes of the four gaussians and the position of the pion peak have been allowed to vary freely in the fit.
Figure 28: $dE/dx$ measurements used to fit the Bethe-Bloch curve. Except for the two points at 45.6 GeV, which come from dileptons, the measurements are made on tracks in hadronic jets.
Figure 29: The measured $dE/dx$ versus particle momentum for a sample of about 40,000 tracks in the ALEPH detector. Each track was required to have at least 150 $dE/dx$ measurements. The fitted parameterization is overplotted for electrons, muons, pions, kaons, and protons. The deuteron band is also visible. The points at high momentum with $dE/dx$ in the range from 2 to 4 result from overlapping tracks which were not resolved by the pattern recognition.
Figure 30: The values for $dE/dx$ as a function of $\beta\gamma$ for $|\cos \theta| < 0.7$ as measured from 1992 data. The points have been fit to the parameterization of equation 14.
at the momentum where the pions are minimum ionizing. To get the appropriate value, we replace the μ-pairs and bhabha electrons in the fit by the low-momentum electrons. The plateau then comes out to be 1.656, which is our best estimate of the relativistic rise.

We have tried to check, using the random number generator, whether the 8%/60% double-sided truncated mean has some effect on the apparent value of the relativistic rise. To simulate two different ionizations, the single random numbers are used for "minimum ionizing," while pairs of random numbers added together are used for the "plateau." For this example, the ratio of truncated means for "plateau" versus "minimum ionizing" varied by only 1% when changing from a 60% single-sided truncated mean to a 8%/60% double-sided truncated mean, so any effect appears to be negligible.

16 Discussion of Effects of Changes Introduced in 1992

The changes to the reconstruction code and the online thresholds that were made for data taken in 1992 and thereafter have profound effects on the dE/dx that take a little bit of thought to understand fully. In order to predict what should be expected, we did some simple experiments with a Landau random number generator. Figure 31 shows a histogram of the output of the generator. From this distribution, N samples are taken and from them a truncated mean is calculated. This procedure is repeated many times in order to generate a distribution of the truncated mean. Then the mean and rms width of that distribution are plotted as a function of N.

First, the 60% truncated mean is on average about 3.7, as seen in figure 32. Note that below about N = 50 it is somewhat unstable, however. That is one reason that the dE/dx generally is not used for tracks with less than 50 dE/dx measurements—the distribution of the mean in such cases is not very gaussian.

Now consider the rms width, which is a measure of the resolution. Figure 33 shows the rms width as a function of N for both the 60% truncated mean and a 2-sided truncated mean where 8% is truncated from the low side of the distribution. The data can be fit by a power-law form:

$$\sigma = A \cdot N^r$$  \hspace{1cm} (16)

from which we find

<table>
<thead>
<tr>
<th></th>
<th>60%</th>
<th>8%/60%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.50 ± 0.06</td>
<td>2.41 ± 0.05</td>
</tr>
<tr>
<td>r</td>
<td>-0.516 ± 0.005</td>
<td>-0.494 ± 0.005</td>
</tr>
</tbody>
</table>

We see that by truncating on the low side we worsen the resolution by about 4% relative, which is not a big effect (this is not as obvious in the table as it is in the figure). This degradation is about what we should expect to see for the change in resolution of electrons in the data. That is because electrons have always produced such a large signal that the threshold never had a large impact. Therefore, for electrons, the only
Figure 31: The random number distribution used to understand the effects of the threshold on the truncated mean.

Figure 32: Truncated mean versus $N$, the number of random-number samples taken from the distribution in figure 31.
$\chi^2 = 30.9 \text{ for } 35 - 2 \text{ d.o.f.}$, \quad C.L. = 57.0%

<table>
<thead>
<tr>
<th>Function</th>
<th>Power Law</th>
<th>Parabolic</th>
<th>Minos</th>
</tr>
</thead>
<tbody>
<tr>
<td>NORM</td>
<td>2.4984</td>
<td>±5.645E-02</td>
<td>0.0000E+00</td>
</tr>
<tr>
<td>POWER</td>
<td>-0.51630</td>
<td>±4.5135E-03</td>
<td>0.0000E+00</td>
</tr>
<tr>
<td>SLOPE</td>
<td>1.0000</td>
<td>±0.0000E+00</td>
<td>0.0000E+00</td>
</tr>
</tbody>
</table>

Figure 33: From a Landau random number generator, the rms width versus number of samples for two types of truncated mean: a single-sided 60% truncated mean, and a double sided 8%/60% truncated mean.
significant change in going from the old to the new algorithm is the addition of the 8% truncation on the low side.

