The general status and recent developments in the subjects of supersymmetry and supergravity are reviewed for the benefit of non-specialists. The report is divided into three parts as follows: I. Basics and General Status; II. Selected Recent Work in Global Supersymmetry; and III. Selected Recent Work in Supergravity.

§1. Basics and General Status

Motivation: If supersymmetry or supergravity is the answer, then what is the question? There is a serious reply to this existential query, namely: is there a symmetry principle powerful enough to raise hope for complete unification of the elementary particles and their interactions?

At the global level, supersymmetry is the only invariance compatible with quantum field theory which unifies particles of different spin and internal quantum numbers. It does this by relating bosons and fermions, the two broad classes of particles found in nature.

Local supersymmetry is a gauge principle of a unique type. It is fermionic—the gauge field is a spin 3/2 Rarita-Schwinger field—and it is a significant extension of the general covariance group of relativity. Local supersymmetry automatically requires gravity. The corresponding supergravity field theories therefore link the concept of space-time geometry with the quantum mechanical notions of spin and statistics.

After two years of active research in supergravity, the following key results have emerged:

1) The graviton can be unified with very restrictive combinations of lower spin particles in irreducible representations of supersymmetry algebras. The highly gauge invariant field theories of these systems raise hope for the unification of gravitation with the other interactions of elementary particles.

2) An early sign of encouragement was the cancellation in these unified theories of some of the divergences that have always plagued previous matter-gravity systems. Specifically, the divergences of b-matrix elements in one and two-loop order cancel.

3) Previous consistency problems of the spin 3/2 field such as a causal propagation and negative probabilities are all overcome in supergravity because the interactions respect a fermionic gauge principle.

Despite these achievements there are difficulties. Non-renormalizability looms at the three-loop level and beyond. Phenomenological applications of both supersymmetry and supergravity have interesting features but presently seem unnatural. This report will describe some of these difficulties and some of the dazzling theoretical features of the formalism which have been the subject of recent work.

Basics: The essential idea of supersymmetry is to construct quantum field theories with conserved (Majorana) spinor charges $Q$. These are elements of graded Lie algebras which also involve the Poincaré group generators $P^\mu$ and $M^{\mu\nu}$ and other operators. The various supersymmetry algebras which have had applications in theoretical physics are listed in Table I.

The fundamental Poincaré supersymmetry algebra contains TV-spinor charges $Q_i$, $i=\ldots \cdot \cdot N$ which satisfy

\[ \{Q^i, \tilde{Q}^j\} = \delta^{ij} (\gamma^\nu)_{\alpha \beta} P^\nu, \]

\[ [M^\nu, Q^i] = -i (\sigma^\nu)_{\alpha \beta} Q^\beta, \]

\[ [P^\nu, Q^i] = 0. \]

The presence of the translation generator in the basic anticommutator (1) already suggests the connection between supersymmetry and the structure of space-time which is full realized in supergravity.

To see that (1-3) imply a symmetry between
Table I. Supersymmetry algebras.

1. Poincaré Supersymmetry: Golfand and Likhtman, Yolkov and Akulov, Wess and Zumino
   \[ P^a, M^\mu, Q_a^i, i = 1, 2, \ldots N \]
   \[ \{Q, Q\} = \gamma^\mu P \]
   Simple: \( N=1 \)
   Extended: \( 7V=2, 3, \ldots \)
   natural SO(7V) internal symmetry
   Central Charges: \( U, F \) Haag, Lopuszanski and Sohnius

2. Conformal Supersymmetry: Wess and Zumino
   \[ P^a, M^\mu, Q_a^i, i = 1, 2, \ldots N \]
   \[ D, K^\mu, A, S_a^i \]
   \[ \{S, S\} = \gamma^\mu K \]
   \[ \text{axial, } \text{conformal, } \text{scale} \]
   required SU(N) internal symmetry: Haag et al.

3. DeSitter Supersymmetry: Keck, MacDowell and Mansouri
   relevant to cosmological terms in supergravity

particles of different spin is extremely simple. One simply applies the spinor charge \( Q \) to a particle state \( \psi \) of definite momentum and helicity. Equation (3) and simple addition of angular momenta then imply

\[ Q_\alpha |p, s\rangle = a |p, s + \frac{1}{2}\rangle + b |p, s - \frac{1}{2}\rangle \]

The right side is a superposition of particles of the same momentum and energy—ergo the same mass—and helicities \( \pm 1/2 \). Supersymmetry therefore relates bosons and fermions.

The algebra with one conserved spinor charge is often referred to as "simple supersymmetry" and algebras with more than one spinor charge as "extended supersymmetries." In the latter case there is a unification with internal symmetry. Extended supersymmetry field theories generally have a natural global \( SO(N) \) internal symmetry, under which the \( \hat{Q} \) transform as a \( jV \)-dimensional vector. A larger \( U(7V) \) internal symmetry involving \( j \) rotations of the \( \hat{Q} \) can usually be defined.

There are now three possible viewpoints, each of which leads deeper into the subject.

1) The algebraic view which emphasizes the particle representations of supersymmetry algebras and properties of the corresponding field theories;

2) The field multiplet view which emphasizes the structure of sets of fields which transform irreducibly under supersymmetry;

3) The geometric view whose major branches are superspace and a group-theoretic approach via curvatures.

We shall adopt 1) in this part of the report because it leads most rapidly to an overall picture of the subject. Recent work in 2) will be described in Part III, while 3) will be discussed by other speakers at this session.

The irreducible representations of supersymmetry always consist of "towers of spins" containing equal numbers of boson and fermion states. They can be derived quite easily using Wigner's method of induced representations. One finds that the little algebras of spinor charges are mathematically equivalent to complex Clifford algebras of \( N \) elements in the massless case and \( 2N \) elements in the massive case. The massless representations (of dimension \( 27V \)) are therefore simpler than the massive representations (of dimension \( 27V \)). The number of distinct spins in a representation increases with \( N \). The lowest spin representation for a given \( TV \) is a tower of spins \( s=0, 1/2, 1, s_{\text{max}} \). In the massless case \( s_{\text{max}}=(1/4)(A+1) \) or \( (1/4)JV \), for odd and even \( N \) respectively. For massive particles \( s_{\text{max}}=(l/2)J \).

Higher spins are a notoriously difficult problem in quantum field theory, and at present consistent local field theories with interactions can be constructed only for spins \( s<2 \). Therefore a practical limitation for supersymmetric field theories is the restriction to representations with \( s_{\text{max}}<2 \). The spin content of all massless representations with \( s_{\text{max}}<2 \) is given in Table II. Particles always appear in totally antisymmetric tensor representations of the \( SO(A^+) \) groups, so that the multiplicities are binomial coefficients. For \( N=1 \) supersymmetry, the representations are doublets \( (s, s+1/2) \) of adjacent spin bosons and fermions.

Almost all theoretical work in the subject concerns properties of the field theories corresponding to the representations in Table II.
Table II. Massless representations of Poincaré supersymmetry.

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the algebra must be doubled to be described by a Lagrangian field theory. In both cases a mass term is allowed. (For \( N=2 \) this gives a supersymmetry algebra with central charges for which the restriction \( s_{\text{max}}=(l/2)N \) need not apply. See Part II).

b) \( \hat{s}_{\text{max}}=1 \). These theories are called the globally supersymmetric Yang-Mills theories. The arbitrary Yang-Mills internal symmetry group appears as a direct product with the supersymmetry algebra. It is completely independent of the unified \( SO(N) \) internal symmetry which is realized globally. The Lagrangians include minimal coupling kinetic terms plus Yukawa and quartic interactions and are therefore conventional renormalizable theories. They possess both Poincare and conformal supersymmetry at the classical level. Some of the many interesting properties of these theories will be discussed below.

c) \( \hat{s}_{\text{max}}=3/2 \). Only free field theories exist for these representations because the rudimentary form of spin 3/2 gauge invariance which remains in the absence of gravity is insufficient for consistent interactions.

d) \( s_{\text{max}}=2 \). The representations include a massless spin 2 particle whose interactions respect the gauge principle of general covariance. The corresponding field theories must possess local rather than only global supersymmetry because the notion of a constant spinor parameter is not covariant. The field theories indeed are the locally supersymmetric unified supergravity theories.

There are more general supersymmetric theories which involve the coupling of different representations. For example, one can couple \( \hat{s}_{\text{max}}=1/2 \) and \( s_{\text{max}}=l \) representations using a supersymmetric generalization of Yang-Mills minimal coupling. Futher globally supersymmetric theories with \( s_{\text{max}}<l \) can be extended to local invariance by coupling to the fields of the supergravity gauge multiplets with \( s_{\text{max}}=2 \).

A Global Supersymmetric Theory. The \( N=1 \) super-Yang-Mills theory corresponds to the \( (1, 1/2) \) doublet representation of simple supersymmetry. The Lagrangian is just the minimal coupling of Yang-Mills potentials \( A^a(x) \) and Majorana spinor fields \( \%(x) \) in the adjoint representation of the arbitrary gauge group \( G \):

\[
\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} i \bar{\%} (D_\mu \%)^a
\]

\[
F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu
\]

\[
(D_\mu \%)^a = \partial_\mu \%^a + g f^{abc} A^b_\mu \%^c
\]

(5)

Before the invention of supersymmetry it would have been very surprising to discover
that this rather conventional theory had additional fermionic symmetries. Indeed, for Majorana spinors of the form $\psi = e + if$, where $s$ and $r$ are constant, the Lagrangian changes by a total derivative under the variations 

$$\delta A_{\mu}^a = \frac{i}{\sqrt{2}} \bar{\psi}^{\nu} \gamma_a \partial_{\mu} \psi_{\nu}$$
$$\delta \psi = \frac{i}{\sqrt{2}} \sigma^\mu \epsilon \partial_{\mu} \phi$$  \hspace{1cm} (6)$$

which embody the bose-fermi character of supersymmetry transformations.

Variations with parameter $e$ correspond to Poincaré supersymmetry transformations with conserved Noether current 

$$\delta V_\mu(x) = -\sigma^\mu \epsilon \partial_{\mu} \phi$$  \hspace{1cm} (7)$$

Variations with parameter $T_J$ are superconformal transformations with Noether current 

$$I^\mu(x) = \gamma_\mu \cdot \epsilon \psi$$ \hspace{1cm} (8)$$

There are no local invariants in global supersymmetry or supergravity because of the close connection with space-time translations. At best there are local densities such as the Lagrangian above. Care must be taken to include the total derivative term in the calculation of Noether currents.

**Basic Supergravity Theory.** The $N=1$ supergravity theory is the gauge theory of the algebra spanned by $P^\nu$, $M^\nu$ and $Q_a$. The spin content is given by the $(2, 3/2)$ representation. The spin 2 graviton is the gauge quantum of the Poincare group, while its partner, the spin 3/2 graviton, is the gauge particle of supersymmetry. The gauge fields are the vierbein and Rarita-Schwinger fields $V_{\mu a}$ and $\phi_\mu$.

The locally supersymmetric field theory of this system can again be expressed as a minimal coupling Lagrangian using the Cartan-Palatini formalism of relativity. Specifically 

$$\mathcal{L} = -(1/4\kappa^2)VV^aV_bV^b + R_{\mu
u ab}$$
$$-\kappa^2 \sigma^\mu (\gamma_\nu \partial_\mu \phi)$$ \hspace{1cm} (9)$$

with curvature tensor 

$$R_{\mu
u ab} = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \omega_{\mu c} \epsilon \omega_{\nu cb}$$
$$-\omega_{\nu c} \epsilon \omega_{\mu cb}$$ \hspace{1cm} (10)$$

and local Lorentz covariant derivative 

$$\partial \bar{\phi} = (\partial_\mu + \epsilon \omega_{\mu ab} \sigma^b) \phi$$ \hspace{1cm} (11)$$

where $\omega_{\mu \nu}$ is the spin connection including torsion terms 

$$\omega_{\mu \nu ab} = \frac{1}{2} [V_{\mu b} (\partial_\nu V_{\alpha a} - \partial_\alpha V_{\nu b}) + V_{\nu a} (\partial_\mu V_{\alpha b} - \partial_\alpha V_{\mu b})] V_{\alpha} + i \kappa^2 (\partial_\mu \psi - \frac{1}{2} (\partial_\alpha \gamma^\alpha \phi_{\nu}) - \partial_\nu \psi_{\alpha}) - [a \to b]$$ \hspace{1cm} (12)$$

while $F \wedge \det V$. The gravitational coupling constant is $K^0 = ATZG$.

The Lagrangian changes by a total derivative under the variations 

$$\delta V_\mu(x) = -i \kappa \epsilon(x) \gamma_\mu \phi$$
$$\delta \phi_\mu(x) = \kappa^{-1} \gamma_\mu \epsilon(x)$$ \hspace{1cm} (13)$$

where $\epsilon(x)$ is an arbitrary Majorana spinor function. The variation of $\phi_\mu$ involves the covariant derivative of the gauge parameter as is the case for the gauge potentials in Yang-Mills theory and ordinary gravitation. This shows that $\phi_\mu$ is indeed the gauge field of supersymmetry transformations.

