A Modern Introduction to Quantum Field Theory

Michele Maggiore

Département de Physique Théorique
Université de Genève

OXFORD UNIVERSITY PRESS
Contents

Preface xi

Notation xii

1 Introduction 1
   1.1 Overview 1
   1.2 Typical scales in high-energy physics 4
   Further reading 11
   Exercises 12

2 Lorentz and Poincaré symmetries in QFT 13
   2.1 Lie groups 13
   2.2 The Lorentz group 16
   2.3 The Lorentz algebra 18
   2.4 Tensor representations 20
      2.4.1 Decomposition of Lorentz tensors under SO(3) 22
   2.5 Spinorial representations 24
      2.5.1 Spinors in non-relativistic quantum mechanics 24
      2.5.2 Spinors in the relativistic theory 26
   2.6 Field representations 29
      2.6.1 Scalar fields 29
      2.6.2 Weyl fields 31
      2.6.3 Dirac fields 32
      2.6.4 Majorana fields 33
      2.6.5 Vector fields 34
   2.7 The Poincaré group 34
      2.7.1 Representation on fields 35
      2.7.2 Representation on one-particle states 36
   Summary of chapter 40
   Further reading 41
   Exercises 41

3 Classical field theory 43
   3.1 The action principle 43
   3.2 Noether’s theorem 46
      3.2.1 The energy–momentum tensor 49
   3.3 Scalar fields 51
      3.3.1 Real scalar fields; Klein–Gordon equation 51
      3.3.2 Complex scalar field; U(1) charge 53
3.4 Spinor fields
3.4.1 The Weyl equation; helicity
3.4.2 The Dirac equation
3.4.3 Chiral symmetry
3.4.4 Majorana mass
3.5 The electromagnetic field
3.5.1 Covariant form of the free Maxwell equations
3.5.2 Gauge invariance; radiation and Lorentz gauges
3.5.3 The energy–momentum tensor
3.5.4 Minimal and non-minimal coupling to matter
3.6 First quantization of relativistic wave equations
3.7 Solved problems

The fine structure of the hydrogen atom
Relativistic energy levels in a magnetic field
Summary of chapter
Exercises

4 Quantization of free fields
4.1 Scalar fields
4.1.1 Real scalar fields. Fock space
4.1.2 Complex scalar field; antiparticles
4.2 Spin 1/2 fields
4.2.1 Dirac field
4.2.2 Massless Weyl field
4.2.3 C, P, T
4.3 Electromagnetic field
4.3.1 Quantization in the radiation gauge
4.3.2 Covariant quantization
Summary of chapter
Exercises

5 Perturbation theory and Feynman diagrams
5.1 The S-matrix
5.2 The LSZ reduction formula
5.3 Setting up the perturbative expansion
5.4 The Feynman propagator
5.5 Wick’s theorem and Feynman diagrams
5.5.1 A few very explicit computations
5.5.2 Loops and divergences
5.5.3 Summary of Feynman rules for a scalar field
5.5.4 Feynman rules for fermions and gauge bosons
5.6 Renormalization
5.7 Vacuum energy and the cosmological constant problem
5.8 The modern point of view on renormalizability
5.9 The running of coupling constants
Summary of chapter
Further reading
Exercises
6 Cross-sections and decay rates
   6.1 Relativistic and non-relativistic normalizations 155
   6.2 Decay rates 156
   6.3 Cross-sections 158
   6.4 Two-body final states 160
   6.5 Resonances and the Breit–Wigner distribution 163
   6.6 Born approximation and non-relativistic scattering 167
   6.7 Solved problems
      Three-body kinematics and phase space 171
      Inelastic scattering of non-relativistic electrons on atoms 173
Summary of chapter 177
Further reading 178
Exercises 178

7 Quantum electrodynamics 180
   7.1 The QED Lagrangian 180
   7.2 One-loop divergences 183
   7.3 Solved problems 186
      \( e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^- \) 186
      Electromagnetic form factors 188
Summary of chapter 193
Further reading 193
Exercises 193

8 The low-energy limit of the electroweak theory 195
   8.1 A four-fermion model 195
   8.2 Charged and neutral currents in the Standard Model 197
   8.3 Solved problems: weak decays
      \( \mu^- \rightarrow e^-\nu_e\nu_\mu \) 202
      \( \pi^+ \rightarrow l^+\nu_l \) 205
      Isospin and flavor \( SU(3) \) 209
      \( K^0 \rightarrow \pi^-l^+\nu_l \) 212
Summary of chapter 216
Further reading 217
Exercises 217

9 Path integral quantization 219
   9.1 Path integral formulation of quantum mechanics 220
   9.2 Path integral quantization of scalar fields 224
   9.3 Perturbative evaluation of the path integral 225
   9.4 Euclidean formulation 228
   9.5 QFT and critical phenomena 231
   9.6 QFT at finite temperature 238
   9.7 Solved problems
      Instantons and tunneling 239
Summary of chapter 241
Further reading 242