The new algorithm has a much more dramatic effect on the minimum ionizing pions. With the old algorithm, the resolution was artificially small because of the threshold which chopped off the lower part of the distribution. This effect can be seen by using the random number generator. The following table was calculated by taking trials of $N = 200$ samples per “track”:

<table>
<thead>
<tr>
<th>method</th>
<th>mean</th>
<th>rms</th>
<th>rms/mean ($\sigma/\mu$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60% truncated mean</td>
<td>3.695</td>
<td>0.1635</td>
<td>4.42</td>
</tr>
<tr>
<td>old algorithm</td>
<td>4.080</td>
<td>0.1560</td>
<td>3.82</td>
</tr>
<tr>
<td>new algorithm</td>
<td>3.929</td>
<td>0.1934</td>
<td>4.92</td>
</tr>
</tbody>
</table>

In this test the “old algorithm” is simulated by simply discarding all samples below a threshold of 2.34 (which on average throws out 8% of the samples). Then a 60% truncated mean is calculated. The “new algorithm” is simulated by setting all samples below 2.34 to unity and then taking an 8%/60% two-sided truncated mean.

The quantity $\sigma/\mu$ is what is generally used to evaluate the quality of the data. This table indicates how misleading it can be. Applying an arbitrary threshold cannot possibly improve the quality of the data, but it does reduce $\sigma/\mu$ by both reducing $\sigma$ and increasing $\mu$. Since we cannot avoid the threshold in our data, then the new algorithm clearly is the better and more honest approach and is necessary if systematic bias from a fixed threshold is to be avoided. A truncation applied as a percentage rather than a fixed threshold has the effect of increasing the rms rather than decreasing it. It also obviously increases the mean. Finally, the ratio of the values for $\sigma/\mu$ for the old versus new algorithms is about 1.3! This is roughly what is observed for the widths of the truncated mean distributions from minimum ionizing pions when going from the old to the new algorithm. One should not conclude from that, however, that the $dE/dx$ has been degraded by 30%! Most of this 30% effect is from a misunderstanding of the pions in the old data. Furthermore, overall the reliability of particle identification has improved. The Bethe-Bloch curve is now more-or-less unique, regardless of the particle’s momentum or track angle. That is what is important for particle identification, not an artificially good resolution with a biased particle identification, as in 1991 data.

Note that the new algorithm does not actually throw away much good data in the case of MIPs, because the 8% of the samples cut out on the low side when calculating the truncated mean are on average just those below-threshold samples that were added by the new algorithm. Electrons do lose out, however, since the 8% of samples cut out of the mean really do contain useful information. The electron resolution (measured with Bhambhas, for example) therefore really does worsen with the new algorithm by about 10% relative. That is the price we pay for avoiding systematics caused by the threshold. Obviously it would be far more satisfactory to have a much lower threshold (i.e. better signal/noise for the TPC wires), such that the lower truncation would not be necessary.
Figure 34: $R_\pi$, the measured $dE/dx$ divided by the expected $dE/dx$, for minimum ionizing pions as a function of $\cos \theta$. The gentle bow in the plot is from residual systematic angular effects, while the sharp dip at $\theta = 90^\circ$ is believed to be due to crosstalk.

17 Cross-Talk

A potential concern is electronic crosstalk. One expects to see an effect in the case where a track is almost parallel to the wire plane. In such a situation, all wires are hit at exactly the same time, and the avalanche on a given wire can partially cancel the signal on the neighboring wires. There may be additional effects within the signal busses and amplifiers, but probably the biggest effect occurs in the amplification region of the wire chamber.

In fact we do see a small decrease in $dE/dx$ for tracks near $\theta = 90^\circ$, as shown in figure 34 for minimum ionizing pions. For ALEPH this is not a big concern, since this is only a very small part of our fiducial region. It is much more serious, to take another example, for OPAL’s jet chamber, which has many planes of wires which can be parallel to tracks of any dip angle. Up to now we have never applied a correction for this small effect. (OPAL, on the other hand, does make a correction for this effect.)