The transformation rules (13) are the simplest realization of the gauge principle of local supersymmetry which has several unusual theoretical features. For example, the commutator $[\partial_\mu (\epsilon_1), \partial_\nu (\epsilon_2)]$ is (when acting on either $V_{\nu a}$ or $\phi_\nu$) 

$$[\partial_\mu (\epsilon_1), \partial_\nu (\epsilon_2)] = [\delta_\mu (\epsilon_1) + \delta_\nu (\epsilon_2) + \epsilon_1 \epsilon_2 \omega_{\mu \nu a} + \delta_\mu (\epsilon_1) + \epsilon_1 \epsilon_2 \omega_{\nu \mu a}]$$
$$-\epsilon_1 \epsilon_2 \omega_{\mu \nu a}$$ \hspace{1cm} (14)$$

It involves a general coordinate transformation with displacement parameter $f^\nu \epsilon e^\nu$ together with field dependent local Lorentz and supersymmetry transformations and equations of motion terms which vanish when $\gamma^\nu = \delta$. At first glance this seems considerably more complicated than the supersymmetry anti-commutator (1). However, global algebraic relations such as (1) are to be understood as acting on the particle states of the theory in an asymptotically flat background geometry. In this limit (14) does imply (1) because the field dependent transformation and equation of motion terms do not contribute. This can be shown using functional Ward identities.

The proof of invariance of global and local supersymmetric field theories (such as (5) and (9)) can be obtained by direct calculation of the variation $hSf$ using the appropriate field transformation laws (such as (6) and (13)). More systematic methods based on field multiplets or super space are available for $N=1$.
theories, and some of them will be discussed in Part III. At the nitty-gritty algebraic level all of these methods involve

a) detailed Dirac algebra, such as the relations

\[ [\gamma^\mu, \gamma^\nu] = i\sigma^{\mu\nu} g^{\rho\sigma} - i\sigma^{\nu\rho} g^{\sigma\mu} \]

and

b) recognition of basic identities such as the gauge field Bianchi identity \( \varepsilon^{\mu
\nu\rho\sigma} \partial_\sigma A_\rho = 0 \) or the gravitational Ricci identity

\[ [\nabla_\mu, \nabla_\nu] \varepsilon(x) = \frac{1}{2} R_{\mu\nu\sigma\tau} \varepsilon^{\sigma\tau}(x) , \]

c) use of Fierz rearrangement to show that expressions such as \( f_{\alpha\beta\gamma\delta} (\gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta) \) vanish because of over-antisymmetrization of spinors.

It is essential, particularly for c), that all spinorial quantities such as \( \varepsilon^\alpha \) or \( \beta^\alpha \) be treated as anti-commuting Grassmann variables. It is here that the connection between spin and statistics enters intimately in supersymmetry and supergravity.

Types of Supergravity Theories: The kinds of theories which have been developed so far include

i) Unified supergravity theories which are field theories of particles in representations with \( \lambda_{\text{max}} = 2 \) of Table II. These representations can be called the gauge multiplets of supergravity.

ii) Matter coupling theories in which the gauge multiplets are coupled to matter multiplets with \( \lambda_{\text{max}} < 2 \). One can hope to obtain such theories for \( N<4 \) where the relevant representations exist.

iii) Conformal supergravity theories which are gauge theories of the superconformal algebras and which can also be viewed as supersymmetric extensions of Weyl's conformal invariant theory of gravitation.

We will now discuss the general properties and status of these theories without detailed reference to Lagrangians, field variations, etc.

Unified Extended Supergravity Theories: These theories first exhibited the promise of the gauge principle of local supersymmetry for the unification of particle interactions and for a renormalizable theory of gravity. There are 8 distinct theories which may be classified according to the number TV of gauged supersymmetry charges. Each theory has manifest \( \text{SO}(10) \) internal symmetry and describes a set of particles where the spin 2 graviton is unified with \( N \) graviton and lower spin particles in antisymmetric tensor representations of \( \text{SO}(TV) \) (see Table II). Complete theories are known for \( \lambda = 2, \lambda = 3, \lambda = 4, \lambda = 8 \) and there are partial results for \( N=6 \).

In each theory all particle states are connected by supersymmetry and \( \text{SO}(N) \) transformations. It is noteworthy that this unification can include spins 1/2 and 0 which are not associated with gauge fields but are necessary to construct field theories which gauge the extended supersymmetry algebras for \( TV>3 \). An approach to supergravity based on algebraic geometry which clarifies the appearance of lower spin particles has recently been proposed by MacDowell.

Unfortunately there is a very complicated non-polynomial structure due to the spinless fields in the extended supergravity theories for \( N>4 \). In recent work we now have two equivalent forms of the \( TV=4 \) theory. The first form has a non-polynomial structure in which functions such as \( \sqrt{-c(A^\dagger B)} \) appear. In the second form there is a manifest \( \text{SU}(4) \) global internal symmetry and a simpler non-polynomial structure with exponential functions \( e^\nu \) of a single scalar field. The equivalence of the two forms involves transformations of the fields which include dual transformation of vector field strengths.

The \( \text{SO}(8) \) extended supergravity theory is thought to be the largest of the class because the lowest spin representations of extended supersymmetry for \( N>9 \) include spins \( 5/2 \) and we are presently unable to describe such high spins in quantum field theory. Theories with \( TV=5,6 \) are expected to be subcases of the \( \text{SO}(8) \) theory while the \( N=1 \) representation has the same particle content as \( TV=8 \) and is expected to be equivalent. The \( N=5 \) theory has therefore been approached as the next barrier beyond \( TV=4 \). Due to the complicated non-polynomial structure, there are at present only partial results which give all terms in the Lagrangian through order \( 1/c \) in the gravitational coupling. However a supergravity theory of the simple supersymmetry algebra in 11 space-time dimensions has recently been constructed by Cremmer, Julia and Scherk.

After compactification of 7 spatial dimensions one expects a 4-dimensional supergravity
theory with SO(7) internal symmetry which should be related to the 80(8) theory.

It may seem that the SO(8) supergravity representation is large enough to accommodate all the elementary particles which are known or suggested by present experiments. However, the opposite is true; the SO(8) theory is too small. In a classification of the states with respect to an assumed exactly conserved SU(3) color subgroup of SO(8), Gell-Mann has found that the spin 1 multiplet breaks up into SU(3) (electric charge) quantum numbers as

28 = 8(0) + 1(0) - 1(0) + 3(-1/3) + 3(-1/3) + 3(2/3) + 3(-1/3) + 3(-2/3).

These particles can be identified with colored gluons, the photon, and weak neutral Z boson plus fractionally charged superheavies of the same type as occur in other grand unification attempts. The spin 1/2 states have the following decomposition as Dirac spinors:

3(2/3) + 3(-1/3) + 3(-1/3) + 3(0) + 1(-1) + 1(X0) + 1(0) and can accommodate the u, d, s, c quarks, a fifth quark flavor which is a color sextet, a neutral octet, one charged lepton (the electron?) and two neutrinos. The particles missing from the scheme are charged vector bosons W± and leptons such as fī and r. The essential reason for this deficiency is that SO(8) is not big enough to contain the product subgroup SU(3)xSU(2)xU(1) of color with weak and electromagnetic flavor. From a purely phenomenological standpoint the A^=9 and 7V=10 supersymmetry algebras have identical y_m=5/2 representations whose lower spin sector is favorable for a grand unification. However, there is so far no indication that a consistent field theory with a "pentatino" and 10 "gravitons" can be formulated.

It is hard to see how to overcome the problem of constructing a unified realistic supergravity theory. Possible lines of approach, with each of its own difficulties, are a) field theories based on several coupled y_m=2 representations; b) superconformai theories which have a natural gauged U(AQ) internal symmetry but which are plagued with ghosts (see below), and c) interpretation of the present set of elementary particles as effective low energy excitations which do not correspond exactly to the fields of supergravity theories. The latter might be evident only near the Planck energy of 10^19 GeV.

We now turn back to the theoretical features of the unified theories and note that the theories were generally constructed in two stages. In the first stage the only coupling constant is the gravitational constant \( tc = (4 \pi G)^{1/2} \times 10^{-18} \) GeV^{-1} and the Lagrangians and transformation rules were found by an iterative procedure in tc. At this stage the SO(7V) internal symmetry is global and there are (l/2)N(N−l) vector fields in the adjoint representation of SO(AQ) (as can be inferred from the particle content given in Table II) which interact non-minimally via field strengths Fji=dAV−dAi. At the next stage of construction SO(7V) is gauged using Yang-Mills minimal coupling with charge e and then determining additional terms necessary to maintain the fermionic gauge invariance. In this way a marriage of the gauge principles of local internal symmetry and local supersymmetry is achieved, but it is a troubled marriage as we will discuss shortly.

The renormalizability properties of the extended supergravity theories are studied (for e=0) by expanding the vierbein V_{αμ} about a flat background geometry and considering the properties of radiative corrections in loop graphs involving virtual gravitons, gravitinos, etc. interacting with strength tc. Since tc carries negative dimension, high powers of virtual momenta occur in Feynman integrals and one cannot hope for traditional renormalizability where divergences are absorbed in redefinition of a finite number of parameters. Instead one hopes for a finite theory in which divergences may be present offshell but cancel in S-matrix elements. Prior to the development of supergravity it was known that such cancellation occurs in one-loop order in pure general relativity (self-coupled gravitons only) but fails when conventional lower spin matter couplings are added.

One-loop finiteness of the unified theories was first demonstrated^20 by arguments which used global supersymmetry to relate 4-point amplitudes with external lower spin to amplitudes with external gravitons for which a generalization of the known proof of finiteness could be applied. These theoretical arguments were confirmed by explicit calculation of the...
Supersymmetry (Including Supergravity)

The divergent part of an amplitude in the $N=2$ theory and, subsequently, by other one-loop calculations. Further arguments showed that the local and non-local divergences at the two-loop level cancel on-shell in simple $N=1$ supergravity. It is still unknown whether this result also holds in pure general relativity.

An approach to renormalizability based on locally supersymmetric counter terms was also initiated at an early stage. A locally supersymmetric counter term is an integral expression of schematic form

$$\int \! d^4x V\{ (R_{...} R) + (\psi_{...} R D\psi) + \cdots \}$$

(15)

involving products of variously contracted curvature tensors $R_{ab}$ and terms involving $\phi$. The integral is invariant under local supersymmetry transformations. When there are $L+1$ factors of $R$ such an integral corresponds by power counting to the possible operator form of a divergence of the sum of all Feynman graphs with $L$ loops in $N=1$ supergravity. It was argued that for $L=1$ and 2 the only possible counter terms vanish when equations of motion are used so that all $S$-matrix elements are finite. However indication was found for an $L=3$ counter term which does not vanish on-shell and is a candidate for a genuine divergence of scattering amplitudes in three-loop order.

There are two more recent developments in the counter term approach. First the $L=3$ counter term mentioned above has been generalized to $N=2$ supergravity, so that unification with internal symmetry does not appear to cure the threatened non-renormalizability. Second, general techniques have been developed for the construction of complete locally supersymmetric counter terms for $N=1$ supergravity (see Part III) which show that there are invariants which correspond to possible divergent scattering amplitudes for any order in perturbation theory beyond $L=3$. At this point it cannot be excluded that the coefficients of the candidate divergent counter terms vanish, but no theoretical reason for such a miracle is apparent.

The difficulties of unified extended supergravity theories with gauged internal symmetry arise from cosmological terms, such as the term $(3/2)Pe/c^2$ which appears in the Lagrangian density of the $SO(2)$ and $SO(3)$ invariant theories with value dictated by the requirement of local supersymmetry. Quantitatively this term cannot be related to a macroscopic cosmological constant because it is more than 100 orders of magnitude larger than the astronomical upper limit. A field theory with cosmological term cannot be quantized in a flat background geometry because Minkowski space is not a solution of the background field equations. In the present case the maximally symmetric background geometry is a de Sitter space of $0(3, 2)$ signature. Since this geometry does not have global Cauchy surfaces, it has been thought that a well-posed initial problem and field quantization are not possible. Recent work indicates that these difficulties may be overcome.

There are even more puzzling features associated with the gauged $SO(2)$ internal symmetry for $N=4$ which are also likely to occur for $N>4$. Because there are spinless fields the cosmological term is no longer constant, but is replaced by a scalar field potential $Vf(A, B)$. The two forms of the $N=4$ theory discussed above which are equivalent for zero gauge charge become inequivalent when the internal symmetry is gauged, and the second form leads to a theory with $SU(2)\times SU(2)$ gauge group. The problem is that the potentials $f(A, B)$ in all of these theories are unbounded from below, and at the naive level this suggests the absence of a stable vacuum state.