One might also expect a crosstalk effect for pairs of tracks at angles away from $\theta = 90^\circ$, since for each point on one track there is a corresponding point on the other track with equal drift time. However, since tracks closer together in $z$ than about 3 cm are not resolved on the wires and are not used for $dE/dx$, then the equal-time points on pairs of tracks are generally separated by a few cm and therefore are not seen on adjacent wires. This would seem to rule out the possibility of crosstalk in such
18 Multi-track Effects and Changes in 1993

A lot of effort was put into the reconstruction program to avoid ever using wire hits that include ionization from more than a single track. In the course of working with Ilias Efthymiopoulos on the search for particles with unexpected ionization[7] we tested this aspect of the code very thoroughly (and made a number of improvements to the code). That is because overlapping tracks caused most of the background in the high ionization region. We are confident that this algorithm never fails unless either there is a track completely missed by the TPC pattern recognition which partially overlaps another track or else there are two tracks which never separate enough on the TPC pads ever to be recognized as such. If this algorithm did not work, one might expect to see an excess \( dE/dx \) for any track which partially overlaps another, since double pulses would be picked up and counted in the truncated mean. That is not observed.

In fact, up to now the \( dE/dx \) has been observed to decrease when tracks get too close together. This is seen most clearly in 3-prong \( \tau \) decays and is partly a result of our zealous discarding of hits from multiple tracks combined with an attempt to maximize the efficiency for picking up single hits. What happens is that the program accepts small, separated pulses from such pairs of tracks, but larger pulses tend to be merged and are discarded. This results in a bias against the large pulses and a corresponding decrease in the truncated mean.

However, the discrepancy between the \( dE/dx \) in 3-\( \pi \) versus single \( \pi \) decays became much worse in the reprocessed '92 data and in '92 Monte Carlo. In fact, the modifications made in '92 do open up some more possibilities for multi-track confusion. If two pulses are merged, the resulting funny pulse shape may produce a time estimate which matches with one of the tracks but not with the other. Such a pulse is nearly always rejected by the cuts on pulse width and pulse shape. The problem is that if one is not careful the other track could be assigned a below-threshold hit.

The program tries to avoid that scenario by checking for each below-threshold hit whether there is any pulse on the wire that overlaps the track, even if the time estimate of that pulse is quite far from the track. To do this, the program obviously needs the pulse lengths, which fortunately were added to the TPP output data stream in 1991 for other reasons (namely to be able to make pedestal corrections offline, which so far has never been done).

After the '92 processing, a bug was discovered which caused those pulses not reduced in the TPP to get the wrong pulse length assigned to them. In fact, they ended up getting the pulse length of the last pulse which was reduced in the TPP, meaning that they tended to get pulse lengths which were too short, thus compromising the effectiveness of the protection against multitrack confusion. In addition, the cut which determined if there was a pulse overlapping the track was not well adjusted and tended to miss many cases where the pulse occurred slightly later or, especially, earlier in drift time compared with the track extrapolation. This bug has been fixed, and the cut has
<table>
<thead>
<tr>
<th></th>
<th>old algorithm</th>
<th>new algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>1-prong</td>
<td>0.556</td>
<td>1.05</td>
</tr>
<tr>
<td>3-prong</td>
<td>0.080</td>
<td>1.30</td>
</tr>
<tr>
<td>1st track</td>
<td>-0.304</td>
<td>1.55</td>
</tr>
<tr>
<td>2nd track</td>
<td>0.107</td>
<td>1.15</td>
</tr>
<tr>
<td>3rd track</td>
<td>0.454</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 1: Means and rms widths of distributions of normalized $dE/dx$ errors from pions in 3-prong $\tau$ decays and 1-prong $\tau$ decays obtained using both the 1992 reconstruction program and the new program intended for 1993 reprocessing. 1st, 2nd, and 3rd refer to the ordering by drift length of the tracks in the 3-prong events. Note that the new algorithm was not yet calibrated to center $\mu$ at zero.

been better adjusted.

In addition, the algorithm for the time estimate of pulses not reduced in the TPP has been changed. Formerly it took a pulse-height-weighted mean of the time of the samples. That works fine for pulses from single tracks but gives rather unpredictable results for doubly-peaked pulses. The new algorithm takes the average of the threshold crossings on both ends of the pulse. This always gives a time which is well centered, such that the beginning and ends of the pulse are well defined in the routine (TFTW2B) which checks for overlap with tracks.

To reduce the bias caused by the rejection of double-peaked pulses, the program has recently been changed, for the '93 processing, such that it tries to split the pulses. If exactly two peaks are found in the pulse, with a distinct valley in between, the pulse is split at the minimum of the valley if the two peaks are at least 3 cm apart. Each of the resulting halves are used in the $dE/dx$ calculation if their peak height is at least 1.9 times the height of the valley. The time estimate for each peak is taken from a parabola algorithm applied to the three highest samples, and the charge is simply taken to be the sum of sample pulse heights, without trying to correct for interference with the companion pulse.

Taken together, these improvements nearly eliminate the major differences between the $dE/dx$ of pions in $\tau^- \rightarrow \rho^- \nu$ decays and pions in 3-prong tau decays. Figure 35 shows the normalized $dE/dx$ errors (called $R5$) for tracks in Monte Carlo 3-prong $\tau$ decays using both the old (with the bug fixed) and new algorithms. The old algorithm resulted in a large non-gaussian tail toward low $dE/dx$ and is in general shifted negative with respect to the $dE/dx$ observed for isolated pions. Table 1 gives the means and rms widths of the $R5$ distributions for the two algorithms, comparing tracks in 3-prong $\tau$ decays with isolated pions in single-prong $\tau$ decays. Also included are the values for the distributions from each of the 3 pions in the 3-prong decays, ordered by increasing drift length. On average, the pions in 3-prong decays get close to the same $dE/dx$ now as do isolated pions, and the non-gaussian tail is gone. However, there still is some dependence
Figure 35: Normalized $dE/dx$ errors ($R5$) for tracks in Monte Carlo 3-prong $\tau$ decays with the old (1992) and new (1993) algorithms for data reduction of long pulses.
Figure 36: Normalized $dE/dx$ errors ($R5$) for tracks in Monte Carlo 3-prong $\tau$ decays, separately plotted for the 1st, 2nd, and 3rd tracks to drift into the proportional chambers. The corresponding means and standard deviations are listed in table 1.

on the track ordering of the mean $dE/dx$. This is most clearly seen in figure 36, which shows the distributions separately for the 1st, 2nd, and 3rd tracks in 3-prong events. It is not yet understood whether this is a residual effect from the problems already considered or is completely new. Evidently there still is some room for improvement in this area.

These Monte Carlo studies have recently been verified using real data from the 1993 reprocessing. The dramatic improvement between the old and new algorithms is most evident in figure 37, obtained from a subset of the 1993 3-prong $\tau$ decays and all of the 1992 3-prong $\tau$ decays. The 1993 data give a mean $dE/dx$ estimator that is independent of the angle between adjacent tracks. That is a big improvement over the 1992 data, where the dependence on that angle displays shifts almost as large as 0.5$\sigma$.

The new algorithm should also reduce the problems with $dE/dx$ measurements in $V^0$ decays and the dependence of electron identification by $dE/dx$ on the track $p_\perp$, with respect to the jet direction. Although this has not yet been fully verified, it also should have the bonus of improving slightly the $dE/dx$ efficiency (the tracks in 3-prong $\tau$ decays now have on average 5 more samples per track than with the old program).

54
Figure 37: The dependence of the $dE/dx$ estimator on the angle between adjacent tracks in 3-prong $\tau$ decays (from work done by Michael Schmitt). The 1993 data are done with the new algorithm.
However, one pays a price in that the '93 data now are quite different from the '92 data, unless the latter are again reprocessed.

19 Conclusion

The TPC $dE/dx$ calibration is complicated, but our understanding of it has improved enormously since data taking began in 1989. The changes made in 1992 to remove the threshold effects was, we believe, a big advance which has greatly improved the systematics with respect to track angles. Unfortunately, it also worsened the multitrack effects. Hopefully that problem has been largely cured for the 1993 run (and for 1992 data if ever they are processed again).

As for the hardware, significant improvements in $dE/dx$ resolution could be obtained by reducing the rather large threshold effect. That could be done either by lowering the TPD threshold even further and accepting more noise on the raw data tapes or else by increasing the TPC gas gain by raising the high voltage, perhaps up to the 1275 volts at which we originally intended to run. (In the first days of LEP running, the TPC high voltage was lowered from the intended 1275 volts, which had been used up to that time, down to 1250 volts, which resulted in about a 30% loss of gain and signal-to-noise.) Neither option is very popular, however, so we are not making any such proposal here.