There is now a glimmer of hope that the formidable problems associated with the cosmological term can be overcome. This hope arises in the approach to quantum gravity taken by Hawking and several collaborators in which the topological aspects of geometries summed in the path integral play a key role. Even for pure general relativity, the Euclidean action is unbounded from below, and it is necessary to make a contour rotation in function space to define the path integral. Further Hawking has been led to introduce a cosmological term in the microscopic Lagrangian as a Lagrange multiplier. A picture of the gravitational vacuum as a "space-time foam" of virtual black holes emerges in which the geometry is highly curved on the microscopic scale but appears nearly flat on length
scales larger than the Planck length. A microscopic cosmological constant of the same sign and order of magnitude as in supergravity is predicted. These ideas must be developed considerably further before the questions associated with the extended supergravity theories can be settled, but it is important that there is a new avenue of approach involving such challenging and elegant concepts.

**Matter Coupling Theories** involving the \((1, 1/2)\) and \((1/2, 0)\) multiplets of \(N=1\) supersymmetry were constructed quite early. The successful extension of field theories from global to local supersymmetry by coupling to the \((2, 3/2)\) gauge fields was important in demonstrating that local supersymmetry is a true gauge principle and in determining the basic terms of supergravity Lagrangians. However, matter and gravity are not unified in such theories and it is known by explicit calculation in several cases\(^{21}\) that one-loop scattering amplitudes are infinite.

Recent progress in the matter coupling includes the discovery\(^{31}\) of models with a super-Higgs effect in which the gravitino acquires a mass as a consequence of spontaneous breakdown of local supersymmetry. Cosmological terms cancel at the minimum of the scalar field effective potential, so that there is no difficulty in the interpretation of the theory at least at the classical level. Readers should refer to the contribution of J. Scherk to these Proceedings which describes a very general formulation\(^{32}\) of the super-Higgs effect. There have also been recent constructions of matter coupling theories for \(N=2\) supergravity. One theory involves an arbitrary number of massless multiplets\(^{33}\) with \(s^2=1\). Another describes a massive multiplet with \(s^2=\sqrt{2}\) which has a central charge in its algebra.

**Superconformal Theories:** The general conviction that symmetry is powerful is sufficient to motivate the study of field theories invariant under superconformal algebras which are the largest permitted in the framework studied by Haag et al (and which for \(N=1\) is an algebra with 24 generators). Global Poincaré supersymmetric field theories of massless particles are automatically superconformai invariant as is indicated by the example of super Yang-Mills theory given earlier. Superconformal invariance fails for the supergravity theories discussed until now because the general relativity Lagrangian is not invariant under local conformal or Weyl transformations.

The construction of local superconformal theories has been considered by Kaku, Townsend, and van Nieuwenhuizen.\(^{35}\) The \(iV=1\) theory is known in closed form and there are partial results for \(N>1\). The construction involved the consideration of potentials and curvatures for the full superconformal group. Constraints on the curvatures were imposed in order to construct the locally invariant action which in final form involves the spin 2 vierbein and self-conjugate spin 3/2 and spin 1 fields \(p_\mu\) and \(A^\mu\). The full Lagrangian is complicated, but the bilinear kinetic terms are

\[
\mathcal{L} \propto \left( R^2 - \frac{1}{2} R^2 \right) - \frac{1}{2} \left( \partial_\mu A_\nu - \partial_\nu A_\mu \right)^2
- \frac{1}{2} \partial_\mu \left( \Box g_{\mu\nu} - \partial_\mu \partial_\nu \right) - \frac{1}{2} \partial_\mu \partial_\nu \left( \partial_\sigma \partial^\sigma g_{\mu\nu} - \partial_\sigma \partial^\sigma \partial_\sigma g_{\mu\nu} \right) \partial^\mu \partial^\nu g_{\mu\nu} \partial^\nu g_{\mu\nu}.
\]

Note that the gravitational terms are those of Weyl's conformal invariant theory, and the superconformal theory can be regarded as the supersymmetric extension thereof.

The full Lagrangian involves one coupling \(e\) which is the \(U(1)\) gauge coupling constant, and one may hope that all forces can be derived in terms of this single parameter. Two problems make it difficult to fulfill this hope. First, there is no known way to recover the macroscopic predictions of general relativity from the Weyl theory and, second, there are negative metric ghosts due to the higher derivative terms, as has been clarified in a recent study of the linearized theory.\(^{36}\) Progress on these points would open a major new avenue for unification in physics.

§11. **Selected Recent Work in Global Supersymmetry**

**Superconformal Anomalies:** The generators \(S_\alpha\) of superconformal transformations satisfy \([S, S]=\gamma K\) where \(Kp\) is the conformal generator. \(Kp\) is conserved classically in a massless field theory of spin 0, 1/2, and 1 but anomalies occur at the quantum level because a length scale is inevitably introduced to define Feynman amplitudes with loops. One should expect analogous anomalies for \(S_\alpha\) and such anomalies were independently discovered by
several groups, though with somewhat confused initial interpretations.

In the supersymmetric Yang-Mills theory discussed earlier, there is a complete parallel between the dilatation and superconformal anomalies as indicated in the following table:

<table>
<thead>
<tr>
<th>Noether Divergence</th>
<th>Dimensional current</th>
<th>Regularization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dilatations</td>
<td>$D^\mu = x^\mu \partial^\mu \theta^a_n$</td>
<td>$\theta^a_n \sim (n-4)F^2$</td>
</tr>
<tr>
<td>Superconformal</td>
<td>$I^\mu = \gamma^{\mu \nu} x^\nu \theta^a_n$</td>
<td>$\gamma^{\mu \nu} \sigma^{a\beta} \gamma^{\nu \alpha} \theta^a_n \sim (n-4)F^2$</td>
</tr>
</tbody>
</table>

The terms which do not vanish for $n^4$ indicate schematically how an anomaly can arise in a dimensionally regulated calculation of one-loop amplitudes. This is merely suggestive because dimensional regularization is invalid in supersymmetric theories. However, one may calculate the one-loop amplitude for $I^\nu$ using a regulator-independent method which gives

$$\gamma^{\mu \nu} \cdot \vec{J} = -(2\sqrt{g})\beta(g)\sigma \cdot F \gamma$$  \hspace{1cm} (17)

to one-loop order, where the renormalization group function $\beta(g) = -\frac{3g}{16\pi^2}$ appears.

The superconformal anomaly may be compared with the axial current and dilatation anomalies

$$\partial \cdot J^\mu = \frac{\kappa^2}{16\pi^2} C_{\muFF}^\nu$$

$$\theta^a_n = \frac{1}{2g} \beta(g) F^2$$  \hspace{1cm} (18)

which are known to all orders in gauge theories of vectors and spinors independent of supersymmetry.

In a class of supersymmetric theories with classical breakdown of conformal invariance via mass terms, the operators above transform together under supersymmetry transformations. For example

$$\delta (\partial \cdot J^\mu) = (1/3) \varepsilon^{\mu \nu \rho \sigma} \theta^a_n \partial \gamma^{\nu \rho} \cdot \vec{F}$$

$$\delta \theta^a_n = -(i/2) \varepsilon^{\mu \nu} \theta^a_n \cdot \vec{F}$$  \hspace{1cm} (19)

At the one-loop level, the quantum anomalies written above also obey these transformation rules. The $\gamma J^2$ anomaly is not known beyond one-loop order but one can already see a conflict between (17), (18) and (19) because the axial anomaly contains no higher order radiative corrections (the Adler-Bardeen theorem) but $I^\mu$ and therefore $\beta(g)$ do. The resolution of this conflict is an interesting open problem which may involve only technical aspects of the renormalization procedure in supersymmetric gauge theories, but may possibly lead to new physics.

Vanishing Two-Loop $\beta(g)$: The largest global supersymmetric theory is based on the $^A=4$ representation with $A_{\mu \nu} = 1$ and describes Yang-Mills potentials $A^\mu$, Majorana spinors $X^a$, and 3 scalars and pseudoscalars $A^\alpha$ and $B^\alpha$ all in the adjoint representation of the gauge group. The Lagrangian was found by dimensional reduction of the dual fermion model in 10 dimensions and consists of the minimal coupling kinetic terms plus Yukawa and quartic coupling terms all involving the single coupling constant $g$. Earlier results in related theories indicated that $\beta(g)$ vanished in one-loop order for this set of fields, but did not vanish in two-loop order, and the question was reopened in connection with the $N=4$ theory with its high degree of symmetry. A vanishing two-loop contribution to $\beta(g)$ was found! The $N=4$ supersymmetric Yang-Mills theory is the only known field theory with no coupling constant renormalization through two-loop order. If this remarkable property remains true to all orders the theory would be exactly conformally covariant.

Central Charges: The concept of central charges in supersymmetry was first discussed by Haag, Lopuszanski and Sohnius who found a consistent modification of the Poincare supersymmetry algebra in which (1) is replaced by

$$[Q^\mu_{\alpha}, Q^\nu_{\beta}] = \delta^{\mu \nu}_{\alpha \beta} P_{\mu} + i \delta_{\alpha \beta} U^{\mu \nu} + \gamma_{\alpha \beta} V^{\mu \nu}$$  \hspace{1cm} (20)

Here $U$ and $V$ are Hermitian scalar and pseudoscalar central charges which are antisymmetric in indices $\mu, \nu$. By definition central charges commute with all elements of the algebra. Since the charges have dimension 1, they can only occur in field theories with a dimensional parameter such as a mass. The $N=2$ massive multiplet with $^A_{\mu \nu} = 1/2$ was the first example of a field theory with central charge.

Our understanding of the physics of central charges has improved greatly recently due largely to work of Witten and Olive. They considered the $A^\mu_{\alpha \beta}$ supersymmetry algebra where both central charges are proportional to the alternating symbol $\varepsilon^\mu$ and (20) takes
the form
\[ \{Q_i, \bar{Q}_j\} = \delta^{ij} \gamma^\mu P + iz^{ij}(U - i\gamma_5 V) \quad (21) \]
One result of Witten and Olive is a lower bound on the mass \( M \) of particles in a representation of this algebra in terms of central charge eigenvalues. The bound
\[ M \geq \sqrt{\bar{U}^2 + V^2} \quad (22) \]
is easily derived in the rest frame using the positivity properties of the anti-commutator.

Further analysis of (21) shows that when the bound is not saturated, the irreducible representations are 16 dimensional as in the case of \( M^0 \) representations of \( N=2 \) supersymmetry without central charges. When the bound is saturated, the 8x8 matrix on the right side of (19) has 4 vanishing eigenvalues, so that the Clifford algebra is smaller. One then finds massive irreducible representations of dimension 4, as in the case of \( m=0 \) representations without central charges. The phenomenon of reduced size massive representations had also been noted by Sohnius\textsuperscript{45} for \( N=2 \) and by Fayet\textsuperscript{46} for both \( N=2 \) and \( N=4 \).

Witten and Olive study the supersymmetry algebra in the \( N=2 \) super Yang-Mills theory with gauge group SU(2). The fields are the gauge potentials \( A^\mu(x) \), two triplets of Majorana spinors \( \gamma^\mu(\gamma(x) \text{ and } \bar{\gamma}(x) \text{. The Lagrangian}
\[
\mathcal{L} = \frac{1}{2} g' \sum_{\lambda=1}^{2} \tilde{\epsilon}^{abc} \tilde{\epsilon}^{ij} (A_{i}^{a} + i\gamma_{5} B_{i}^{a}) \phi_{ij} - i g^2 \sum_{\lambda=1}^{2} \tilde{\epsilon}^{abc} \tilde{\epsilon}^{ij} (A_{i}^{a} + i\gamma_{5} B_{i}^{a}) \phi_{ij} \quad (23)
\]
consists of covariant kinetic terms and Yukawa and quartic couplings. They calculate the anticommutator \( \{Q, \bar{Q}\} \) and find that the central charges appear as surface terms of the form
\[ U = \int d^3x \partial_\mu (A^\mu F^a_{\mu} + \cdots) \]
\[ V = \int d^3x \partial_\mu (A^\mu \tilde{\epsilon}^{ijk} F^a_{ijk} + \cdots) \quad (24) \]
Spontaneous breakdown of the SU(2) gauge symmetry can occur in this model preserving a U(1) subgroup, which defines an electromagnetism, and preserving supersymmetry. It is also known\textsuperscript{15} that the model contains classical soliton solutions which are magnetic monopoles and dyons and zero energy fermion solutions which are their supersymmetric partners and can be called solitinos. In this situation the central charges become
\[ U = \langle A \rangle E \]
\[ V = \langle B \rangle G \]
where \( \langle A \rangle \) is the vacuum expectation value and \( E \) and \( G \) are electric and magnetic charges. Central charges are therefore related to topological charges!

The final point of Witten and Olive derives from the observation that for all known Higgs mechanism and soliton states the mass formula
\[ M = \langle A \rangle \sqrt{E^2 + G^2} \quad (26) \]
holds through first order in quantum corrections. This corresponds to saturation of the lower bound discussed earlier, and the massive states therefore span reduced size supersymmetry representations. Higher order quantum corrections are unlikely to introduce new particle states. The reduced size multiplets persist and the mass formula must be exact to all orders, which is a remarkable result in any quantum field theory!

There are several other subjects of recent research in global supersymmetry which cannot be discussed in detail because of the page limitations on this report. Among them are interesting work on two dimensional supersymmetric theories\textsuperscript{50} especially the construction of some exact S-matrices\textsuperscript{51} the derivation of a supersymmetric non-linear σ-model\textsuperscript{52} in four dimensions, and an extensive research program\textsuperscript{53} on realistic models for hadron and lepton phenomenology. Many problems have been overcome in constructing phenomenological models and, although complicated, they make characteristic predictions for a new class of particles which carry an additive quantum number \( R \).

§111. Selected Recent Work in Supergravity

Many new results have already been mentioned in Part I, and we discuss a few additional topics here.

Auxiliary Fields and Multiplet Calculus for \( N=1 \) Supergravity: The main results here are systematic procedures to construct locally supersymmetric invariants for simple supergravity. There have been applications to matter coupling theories and to locally supersymmetric counter terms.
The recent development was motivated by the calculation of the commutator of two supersymmetry variations, see eq. (14), in which equation of motion terms appeared. Although supergravity can be perfectly well formulated in terms of the physical fields \( V_a(x) \) and \( \dot{\rho}_a(x) \), the equation of motion terms mean that \( V_a(x) \) and \( \dot{\rho}_a(x) \) are an incomplete multiplet of fields, which do not realize a closed algebra of local supersymmetry, Lorentz and coordinate transformations independent of dynamics. Similar situations occur in global supersymmetry. For example, the commutator algebra of the super-Yang-Mills theory does not close on the physical fields \( A_p(x) \) and \( \dot{\sigma}_a(x) \) but an auxiliary pseudoscalar \( D(x) \) can be introduced to form a complete multiplet.

In supergravity three groups have recently found that a complete multiplet of fields can be formed from \( V_a \), \( \dot{\rho}_a \) and auxiliary axial vector, scalar and pseudoscalar fields called \( A, S \) and \( P \). This set of 6 auxiliaries supersedes a larger set found earlier. The local supersymmetry variations of the new multiplet are

\[
\begin{align*}
\delta V_a &= \kappa \bar{\nu} \gamma_\mu \dot{\nu}_\mu \\
\delta \dot{\nu}_\mu &= -2 \xi^{-1} \gamma_\mu \dot{\nu} + \frac{1}{2} \gamma_\mu (S - \dot{\nu}_3 P) \\
&+ i(A_{\cdot} - \frac{1}{2} \bar{\gamma}_\mu A) \bar{\nu}_\mu \\
\delta \dot{\rho}_a &= -\frac{1}{2} \bar{\nu} \gamma_\mu \bar{\nu} \dot{\mu} + \frac{1}{3} \gamma_a \gamma_\mu \gamma_\nu \phi \phi P \\
&+ (i/2) \bar{\gamma}_{\mu \nu} \gamma_\lambda \phi S + \frac{1}{2} \kappa \bar{\gamma}_a A \cdot \phi \\
\delta S &= \frac{1}{2} \bar{\nu} \gamma_\mu \gamma_\nu \phi P - \frac{1}{2} \kappa \bar{\gamma}_a A \cdot \phi \\
&- (i/2) \bar{\gamma}_{\mu \nu} \gamma_\lambda \phi S + (i/2) \bar{\gamma}_{\mu \nu} \gamma_\lambda \phi A \cdot \phi \\
\delta A_{\cdot} &= \frac{3}{2} \bar{\nu} \gamma_\mu \gamma_\nu \phi P + (i/2) \bar{\gamma}_{\mu \nu} \gamma_\lambda \phi S - \frac{1}{2} \kappa \bar{\gamma}_a A \cdot \phi \phi P \\
&+ (i/2) \bar{\gamma}_{\mu \nu} \gamma_\lambda \phi S + \frac{1}{2} \kappa \bar{\gamma}_a A \cdot \phi \\
&- \frac{1}{2} \kappa \bar{\gamma}_a A \cdot \phi \phi P \\
\end{align*}
\]

(27)

where \( \bar{\omega}^{\mu \nu} = V^{-1} \bar{\omega}^{\mu \rho \sigma} \gamma_\rho \gamma_\sigma \) is the spin 3/2 Euler variation in supergravity. The commutator of two of these transformations again takes the form of (14) with no equation of motion terms and a more complicated local Lorentz parameter involving the auxiliary fields. Thus a non-linear but closed local algebra has been obtained. It is the universal algebra of all \( \mathcal{N}=1 \) supergravity theories encompassing pure supergravity and matter coupling models.

The Lagrangian

\[
\mathcal{L}_{SG}^{\mathcal{N}=1} = -(1/\kappa^2) VR - \frac{1}{2} \kappa \bar{\nu} \gamma_\mu \gamma_\nu A^\mu \\
+ \frac{1}{3} \kappa (A_{\cdot}^2 - S^2 - P^2) \\
\]

(28)

is invariant under (27). From the equations of motion one finds that all auxiliaries vanish, and everything reduces to the form of supergravity discussed in Part I (with new conventions). Matter coupling theories can be reformulated using auxiliary fields with some simplification of structure. The auxiliary field equations relate \( A, S, \) and \( P \) to the matter fields.

Further progress has emerged from the derivation of the "tensor calculus" based on scalar multiplets by Ferrara and van Nieuwenhuizen and based on vector multiplets by Stelle and West. Both are direct generalizations of well-known aspects of global supersymmetry. We can describe only the basic notions of the scalar multiplet calculus. The vector multiplet calculus is entirely parallel, and has advantages in some applications.

A local scalar multiplet is any set of component fields \( A, B, x \times F \times G \) with the following transformation rules

\[
\begin{align*}
\delta A &= \xi A \\
\delta B &= -i \bar{\gamma}_5 \dot{\xi} A \\
\delta \xi &= \bar{D}[A - i \gamma_5 B] + (F + i \gamma_5 G) \xi \\
\delta F &= \bar{D} \left( -\frac{3}{2} \dot{\sigma}(S - i \gamma_5 P) - (i/2) \gamma_5 \gamma_5 A \right) \\
\delta G &= i \gamma_5 \left( \bar{D} + \frac{3}{2} \dot{\sigma}(S - i \gamma_5 P) + (i/2) \gamma_5 \gamma_5 A \right) \\
\end{align*}
\]

(29)

where \( D \) is a supercovariant derivative defined to transform without derivatives of \( e(x) \). For example \( DA - dA - i\bar{\gamma}_5 \dot{\xi} B \). Scalar multiplets can be formed from matter fields, or from any combinations of \( V_{\mu \alpha}, A_{\cdot}, S \) and \( P \), for which the transformation rules (29) can be established.

A calculus of scalar multiplets can be set up in which the product of two multiplets, the derivative of a multiplet, and a local scalar density can be defined. The density formula tells us that, given the components of any scalar multiplet, the integral

\[
I = \int d^4x V \left( F + \frac{1}{2} \kappa \bar{\nu} \gamma_\mu \gamma_\nu A \cdot \phi \phi P + \frac{1}{3} \kappa (A_{\cdot}^2 - S^2 - P^2) \right) \\
\]

(30)

is invariant under local supersymmetry transformations. Therefore the problem of constructing supergravity Lagrangians is transformed into the problem of finding local scalar multiplets and applying (30). This is not
intrinsically easier, but it does permit input from other approaches. For example, scalar multiplets involving matter fields have been found using global supersymmetry results, while other multiplets involving various combinations of the supergravity gauge multiplet $V_{a}$, $A_{a}$, $S$, $P$ have been constructed using results of the superspace approach.\textsuperscript{57}

One such multiplet, called the scalar curvature multiplet, has components

$$\tilde{A} = S \quad \tilde{B} = P$$

$$\tilde{E} = \frac{1}{2} R + \frac{1}{2} \tilde{\alpha} \tilde{\beta} \gamma_{\mu} \tilde{\chi}^\alpha \gamma^\mu (S - i \tilde{\gamma}^5 P)$$

$$+ (i/4) \tilde{\alpha} \tilde{\beta} \gamma^5 (S - i \tilde{\gamma}^5 P) \phi^\alpha \phi^\beta$$

$$\tilde{G} = -V^{-1} \partial_\mu (\bar{V} A^\mu) + \frac{1}{2} \bar{\phi} \gamma^\mu \tilde{\alpha} \bar{\phi}$$

$$- \frac{1}{2} \bar{\phi} \gamma^\mu \gamma_5 \tilde{\alpha} \bar{\phi} + \frac{1}{4} \bar{\phi} (S - i \tilde{\gamma}^5 P) \phi^\alpha \phi^\beta$$

$$+ \frac{1}{4} \bar{\phi} \gamma^5 (S - i \tilde{\gamma}^5 P) \phi^\alpha \phi^\beta$$

(31)

The scalar density formed from this multiplet according to (30) is $\mathcal{S}$ of (28). From similar multiplets containing the Weyl tensor $C_{\mu}^\nu$, one can construct the locally supersymmetric counter terms discussed earlier in connection with the renormalizability question.

The several applications of the multiplet calculus formalism, both in the scalar and vector case, illustrate the power of the formalism. On the other hand, it is clear from (31) that it is not entirely simple. It seems unlikely to me that any simpler procedure will be found because of the close relation of the multiplet calculus to global supersymmetry and superspace.\textsuperscript{57} The formalism is presently limited to simple supergravity and we must hope that extensions to $N>2$ can be found.

New Ghost Couplings: Another curious consequence of the non-closure of the commutator algebra (14) is that the naive generalization of the Faddeev-Popov procedure to supergravity formulated with physical fields $V_{a}$ and $\phi$, is incorrect. The usual gauge fixing term\textsuperscript{31} is $F(\phi) = (\phi \gamma^5 \phi)$, and both fixed flat space and covariant matrices may be used. Naively, one would expect a ghost Lagrangian of the form $\frac{1}{2} \bar{\phi} \gamma^5 \phi^\alpha \gamma^\mu (S - i \tilde{\gamma}^5 P) \phi^\beta$. It is now known that one must add the quartic coupling of the ghost fields $(\phi \gamma^5 \phi^\alpha)(c \gamma^5 \phi)$. This term has now been derived from the viewpoints of the canonical quantization formalism,\textsuperscript{59} BRS identities and unitarity,\textsuperscript{60} and from auxiliary fields.\textsuperscript{61} In fact, the necessity for quartic coupling terms is quite easily seen from the auxiliary field viewpoint. Naive covariant gauge quantization is correct for the Lagrangian (28), if the auxiliary field terms in the transformation $\delta \phi$, in (27) are included in the computation of the Fadeev-Popov determinant. However, it is then clear that quartic ghost couplings will arise when auxiliary fields are eliminated, using the equations of motion of the effective Lagrangian.

The Lagrangian of a Rarita-Schwinger field in a fixed external background metric which satisfies the Einstein equations $\Box \phi = 0$ gives a consistent spin $3/2$ field theory. The covariant gauge condition $F(\phi) = \gamma^5 (c \gamma^5 \phi)$, and weight term $\partial_\mu \gamma^\mu \phi$ are then very natural. Nielsen\textsuperscript{62} has shown that a third spin $1/2$ ghost is then necessary for the unitarity of one-loop amplitudes.

Anomalies: The conformai\textsuperscript{63-65} and axial\textsuperscript{64-67} anomalies have now been calculated for particles of spin $3/2$, and even for arbitrary spin. Tabulations of the conformai anomaly for various supergravity theories have been made. Curiously the anomaly vanishes in the unified SO(3) theory indicating that this theory will have finite one-loop "S-matrix" even in a background geometry with non-trivial Euler characteristic.

The form of the gravitational axial anomaly is

$$\partial_\mu J^\mu = \frac{1}{768 \pi^2} \epsilon_{\mu \nu \rho \sigma} R^{\mu \nu \rho \sigma}$$

and the value $l=1$ obtains for a Majorana spin $1/2$ particle. Several groups using different methods have found the value $A=-2$ for a Majorana gravitino in an external field with covariant gauge fixing. It has also been found that this result is correct for supergravity,\textsuperscript{68} and that the value $Z=4$ found earlier is in error.\textsuperscript{69} Finally, there has been a recent study\textsuperscript{70} of eigenmodes of wave operators up to spin $2$ in a self-dual gravitational instanton background, which have an interesting relation to supersymmetry.

References

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Supersymmetry (Including Supergravity)
Supersymmetry assigns equal masses to bosons and fermions in the same multiplet. Since such a degeneracy is not observed in Nature, it is important to break supersymmetry either spontaneously or explicitly. We opt for spontaneous symmetry breaking since the introduction of non-symmetrical terms would make the theory lose all predictive power. The recent advances in supergravity, namely the discovery of a minimal set of auxiliary fields, and the establishment of a tensor calculus, allow us to construct the most general coupling of supergravity $(2, 3/2)$ to the scalar multiplet $(1/2, 0^+, 0^-)$. We recover as special cases all the previously derived couplings and show that the model depends upon an arbitrary function $G(A, B)$ of the scalar $(A)$ and pseudoscalar $(B)$ fields.