For the moment, the actual performance of the $dE/dx$ is best summarized in terms of average separation between particle types in hadronic jets. That can be seen in figure 38, which was obtained simply by taking the parameterizations of resolution and the Bethe-Bloch curve as determined above and applying them to all tracks with at least 50 $dE/dx$ measurements in hadronic $Z$ decays (88% of the tracks). For each momentum bin, then, the average particle separation is plotted, and a smooth curve is drawn. Clearly the separation must be considerably better for isolated tracks, such as those in single-prong tau decays, since isolated tracks generally have the most measurements and therefore the best $dE/dx$ resolution. The tracks included in figure 38 have on average only 60% of the maximum possible number of $dE/dx$ measurements.

20 Appendix: Derivation of the Variance of a Truncated Mean

Equation 4 has been derived by a general method outlined in [12]. The derivation goes as follows. A general result, valid for large $n$, gives the covariance between any two ordered measurements from a distribution $f(x)$ with cumulative distribution $F(t)$:

$$\text{cov}(x_i, x_j) = \frac{i}{n+1} \frac{1 - \frac{j}{n+1}}{n \left( \frac{1}{n+1} \right) \left( \frac{j}{n+1} \right)}$$  \hspace{1cm} (17)$$

where $h(s) \equiv f(F^{-1}(s))$. The variance of the truncated mean, $\sigma_{\bar{x}_t}^2$, involves a sum of the covariances, $\text{cov}(x_i, x_j)$, which can be approximated by an integral. Define the
Figure 38: The average $dE/dx$ separation in standard deviations between particle types, as computed using all tracks in hadronic $Z$ decays which have at least 50 $dE/dx$ measurements. The $e-\pi$ separation, for example, is computed from $S = (I_e - I_\pi)/\sigma_\pi$, where $I_e$ and $I_\pi$ are calculated from our parameterization of the Bethe-Bloch curve, while $\sigma_\pi$ is from our parameterization of the resolution, computed for the hypothesis that the track is a pion. For the $p-K$ separation, the resolution in the denominator is taken to be that of the kaon hypothesis, while for the $\pi-K$ separation, it is taken to be that of the pion hypothesis.
truncation point \( x_c \) by \( h(p) = f(x_c) \), where \( p \) is the fraction of measurements included in the truncated mean. Then, with a weight function \( b(s) \) defined as

\[
b(s) = \begin{cases} 
1/p & 0 \leq s \leq p \\
0 & p < s \leq 1 
\end{cases},
\]

the variance of the truncated mean can be expressed as

\[
\sigma^2_{\bar{x}_t} = \frac{2}{n} \int_0^1 \int_0^t \frac{s(1-t)}{h(s) h(t)} b(s) ds b(t) dt.
\]

(19)

Here, \( n \) is the total number of measurements, before truncation, so \( m \equiv p \cdot n \) is the number of values actually included in the truncated mean. Equation 19, after a nontrivial integration by parts, yields

\[
n \cdot \sigma^2_{\bar{x}_t} = \int_0^1 C^2(t) dt - \left( \int_0^1 C(t) dt \right)^2,
\]

(20)

where the function \( C(t) \) is defined by

\[
C(t) = \int_0^t \frac{b(s) ds}{h(s)}.
\]

(21)

To evaluate the integrals in equation 20, we make a transformation of variables in the integrand from \( t \) to \( y \), where \( t = F(y) \). Define the function \( D(z) \) such that

\[
D(z) = C(F(z)) = \int_0^z \frac{b(F(y)) f(y) dy}{h(F(y))}.
\]

(22)

Since \( h(F(y)) = f(y) \), this becomes simply

\[
D(z) = \int_0^z b(F(y)) dy = \begin{cases} 
z/p & 0 \leq z \leq x_c \\
0 & x_c < z \leq \infty 
\end{cases}.
\]

(23)

Finally, one ends up with the rather easy integrals

\[
n \cdot \sigma^2_{\bar{x}_t} = \int_0^1 D^2(z) f(z) dz - \left( \int_0^1 D(z) f(z) dz \right)^2,
\]

(24)

and the final result is

\[
\sigma^2_{\bar{x}_t} = \frac{1}{np} \left[ s_t^2 + (1 - p)(\bar{x}_t - x_c)^2 \right],
\]

(25)

where \( s_t^2 = \bar{x}_t^2 - \bar{x}_c^2 \) is the square of the standard deviation of the truncated sample, and \( \bar{x}_t \) is the truncated mean itself.
References