Further we show that for a very large class of such functions, spontaneous symmetry breaking of supersymmetry takes place. The spinor $\gamma$ field of the scalar multiplet plays the role of a Goldstone fermion of supersymmetry (Goldstino). It is then absorbed by the spin $3/2$ gauge field of supergravity (gravitino), just like in the Higgs model, after which banquet the gravitino becomes massive. In addition, this can occur without developing a cosmological constant, due to a cancellation between terms of opposite signs.

When supergravity was first discovered, it was remarked that the algebra of local supersymmetry transformations did not close unless one used the equations of motion of the spin $3/2$ field, a phenomenon which occurs also in flat space supersymmetry when auxiliary (non-propagating) fields are eliminated by use of their equations of motion. This situation was cured by the discovery of a very simple set of 6 auxiliary fields, consisting of an axial vector $A$, a scalar $S$ and a pseudoscalar $P$.

This led in turn to the development of a tensor calculus which generalizes to curved space the results originally obtained in flat space by Wess and Zumino. Tensor calculus applies both to scalar and vector multiplets. Since here we are interested in the coupling of supergravity to a scalar multiplet, we give a very short summary of the tensor calculus for scalar multiplets.

A scalar multiplet is a set of 5 objects $\phi(A, B, x, F, G')$ which have well defined properties under local supersymmetry transformation. For instance $dA = e(x)A$, $3B = -\partial x\phi x$. The fields $A, F'$ $(B, G')$ are scalars (pseudoscalars) and $\gamma$ is a Majorana spinor.

The tensor calculus consists of two basic operations. The first is the multiplication, which to two scalar multiplets $l, 2$, associate their product $2 \otimes 2$. In component form, one has $A = AA, -BB$, and so on for the fields $\gamma$.
other components. This operation is commutative and does not involve any derivatives. It is purely algebraic.

The second operation is called derivation and associates to a multiplet $I$ its derivative $T(I)$. The third component of $T(I)$ contains a supercovariant generalization of the Dirac operator applied on $x \cdots ^i$ the fourth and fifth components supercovariant generalizations of the d’Alembertian operator applied on $A$ and $B$.

Finally an invariant action can be obtained from any scalar multiplet $S$ by the formula:

$$I(\Sigma) = \int d^4x \epsilon \left[ F^I + \frac{1}{2} \epsilon \cdot \gamma \xi \right]$$

$$\quad + \frac{1}{2} \epsilon \gamma a^m_{\nu \rho} (A - i B) \phi_{\nu} + S A + P B$$

In this formalism, the construction of invariant actions under supersymmetry transformations is very straightforward and is analogous to the construction of covariant action in curved space.

The most general interaction between supergravity and a scalar multiplet which involves no more than one derivative on the fermi field and 2 derivatives on the bose field can thus be written as:

$$I(\Sigma) = \sum_{\alpha, \beta, \gamma} a_{\alpha} \sum_{\gamma} T(\Sigma^\gamma) + \sum_{\alpha} b_{\alpha} \sum_{\gamma} \Sigma^\gamma$$

$a_{\alpha}$ can be taken real and symmetric as one can show that $I(\Sigma; T(A) - A \otimes T(Z)) = 0$ and we wish to have a parity conserving model. The action will thus depend a priori on two functions of the spin 0 fields.

$$\phi(z, \bar{z}) = a_{\alpha} \sum_{\gamma} \Sigma^\gamma$$

$$g(z) = \sum_{\alpha} b_{\alpha} \sum_{\gamma} \Sigma^\gamma$$

where $z = A + i B$.

First one computes explicitly $I$ as a function of $e_{\alpha}, \phi_{\alpha}, A, S, P, A, B, \ldots$ of $G$. The fields $A, S, P, \ldots$ of $G$ are auxiliary fields and appear only quadratically in the result. Thus they can be eliminated by solving a set of linear equations and one obtains a reduced action depending only on the physical fields $e_{\alpha}, \phi_{\alpha}, x, A, B$.

The reduced action needs still to be put in a canonical form, as for instance, at that stage, the Einstein scalar curvature $R$ and the Rarita-Schwinger Lagrangian appear multiplied by the function $0(z, \bar{z})$.

Thus one redefines the fields by a Weyl rescaling such that the scalar curvature $R$ appears in the pure Einstein form with the correct normalization. Similarly one redefines the fields $<p_{\alpha}$ and $x \cdots ^i$ that the kinetic terms of the spin 3/2 and spin 1/2 fields are the canonical correctly normalized terms minimally coupled to gravity.

These transformations are compatible with the reality of the vierbein and the Majorana property of the spinor fields under very general conditions, namely:

$$\Lambda(z, \bar{z}) < 0 \quad \text{and} \quad G_{\alpha \beta} \bar{z} < 0$$

where $G_{\alpha \beta} = 3 \ln \left( \frac{\Lambda(z, \bar{z})}{\Lambda(z, \bar{z})} \right) - \frac{1}{2} \ln \left( \frac{\Lambda(z, \bar{z})}{\Lambda(z, \bar{z})} \right)$

Remarkably enough, the final action and transformation laws involve only the function $<p(z, \bar{z})$ rather than $<p$ and $g$ separately. Further the condition $G_{\alpha \beta} < 0$ also implies that the $A, B$ fields kinetic terms have the right sign, $/>$, that these fields have positive metric and are not ghost-like.

The final action is given by:

$$I = I_{\text{sg}}^0 + I_{\text{matter}}^0$$

where the first term is the supergravity action and

$$I_{\text{matter}}^0 = I_{\bar{\nu}}^0 + I_{\text{nt}}^0$$

where the bosonic action is given by:

$$I_{\bar{\nu}}^0 = \int d^4x \left[ G_{x x, x} \bar{\partial}_{\bar{\nu}} \bar{\partial}_{\bar{\nu}} - V(z, \bar{z}) \right]$$

and the potential Kby:

$$V = - \left( \exp - G \right) \left[ 3 + \left( \frac{|G_{x, x}|^2}{G_{x, x}} \right) \right]$$

The fermionic action $I_{\mu}^0$ contains the kinetic term of the $x$ field and terms bilinear in the fermi fields $<p_{\alpha}$ and but does not contain derivatives of the scalar fields:

$$I_{\bar{\nu}}^0 = \int d^4x \left[ - \frac{1}{2} \partial_{\bar{\nu}} \partial_{\nu} - \left( \exp - G \right) \left( \bar{\partial}_{\bar{\nu}} \partial_{\nu} \phi_{\nu} \right) \right.$$

$$\left. - \left( -2G_{x}, x \right)^{-1} \gamma \hat{G}_{x}, \bar{\partial}_{\bar{\nu}} \bar{\partial}_{\bar{\nu}} \right]$$

$$\times \left( \hat{G}_{x, x} \left[ \bar{\gamma} \bar{G}_{x, x} \bar{G}_{x, x} - \hat{G}_{x, x} \right] \right)$$

where by definition if $M = \text{Re} M + i \text{Im} M$.

Finally $I_{\text{nt}}$ contains quartic terms in the fermi fields $<p_{\alpha}$ and $x$ and bilinear of the fermi fields multiplied by derivatives of the scalar fields, for instance of the type $<G_{x, x} - x^2$.

This action is the most general coupling
of supergravity to the scalar multiplet and depends upon one arbitrary function \( G(z, z) \) of two real variables. It includes as special cases all previously derived couplings.\(^{6-11}\)

The transformation laws of the redefined fields can also be computed. For the discussion of the super-Higgs effect, we shall only need the transformation law of \( y \):

\[
\tilde{\partial}x_L = \tilde{\partial}x_R ( -2G, z z ) \hat{\partial} \left( \exp \left( \frac{G}{2} \right) \frac{G, z z}{G, z z / 2} \hat{\partial} \right) \tilde{\partial}x_L + \cdots
\]

where the dots indicate cubic terms in the fermi fields.

Using this very general result we can discuss the super-Higgs effect\(^{12,13}\) in a model independent way. In flat space a necessary condition for spontaneous supersymmetry breaking to occur is that the auxiliary field \( F' \) (resp. \( D \) in the vector multiplet) can pick up a non-zero vacuum expectation value. It then follows that \( d\% \) contains a term \( \frac{1}{6a} = (-\alpha)e^\frac{1}{6a} \) where \( a \) is a constant.

The supersymmetry charge \( Q \) does not annihilate the vacuum, and a zero mass excitation of spin 1/2 (Goldstino) is seen to be present in the theory. In addition, a cosmological constant of fixed sign (+) is induced by the supersymmetry breaking.

In curved space, Deser and Zumino\(^{13}\) used as a model for spontaneous breaking of supersymmetry the coupling to supergravity the non-linear Volkov-Akulov Lagrangian\(^{14}\) which contains only one fermi field \( \psi \) transforming as \( 0 = \chi^X = \frac{1}{\sqrt{2}} e^{i/\sqrt{2} G} \). As \( d\% \) contains a \( (\alpha/6) e \) term, \( % \) is a candidate to represent a Goldstino field. The presence of this term implies a negative cosmological constant; however if one introduces a mass term for the spin 3/2, another cosmological constant of positive sign arises. Since experimentally the cosmological constant is very small, one imposes that the net cosmological constant vanishes. One then finds\(^{10}\) that \( m = k/6a \) and is very small. Since \( 8\% \) contains a constant term proportional to \( e(x) \) and we have one spinor gauge degree of freedom, \( x \) can be gauged away completely and is absorbed by the gravitino which becomes massive. A massive gravitino indeed has 4 helicity states \( \pm 3/2, \pm 1/2 \).

In our general model we can see the same phenomenon occurring for a large class of functions \( G(z, z) \).

The potential \( V(z, z) \) reaches its minimum for a certain value \( z \), such that:

\[
V, z |_x = V, z |_z = 0
\]

If we require that the final theory does not violate parity \( z < (x) \) must be real.

Further we can impose that the absolute minimum of \( V \) is reached for \( V(z, z) = 0 \) which implies the absence of a cosmological constant, and that \( V > 0 \) everywhere. It is indeed possible for \( V \) to be non negative as it contains two terms of opposite sign (remember that \( G, z < 0 \) is a necessary condition for the \( A, B \) fields not to represent ghost particles of negative metric).

Finally the condition for spontaneous breaking of supersymmetry is that at the minimum, \( \hat{\partial}x \) contains a constant term times \( e(x) \). Looking at \( 8\% \) we see that the requirement is that \( 1/a = (\exp -G/2) \{ G, G X \} U = \gamma \ast 0. \)

The condition that \( V(z, z) = 0 \) are compatible as 0 vanishes precisely if

\[
\left[ G, \chi \right] \right|_x = -3G, 2z
\]

in which case \( 1/a = 6 \exp (G) \) looking at \( 8\% \) above we see that the requirement is that \( 1/a = (\exp -G/2) \{ G, G X \} U = \gamma \ast 0. \)

The condition that \( V(z, z) = 0 \) are compatible as 0 vanishes precisely if

\[
I = \int d^4 x \left[ -e^\frac{1}{2} R - \frac{1}{2} e^{i\mu \nu} \gamma^\mu \gamma^\nu D \psi \hat{\partial} \right]
\]

In general \( A \) and \( B \) acquire different masses. In the simple case where \( G = -1/2 \) (canonical
kinetic term for $A, B$) one can establish the general mass formula:

$$m_A^2 + m_B^2 = 4m_\phi^2$$

which is independent upon the remaining arbitrary function of one variable $g(z)$. Particular examples of functions exhibiting the super-Higgs effect are easily found.

Our investigation shows thus that the super Higgs effect can take place in a large class of models, and that realistic models where supersymmetry is spontaneously broken even in curved space can be constructed, without having huge cosmological constants. It would be interesting to extend these result to the $O(N)$ supergravity theories which once they are gauged, create their own potential for $N>4$ since they contain spin 0 fields. Unfortunately the potentials found in the $O(4)$ and $SU(4)$ theories do not lead to spontaneous symmetry breaking, being unbounded below, which is a disappointing result. However, the discovery of auxiliary fields for $O(N)$ supergravity theories may reveal more flexibility in this construction than originally thought, as similarly it was believed that the coupling of supergravity to the scalar multiplet was unique.

References


C6  Ghost-Fields, BRS and Extended Supergravity as Applications of Gauge Geometry

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We review the methods of Gauge Geometry, including the sequence Lie group/Fiber Bundle/Gauge Manifold. We derive the Ghost Fields and BRS transformations geometrically (and classically), showing that the Ghost fields are nothing but the vertical components of the gauge-potential one-forms. We state the theorem of Spontaneous Fibration of a Weakly-Reducible and Symmetric Group Manifold and apply the method to the derivation of Extended Supergravity (Chiral or Parity-conserving) for any TV.

§1. Geometrical Introduction

The geometrical treatment of gauge theories has provided numerous insights and precise results at both the classical and quantum levels. In this note, we present two new applications, one for any Internal Gauge theory, and the other in the realm of the Non-internal ones. In the first, we derive the existence of the Ghost-fields, needed for Unitarity and Renormalization, as a classical geometrical result on a Principal Bundle. The BRS transformations come cut naturally, and one may hope that this approach will produce further insights in the study of the renormalization of Gravity and other theories. Our second example consists in a direct derivation of all allowed “geometrical” theories of Extended Supergravity, using a theorem of Spontaneous Fibration, in the method of gauging on a Group Manifold which we recently introduced.

It is instructive to compare a Lie group G, a Principal Bundle P with fiber F=G, and the Group Manifold G. A Lie group G is a space in which finite motions are specified, up to global topological considerations, by its Lie algebra A. The connection forms over G are the Cartan Left-invariant forms w. They provide a rigid triangulation and a vanishing curvature

\[ d\omega - \frac{1}{2} [\omega, \omega] = 0. \] (0.1)

The Left-invariant algebra \( D \) is an Orthonormal basis to the \( w \).

\[ \omega^i(D^i_j) = \delta^i_j. \] (1.2)

\[ [D^i_j, D^i_k] = D^i_{[j,k]}. \] (1.3)

In the Principal Bundle \( P(P, \xi, TC, G, -) \), with a horizontal base space \( B \) in addition to \( G \), and a projection \( TZ \) onto the base space,

\[ \forall \rho \in P, g \in G: \pi(p \cdot g) = \pi(g) \] (1.4)

the transformations of G are vertical by (4). The connection forms \( m \) are only vertical, and map tangent vectors onto \( A \). The curvature two-forms \( Q \) are purely horizontal:

\[ \Omega = d\omega - \frac{1}{2} [\omega, \omega]. \] (1.5)

\[ \forall y \in A, D_y \cdot \Omega = 0. \] (1.6)

\( P \) is specified by the knowledge of the fields over a section \( I \).

The Group Manifold G is punctually identical to G. However, the connection one-forms \( p \) are locally independent. In G any direction is a gauge direction (as in \( G = F_cP \)), but all directions are also curved (as in \( J5cP \)). The Killing Lie algebra is not isomorphic to \( A \).

\[ [\tilde{D}_y, \tilde{D}_y'] = \tilde{D}_{[y, y']} \cdot (\tilde{D}_y - R)\tilde{D}_y' \] (1.7)

\[ \rho'(\tilde{D}_y) = \tilde{\delta}_y. \] (1.8)

The curvature

\[ R = d\rho - \frac{1}{2} [\rho, \rho] \] (1.9)

is unrestricted. However, the Jacobi identity does give rise to Bianchi identities

\[ DR = 0. \] (1.10)

Gauging corresponds to a local map in the group’s right action, with

\[ xy = z, x \rightarrow y, x, y, z \in G, \]

\[ \tilde{\delta}_\rho \rightarrow \tilde{D}_\lambda, \tilde{\delta}R = [\tilde{\lambda}, \tilde{\rho}] = \tilde{D}(\tilde{\delta}_\rho). \] (1.12)
§2. Ghosts and BRS

Take first an internal gauge, described by \( P \). We may decompose \( \omega \) with respect to a section \( I \) into the sum,

\[
\omega = \phi \cdot \chi, \quad D_\chi \cdot \phi = 0, \quad \partial_\mu \cdot \chi = 0 \tag{2.1}
\]

\(<\phi>\) is thus the (horizontal) Yang-Mills potential. \( X \) is the Feynman-De Witt- Faddeev-Popov ghost. It anticommutes just because it is a one-form. A differential \( df \) can be expanded as

\[
df = \partial_\mu f dx^\mu + \partial_i f dy^i = \Phi + s \tag{2.2}
\]

Equations (5)-(6) can be rewritten as

\[
\begin{align*}
\mathbf{s}_\chi - 1/2 [\chi, \chi] &= 0 \\
\mathbf{s}_\phi - B_\chi &= 0 \\
\Omega &= \mathbf{b}_\phi - 1/2 [\phi, \phi] \tag{2.3}
\end{align*}
\]

with

\[
B_\chi = b_\chi + [\phi, \chi] \tag{2.6}
\]

These are the BRS equations. The application to path integrals and renormalization will be described elsewhere.

§3. Gauging a Non-Internal Group \( G \)

3.1. Quadratic (simple \( G \)) and linear (\( G \) contracted) Lagrangians

For a simple \( G \), use the quadratic Lagrangian introduced in ref. 8),

\[
L = R^a \wedge \tau_{ab} R^b, \quad a, b \in F, \quad F \subseteq A \tag{3.1}
\]

Let \( G \) be Weakly-reducible and symmetric decomposition of its algebra \( A=F\mathbb{Q} H; [F, F] \subseteq F \). The F-metric \( c_a \) is the most general tensor satisfying, \( [F, H]aH; [H, H]dF \)

\[
D^F = 0, \quad \tau_{ab} = \tau_{ac} = 0, \quad h, \ i, j \in H \tag{3.2}
\]

The equations of motion are,

\[
\begin{align*}
E_a &:= \tau_{ab} D^F R^b - \tau_{ab} f_{k}^{b} g_{k}^{a} = 0 \\
E_h &:= \tau_{h} g_{k}^{a} (\partial^h \chi) \wedge R^h = 0 \\
D^F L &:= 0 \tag{3.3}
\end{align*}
\]

where \( \Rightarrow \) implies application of the equations of motion ("pseudo-invariance").

Under contraction over \( F, \ H \rightarrow \lambda H, \ \zeta_a \) is a "pseudo-curvature":

\[
L_{\zeta a} = R^a \wedge \zeta_a, \quad \zeta_a = \tau_{ab} (\partial g_{k}^{a} / \partial \lambda) \mid_{\lambda=0} (\partial^h \chi) \wedge R^h, \quad \zeta_{\lambda h} = 0 \tag{3.5}
\]

where the latter condition guarantees that the "flat" Lie group \( G \) will satisfy the equations of motion, or in the language of General Relativity, that the vacuum will coincide with the "tangent manifold." The equations of motion will be

\[
E_a := D_{\zeta a} = 0 \quad E_h := R^a (\partial_{\zeta a} / \partial \rho^h) = 0 \tag{3.7}
\]

Eq. (3.4) plays the role of BRS in establishing the Quantum version.

3.2. The spontaneous fibration theorem

(The proof is given in ref. 4)

Given

1. A semi-simple Group or Supergroup
2. \( G \) is Weakly reducible and Symmetric (WRS), i.e., there exists a decomposition of its Lie algebra

\[
A = F \oplus H, \quad [F, F] \subseteq F, \quad [F, H] \subseteq H, \quad [H, H] \subseteq F \tag{3.8}
\]

3. The complete \( F \) is generated by \( [H, //] \), and the dimension of \( F \)

\[
d(F) \geq (d(H) - 1)^2 / 3, d(H) \quad \text{the dimension of } \ H.
\]

4. The Lagrangian is \( F \)-gauge invariant but not \( v_4 \)-gauge invariant, Then, \( G \) undergoes Spontaneous Fibration with respect to \( F \), i.e., up to global topological considerations,

(1) the \( F \) variables factorize, in the sense of ref. 6.

(2) all torsions (curvatures in the \( H \) direction) vanish,

\[
R^i \wedge 0 \tag{3.9}
\]

(3) all curvatures become horizontal, namely in the decomposition over a cobasis

\[
R_i = \frac{1}{2} R^3_{ij} \rho^j \wedge \rho^i = R^3_{ij} + R_{ij} + R_i \tag{3.10}
\]

\[
R^h_{(F)} = R^h_{(F')} = 0 \tag{3.11}
\]

where \( (\cdot) \) denotes \( 2 \), \( C \in F, \ (\cdot) \) stands for \( B, C \in H \) and \( (\cdot) \) for the mixed case.

§4. Extended Supergravity

4.1. \( OSp(N/4) \)

Take \( G = OSp (N/4) \), the Supergroup preserving a metric \( ^0 (\wedge 3 \wedge 2 \wedge 0 \cdot 0 \cdot 0) \cdot \). where \( v_i \) and \( a_i \) stand for the identity in \( N \) and 2 dimensions respectively.

We use \( a, \beta, \cdots \) for space-time directions; \( a, /3, \cdots \) and \( d, /3, \cdots \) for spinors, \( ij, \cdots \) for \( Q(A0) \) TV-vectors.
4.2. Extended left-Chiral supergravity

Take \( F_i : (J_{ijk}, I_{i}, S_{n}), H_i(S_{n}, T) \). Using (3.2) we find,\(^4\)

\[
L_L = \varepsilon_{abcd} R^{[ed]} \otimes R^{[cd]} + i \varepsilon (R^{[a]} \otimes R^{[b]} - R^{[b]} \otimes R^{[a]})
\]

\[
- R^{\beta i} \otimes R^{\beta j} + R^{[i]} \otimes R^{[j]} \quad (4.1)
\]

Contract \( H_i \rightarrow \lambda H_i \)

\[
L_f (\tilde{\lambda}^2) = \varepsilon_{abcd} \theta^{a} \otimes \theta^{b} \otimes \tilde{R}^{[ed]}
\]

\[
+ i \rho^{a} \otimes \rho^{b} \otimes \tilde{R}^{[ab]} + i \sigma_{a \beta}^{\mu} \theta^{a} \otimes \rho_{\mu}^{\beta} \otimes \tilde{R}^{[i][j]}
\]

\[
+ i \tilde{R}^{[i][j]} \otimes \rho_{ij}^{\alpha} \otimes \rho^{\alpha} \quad (4.2)
\]

where the dash stands for a restriction of (1.9) to \( F \). Both theories violate parity even in the gravitational sector. \( L \) vanishes for the flat \( G=G \) and \( L (\tilde{A}) \) obeys (3.6).

Spontaneous fibration guarantees that

\[
R_{a} = R_{a}^{\lambda} = 0
\]

\[
R_{\mu}^{[ed]} = R_{\mu}^{[i][j]} = 0
\]

\[
R_{\mu}^{[i][j]} = R_{\mu}^{[i][j]} = 0
\]

(4.3)

To recover supersymmetry we still have to contract inside \( F \). We contract over \( F \), with \( H_i = H_i^{(S)} \). Taking \( H_i + juH \), we find (\( R \) denotes a restriction of (1.9) to \( F.nF \))

\[
L_L (\tilde{\lambda}^2 \mu^2) = \varepsilon_{abcd} \theta^{a} \otimes \theta^{b} \otimes \tilde{R}^{[ed]}
\]

\[
+ i \rho^{a} \otimes \rho^{b} \otimes \tilde{R}^{[ab]} + i \sigma_{a \beta}^{\mu} \theta^{a} \otimes \rho_{\mu}^{\beta} \otimes \tilde{R}^{[i][j]}
\]

\[
+ i \tilde{R}^{[i][j]} \otimes \rho_{ij}^{\alpha} \otimes \rho^{\alpha} \quad (4.2)
\]

The cosmological term violates (3.6) and in fact the Lagrangian is not of the Gauge type (3.1) or (3.5). The \( E \) and equations will have source terms which do not vanish for the flat group, i.e., the vacuum does not correspond to the tangent manifold.

There is no \( \nu=8 \) limit in this formalism which deals with the gauge fields in the adjoint representation (the usual "field content" are holonomic composite constructs of the gauge fields).

Note that (4.3) does not contain either \( R^{\cdots} \) or \( p^{\cdots} \), i.e., the internal symmetry is not locally gauged.

4.3. Parity conserving extended supergravity

To recover Parity conservation, construct first Right-Chiral Extended Supergravity by taking a decomposition \( F_i : (J_{ijk}^{(e)}, I_{i}^{(e)}, S), H_i(S, T) \). Construct \( L \) and contract \( H_i \rightarrow H_{i}^{(S)} \) and find \( L_n (\Lambda H) \). Add to (4.16) and find,

\[
L = \frac{1}{2} L_L + \frac{1}{2} L_R = \varepsilon_{abcd} \theta^{a} \otimes \theta^{b} \otimes \tilde{R}^{[ed]}
\]

\[
+ (i/2) \sigma_{a \beta}^{\mu} \theta^{a} \otimes \tilde{R}^{[i][j]} \otimes \rho_{ij}^{\alpha} \otimes \rho^{\alpha} \quad (4.4)
\]

This Lagrangian appears to be of the type (3.5). However, the action of the internal symmetry in calculating for instance makes a "cosmological" term reappear.

The group is not a WRS (since \([H,H](\tilde{H})\)) and the equations of motion are of the type studied in refs. 5, 6 and not as in equation (3.7). Note that for \( N=1 \), the term vanishes identically and the vacuum fits \( G=G \).

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The Superspace Method for Supergravity

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Superspace was introduced by Salam and Strathdee as a technique for the study of representations of the supersymmetry algebra in terms of supermultiplets of fields. The points of superspace are parametrized by coordinates \( z^a = (x^a, \theta^a) \) where \( x^a \) are the commuting space-time coordinates while \( \theta^a \) are anticommuting variables (which are Majorana spinors). Supersymmetry transformations can be realized as rigid motions in superspace

\[
\begin{align}
\delta x^a &= -i \xi^a \theta \\
\delta \theta^a &= \zeta^a,
\end{align}
\]

where \( \xi^a \) are the infinitesimal anticommuting spinorial (Majorana) parameters. The commutator of two transformations (1) of parameters \( \xi \) and \( \eta \)

\[
[\delta_{\xi}, \delta_{\eta}] x^a = -2i \xi_\beta \eta^\beta \zeta^a,
\]

\[
[\delta_{\xi}, \delta_{\eta}] \theta = 0
\]

is a translation of parameter \(-i \xi^a \eta^a\) as required by the supersymmetry algebra. A superfield \( V(x, \theta) \) is a function in superspace. Since there are four components \( \theta^a \) anticommuting, in particular the square of each one is zero, a power series in \( \theta^a \) is at most a polynomial of fourth degree. Therefore a superfield is equivalent to a finite collection of ordinary fields (a supermultiplet), carrying various spinorial indices and satisfying corresponding statistics. When the superfield transforms like a scalar

\[
(\partial_a - \partial_\theta \partial^a) \partial V(z^a(\partial_\theta \partial^a) - i \xi^a \partial_\theta \partial_a) V
\]

the coefficients of its expansion in \( \theta \) is at most a polynomial of fourth degree. Therefore a superfield is equivalent to a finite collection of ordinary fields (a supermultiplet), carrying various spinorial indices and satisfying corresponding statistics. When the superfield transforms like a scalar

\[
D_a - \partial_\theta \partial^a + i \xi^a \partial_\theta \partial_a, \quad D_a = \partial_a
\]

commute with the infinitesimal operator in (3) and can be used to impose supersymmetric constraints on superfields. They satisfy

\[
\{D_a, \tilde{D}_b\} = -2i \xi^b \delta_a \tilde{D}_b,
\]

showing that superspace has torsion, even though its curvature vanishes. This "flat" superspace has been extensively used in the study of supersymmetric field theories in Minkowski space. Just as Einstein's theory is obtained when one generalizes from flat Minkowski space to curved space-time, one expects supergravity (the supersymmetric theory of gravitation) to arise from the differential geometry of curved superspace. Indeed this superfield approach to supergravity preceded that based on ordinary fields.

The first work on curved superspace was that of Arnowitt and Nath, who developed the differential geometry of a super-Riemannian superspace. However, the flat superspace of rigid supersymmetry does not fit naturally as a special case of a super-Riemannian geometry. For this reason a different geometry of superspace was introduced by Wess and the author and by Akulov, Volkov and Soroka. In this geometry the tangent space group at each point of superspace is not taken to be the graded Lorentz group (orthosimplectic \((4, 4))\), but just the ordinary Lorentz group. Furthermore the superspace is allowed to have torsion. More recently this geometry has been adopted by other authors. In this lecture we shall follow our own work, as developed in refs. Ours is a second order formulation, in that we impose on the super-torsion certain conditions ("constraints"), which are sufficient to express the superconnexion in terms of the supervielbein. These conditions also restrict to a certain extent the supervielbein itself. The dynamics is given in terms of a superspace action which must be varied keeping the constraints on the torsion satisfied. Our formulation is exactly equivalent to the recently developed formalism for supergravity with a minimal set of auxiliary constraints. It would be preferable to have a first order action in superspace, from which both constraints and equations of motion follow. A possible choice for a first order action was suggested in ref. 8 and is also discussed in the lecture by J. H. Schwarz in these Proceedings. However, dimensional arguments indicate that \( \delta \) integration of that superspace action does not reproduce the correct supergravity action in coordinate space.
fields. Indeed this auxiliary field formalism can be derived from our geometrical formulation.

A non-geometric description of supergravity in superspace is favored by Ogievesky and Sokatchev and by Warren Siegel. The supergravity fields are contained in a superfield $U(x, \theta)$ endowed with a vector index $m$. The theory, based directly on this superfield, has a certain appeal because it is free of constraints. However, the lack of a complete geometrical structure in such a theory makes the construction of invariants a difficult task. On the other hand, from our point of view, the superfield $U^m$ emerges in solving the constraints on the torsion. The supervielbein and the superconnection are expressed in terms of it and we can use the powerful techniques of differential geometry for the construction of invariants.

The geometric superspace approach gives a complete and satisfactory method for the study of simple supergravity and its couplings to supersymmetric matter (both for the vector-spinor and for the spinor-scalar-pseudoscalar multiplets). All possible invariants of given dimension are easily constructed and their knowledge can be used to discuss the divergences which arise in perturbation theory.

The situation is quite different in the case of extended supergravity. Clearly one must use here a larger superspace in which the fermionic coordinates carry, in addition to the spinorial index, an internal symmetry index, and the tangent space group should be the product of the Lorentz group with the internal symmetry $O(A \cdot 0)$. The difficulty consists in finding the correct separation of the equations into kinematical constraints (which are part of the specification of the geometry) and equations of motion (which should follow from the variation of an action, subject to the constraints). The results obtained so far for $A'=3$ extended supergravity in superspace do not make this separation. These difficulties in the superspace method correspond to our present ignorance as to the correct auxiliary fields for extended supergravity.

**Affine superspace**

The points of superspace are parametrized by coordinates $z^m = (x^i, \theta^a)$. Latin letters denote vectorial (bosonic), Greek letters spinorial (fermionic) indices. The supervielbein matrix $E^i_m(z)$, where $A=(a, a')$, and its inverse $E_m^i(z)$, can be used to transform world tensors into tangent space tensors and vice versa (early alphabet letters denote tangent space indices). Covariant derivatives involve the superconnection $\Theta^i_{ABm}$, as in

$$\sum_{AB} \partial \Phi_{ABm} = \Phi_{ABm}^i \Theta^i_{ABm}.$$  \hspace{1cm} (6)

Covariant derivatives with tangent space indices, $\partial = \partial_{\mathcal{F}}$, satisfy the graded commutation relations

$$[\partial_{\mathcal{F}}, \partial_{\mathcal{G}}] = -R_{AB} + T_{AB} \partial_{\mathcal{F}} \partial_{\mathcal{G}}.$$ \hspace{1cm} (7)

where $T_{AB}^m$ are the supertorsion coefficients and $R_{a}$ is the Lie algebra valued super-curvature operator. The bracket in eq. (7) is an anticommutator if both indices $A$ and $B$ are spinorial, a commutator otherwise. Torsion and curvature satisfy the Bianchi identities

$$\sum_{ABC} \left( \partial_{\mathcal{F}} T_{ABC}^m - \partial_{\mathcal{G}} T_{ABC}^m - R_{ABC} \partial_{\mathcal{F}} \partial_{\mathcal{G}} \right) = 0$$ \hspace{1cm} (8)

and

$$\sum_{ABC} \left( \partial_{\mathcal{G}} R_{ABC}^m + T_{ABC} \partial_{\mathcal{F}} \partial_{\mathcal{G}} \right) = 0.$$ \hspace{1cm} (9)

The cyclic sum is taken with appropriate signs: a permutation of two vectorial or one vectorial and one spinorial index gives rise to a change in sign, while for two spinorial indices there is no change in sign.

**Specification of the geometry (kinematics)**

We now restrict the tangent space group from the general graded linear group to the $(x$ and $\theta$ dependent) Lorentz group. Since the curvature operator belongs to the algebra of the tangent space group, this implies restrictions on the curvature coefficients ($\sum_{ab} = \frac{1}{4} \left[ \varepsilon_{ab}, \varepsilon_{cd} \right]$)

$$R_{\mathcal{F}, \mathcal{G}} = \frac{1}{2} R_{\mathcal{F}, \mathcal{G}}$$ \hspace{1cm} (10)

(similarly for the superconnection). We also impose the constraints on the supertorsion

$$T_{\alpha a} = -2i(\partial_{\mathcal{F}} \partial_{\mathcal{G}}), T_{\alpha a} = 0,$$

$$T_{\alpha a} = 0,$$ \hspace{1cm} (11)

but leave $T_{\alpha a}$ and $T_{\alpha a}$ undetermined. Using the Bianchi identities (8) and (9) one can show...
that (10) and (11) imply that all components of the supercurvature and \( R \) and their conjugates. Here we have used two-component spinor notation, \( G, p \) is\footnote{ This notation is used for convenience.} hermitean, \( W \) totally symmetric. For instance
\[
R_{\alpha\beta\gamma} = -4(\varepsilon_{\alpha\beta\gamma} - \varepsilon_{\beta\gamma\alpha})R^\alpha, \\
T_{\alpha\beta\gamma} = T_{\alpha\beta\gamma} = (i/4)(\varepsilon_{\alpha\beta\gamma}G_{\beta\gamma} - 3\varepsilon_{\alpha\beta\gamma}G_{\gamma\beta} - \varepsilon_{\alpha\beta\gamma}G_{\beta\gamma}).
\]

The remaining content of the Bianchi identities is expressed by the differential relations
\[ \partial_\alpha W_{\alpha\beta\gamma} = \partial_\beta G_{\alpha\beta\gamma} + \partial_\gamma G_{\beta\alpha\gamma} - \partial_\gamma R^\alpha, \]
\[ \partial_\alpha W_{\alpha\beta\gamma} = 0, \partial^\alpha R = 0, \partial_\gamma \equiv (\sigma^\alpha)_{\gamma\alpha}. \]

The superfield \( G \) has a simple physical meaning, which can be exhibited by considering the coefficients of its expansion in \( \sigma \). For instance, at the 00 level one finds a tensor which contains the Einstein tensor, at the 000 level a spinor which is the Rarita-Schwinger operator. Similarly, \( R \) contains the contracted (scalar) curvature tensor, at the 00 level. \( W \) contains the Weyl conformal spinor, at the \( \delta \) level; for \( \delta = 0 \), it contains the Rarita-Schwinger field strength. Observe that some components of \( R \) are obtained from those of \( G \) by tracing over certain indices: in superspace this is expressed by the differential identities (13) in \( \delta \).

Finally we observe that the constraints (11) on the torsion imply
\[ \hat{\omega} dx d\theta E^\alpha \gamma = 0, \]
for any vector \( \gamma \), where
\[ E = \text{det} E^\alpha \gamma \]
is the graded determinant of the supervielbein as defined in the first of ref. 6. This formula permits integration by parts with covariant derivatives carrying tangent space indices.

Conformal transformations

The correct conformal transformations in superspace must preserve the constraints (11). These transformation involve a covariantly chiral infinitesimal parameter function
\[ \sigma^\alpha \equiv \sum = 0, \]
and are (Howe and Tucker)
\[ \begin{aligned}
\partial E^\alpha \gamma &= (\sum - \sum^* E^\gamma \alpha) \\
\partial E^\alpha \gamma &= (2\sum - \sum) E^\gamma \alpha \\
\partial E^\gamma \alpha &= -(i/2) E^\alpha \gamma (\sigma^\alpha)_{\gamma\beta} \sum^* \\
\partial E^\gamma \alpha &= (i/2) E^\alpha \gamma (\sigma^\alpha)_{\gamma\beta} \sum \\
\delta \Omega_{\alpha\beta\gamma} &= E^\gamma \alpha \delta \sum^* E^\gamma \beta \delta \sum + (\sigma^\alpha)_{\alpha\beta} E^\alpha \gamma (\sum + \sum^*).
\end{aligned} \]

For the determinant of the supervielbein, (17) imply
\[ \hat{\omega} dE = 2(\sum + \sum^*)E, \]
while curvature and torsion behave as given by
\[ \begin{aligned}
\partial R &= -(2\sum^* - 4\sum) R - \frac{1}{2} \sigma^\alpha \partial^\alpha R^\gamma \\
\partial G_{\alpha\beta\gamma} &= -(\sum + \sum^*) G_{\alpha\beta\gamma} + i \partial^\alpha (\sigma^\alpha)_{\gamma\beta} \sum \\
\partial W_{\alpha\beta\gamma} &= -3\sum W_{\alpha\beta\gamma}.
\end{aligned} \]

Since the constraints are invariant under (17) and (18), we can say that the geometry (kinematics) is invariant under conformal transformations. The dynamics may or may not be, depending on the choice of Lagrangian. For instance, the action of supergravity (21), given below, is only invariant under general coordinate transformations in superspace and under local Lorentz transformations, but not under conformal transformation. The action of conformal supergravity (28), instead, is also conformally invariant.

Dynamics

The supergravity equations of motion are derived from the action
\[ I = \int dx d\theta \text{det} E^\gamma \gamma. \]

Varying (21) subject to the constraints on gives the equations
\[ G_{\alpha\beta} = 0, R = 0. \]

Because of the identities (13) they give
\[ \partial^\alpha W_{\alpha\beta\gamma} = 0 \]
and this, in turn, implies
\[ \sigma_{\gamma\beta} W_{\alpha\beta\gamma} = 0. \]

On the supergravity mass shell all components of the supertorsion and of the supercurvature change, hence (22) imply
\[ \sigma_{\gamma\beta} W_{\alpha\beta\gamma} = 0. \]

The supergravity equations of motion are derived from the action
\[ I = \int dx d\theta \text{det} E^\gamma \gamma. \]
are expressed in terms of the symmetric, chiral spinor superfield $W^a$, which now also satisfies (23), and of its derivatives (and, of course, of the complex conjugate quantities). It is therefore very easy to list all invariants of given dimension and to discuss questions of renormalizability.

**Chiral superfields as Lagrangians**

In flat superspace any chiral superfield $0$ can be written in the form $0 = \mathcal{D} \Phi$. One can use $0$ as a Lagrangian by integrating only over $Q$ (and $x$) but not over $\mathcal{D}$. Alternatively, one can integrate $U$ over all four $0$ components (and $x$) with the same result. The first procedure has no analogue in curved superspace in a general gauge, the second does. One can show that any chiral superfield $0$

$$\mathcal{D} \Phi = 0$$

(25)

can be written in the form

$$\Phi = (\mathcal{D} \mathcal{D}^a - 8R)U,$$

(26)

where the new term comes from the commutation relations among the covariant derivatives. The action obtained from $U$ can be transformed as follows

$$\int EU = -\frac{1}{8} \int \frac{E}{R} (\Phi - \mathcal{D} \mathcal{D}^a U) = -\frac{1}{8} \int \frac{E}{R} \Phi,$$

(27)

where the last equality follows by partial integration because $R$ is chiral. We see that, in order to construct a Lagrangian density from a chiral superfield, we must multiply it by $-E/8R$, not by $E$. As an example, consider the chiral superfield $W^a W^a$. The corresponding action, proportional to

$$\int \frac{E}{R} W^a W^a$$

(28)

is invariant under the conformal transformations (19) and (20). It is the superspace form of the action for conformal supergravity.

**Coupling to vector-spinor matter**

In analogy with flat superspace, this matter supermultiplet is described by a real scalar superfield $V$, and a supersymmetric gauge transformation by

$$V \rightarrow V + i A - i A^*, \mathcal{D} \Phi = 0.$$

(29)

The gauge invariant superfield which contains the electromagnetic field is defined as

$$W_a = (\mathcal{D}_a \mathcal{D}^a - 8R)U.$$

(30)

It satisfies identically

$$\mathcal{D}_a W^a = 0$$

(31)

and

$$\mathcal{D}^a W_a = \mathcal{D}^a W^a$$

(32)

which is the supersymmetric form of the electromagnetic Bianchi identities. We can take the action, up to a proportionality constant.

(observe that we use the density $E/8R$ appropriate to a chiral Lagrangian). The corresponding equation of motion is

$$\mathcal{D}^a W_a + \mathcal{D}^a W^a = 0$$

(34)

which, combined with (32), gives

$$\mathcal{D}^a W_a + \mathcal{D}^a W^a = 0.$$

(35)

The action (33) is invariant under the gauge transformation (29). It is also invariant under conformal transformation provided one assumes the conformal property

$$\delta V = 0,$$

(36)

which implies

$$\delta W_a = -3 \sum W_a.$$

To see it, combine (37) with (19) and (20) and integrate by parts.

**Coupling to spinor-scalar-pseudoscalar matter**

Let $0$ be a chiral superfield

$$\mathcal{D}_a \Phi = 0, \mathcal{D}_a \Phi^* = 0.$$

(38)

One can take the action

$$\int \left( E \Phi \Phi^* + E \left[ \frac{F(\Phi)}{R} + \frac{F(\Phi^*)}{R^*} \right] \right),$$

(39)

where $F(0)$ is a (real) function of $0$ (potential). In order to derive the equations of motion it is simplest to satisfy the conditions (38) by setting

$$\Phi = (\mathcal{D}_a \mathcal{D}^a - 8R)U,$$

$$\Phi^* = (\mathcal{D}^a \mathcal{D}_a - 8R^*)U^*.$$

(40)

The superfields $U$ and $J^*$ can be subjected to arbitrary variations. One finds the equation of motion

$$\mathcal{D}^a (\mathcal{D}_a \mathcal{D}^a - 8R^*) \Phi - 8F' \Phi^* = 0$$

(41)

and its complex conjugate. It is not difficult to verify that (38) and (41) imply that the supercurrent
Supersymmetry (Including Supergravity) 559

\[ J_{\alpha \dot{\alpha}} = i \Phi \left( \overrightarrow{\partial}_{\alpha \dot{\alpha}} - \overleftarrow{\partial}_{\alpha \dot{\alpha}} \right) \Phi^* + \frac{1}{2} \partial_{\alpha \dot{\alpha}} \Phi \partial_{\alpha \dot{\alpha}} \Phi^* + 2G_{\alpha \dot{\alpha}} \Phi \Phi^* \]  \hspace{1cm} (42)

satisfies

\[ \partial^\alpha J_{\alpha \dot{\alpha}} = \partial_{\dot{\alpha}} S^*, \] \hspace{1cm} (43)

where

\[ S = 6F(\Phi) - 2 \Phi F'(\Phi) \] \hspace{1cm} (44)

The matter superfields \( J, S, S^* \) (which contain the energy momentum tensor and the spinor current) satisfy an identity analogous to that [see (13)] satisfied by the supergravity superfields \( G, R, R^* \) (which contain the Einstein tensor and the Rarita-Schwinger operator). Indeed the supergravity equations state the proportionality of these two sets of fields

\[ G_{\alpha \dot{\alpha}} = cJ_{\alpha \dot{\alpha}}, \quad R = cS. \] \hspace{1cm} (45)

If one attributes to the superfield \( \partial \) the conformal property

\[ \partial \Phi = -2 \Sigma \Phi, \] \hspace{1cm} (46)

one finds that the first (kinetic) term in the action (39) is conformally invariant. For the potential term, one obtains

\[ \partial \left( \int \frac{E}{R} F(\Phi) - 2 \int \frac{E}{R} S \Sigma \right). \] \hspace{1cm} (47)

The superfield \( S \), given by (44), appears as a measure of the violation of conformal invariance in the matter action. For \( F(\partial \partial \Phi) \), one has \( S = 0 \) and the entire matter action is conformally invariant. In this case the supergravity equations of motion give \( R = 0 \). In general, however, for instance if the matter supermultiplet has a mass \( (\partial \partial \Phi) R \) will not vanish.

A question which deserves consideration in future work is that of the quantization of the theory directly in terms of superfields. The natural objects here appear to be the unconstrained superfield \( U \) and its superspace propagator. A perturbation theory in superspace will be simpler than that in ordinary space, in that the number of indices and of diagrams will be much smaller. In this way one may hope to find a reduction in the divergences of the theory beyond that established so far on the basis of invariance arguments.

References

The equations of \( N=8 \) supergravity in superspace, with \( e^{-i\phi} \) and with external vector-spinor matter, were derived previously, but in redundant form (420 equations for 112 independent superfields \( V_{AA} \) and \( h_{rs} \)). Here we derive a minimal set of 112 equations from a Lagrangian linear in the torsions and curvatures. The remaining 308 relations can presumably be derived using Bianchi identities. The linear Lagrangian is found by a systematic method from the structure constants of the relevant superalgebra, and the same method can be used for \( 7V=0 \) to derive the Einstein theory of gravitation.

The reformulation of supergravity theories in superspace is motivated in part by the hope that the systematics of those theories will become clearer, that a simple formula can be found to describe all of them, and that more will be learned about the nature of the apparent restriction \( 7V<8 \). Furthermore, the great success of the geometrical interpretation of Einstein's theory of gravitation indicates that a full geometrical interpretation of supergravity in superspace will be illuminating. Finally it is possible that such further insights might suggest some generalization of supergravity that is not now apparent and that might come closer to describing physical reality.

Equations of motion for the curvatures and torsions in superspace have been given for \( N=1 \) supergravity, including self-coupling parameter \( e^{i\phi} \) and vector-spinor external matter, and also for \( 7V=3 \) supergravity. In this communication we restrict ourselves to the case \( N=1 \) and employ the notation of ref. 2, which we briefly review and extend.

Latin indices refer to the tangent space and Greek indices to the base space. Early letters \( (a, b, \ldots) \) and \( v, \ldots \) are spinorial and late letters \( s, \ldots \) are spatial. Capital Latin indices up to \( U \) represent all eight coordinates of the tangent space, while \( V-Z \) take 14 values and run over the six Lorentz rotations as well. Capital Greek indices \( (A, \ldots) \) represent all eight coordinates of the base space.

The superfields we employ are the vielbein \( V_j \), with inverse \( V^\alpha \) satisfying the relation

\[
V_{\alpha}^\beta V^\beta_{\alpha} = \delta^\alpha_{\beta}, \quad \text{a)}
\]

and the connection \( h_J, \) antisymmetric in \( r \) and \( s \), which gauges local Lorentz transformations. We define the tangent space derivative \( L/ \), covariant with respect to these transformations:

\[
\gamma_\alpha = V^\beta_{\alpha} \left( \partial - \frac{1}{2} h^r_s X_{rs} \right) = \partial - \frac{1}{2} h^r_s X_{rs}, \quad \text{b)}
\]

where \( X_{rs} \) are the generators of the Lorentz transformations. In "flat" superspace with no supergravitational disturbances, the fourteenth operators \( E_Y \) and \( X \) (collectively called \( G \)) generate the superalgebra \( \text{OSp}(1,4) \):

\[
\{G_X, G_Y\} = C^Z_{\alpha} G_Z, \quad \text{c)}
\]

where the bracket is a commutator unless \( X \) and \( Y \) are both fermionic, in which case it is an anticommutator, and the non-zero components of the structure constants \( C^Z_{\alpha} \), are given in Table I.

In the general case of supergravity, we can define superspace-dependent quantities \( i?J_i(z) \) by the analogous formula

\[
\{G_X, G_Y\} = R^Z_{\alpha} G_Z, \quad \text{d)}
\]

where \( R^\alpha \) are the curvatures defined in ref. 2, \( R^\alpha \) are minus twice the torsions defined in ref. 2, and the remaining \( i?J_5 \), are just equal to the corresponding constants \( C^5_{\alpha} \).

An intermediate situation is also worth considering, in which supergravitational disturbances are turned off except for the coupling of supergravity to external vector-spinor matter, described by an axial vector superfield \( J^a_5(z) \), satisfying the conservation condition

\[
\partial_{i} J_{i} = 0, \quad \text{e)}
\]
Table I.

The $J$-dependent structure "constants" $f^a_{b\ell}$.

The cases not listed are either zero or related to ones in the list by $f^a_{b\ell} = (-1)^a \ast f^a_{b\ell}$.

(The ordinary structure constants $C_{b\ell}$ are obtained by putting $J = 0$.)

We use the metric $\gamma^a_{b\ell}$ and Dirac algebra conventions in which $\gamma^a_{b\ell} = \delta^a_{b\ell} + 2\alpha \gamma^a_{b\ell}$ supplemented by the rule that Dirac indices are raised and lowered with the charge conjugation matrix. Thus, for example, $\gamma^a_{b\ell} = \gamma^a_{b\ell} C^c_{b\ell}$ is a symmetric matrix. The dimensionless constant $e$ is related to the de Sitter radius $R$ by $e = i\omega R$, where $\omega = iATZG$ is the gravitational coupling (Planck length), which we set equal to one.

In our general case ($N=1$ supergravity with $e = i\omega$ and external vector-spinor matter), the non-trivial components $R_{\alpha\beta}$ obey the 420 equations of ref. 2, which can be transcribed as follows. The components $R_{\alpha\beta}$, $R^{\alpha\beta}$, $R_{\ell\beta}$, $R^{\ell\beta}$, and $i^*_{\alpha\beta}$, are just equal to the corresponding components of $f^a_{b\ell}$ as given in Table I. The remaining components obey weaker equations than those for the corresponding $\gamma_{b\ell}$, namely

$$R_{\alpha\beta} = \gamma_{\alpha\beta} R_{\gamma\beta}$$
$$R^{\alpha\beta} = \gamma^{\alpha\beta} R^{\gamma\beta}$$
$$R_{\ell\beta} = \gamma_{\ell\beta} R_{\gamma\beta}$$
$$R^{\ell\beta} = \gamma^{\ell\beta} R^{\gamma\beta}$$
$$R^*_{\alpha\beta} = \gamma^*_{\alpha\beta} R_{\gamma\beta}$$
$$R^{*\beta}_{\alpha\beta} = \gamma^{*\beta}_{\alpha\beta} R^{\gamma\beta}$$
$$R_{\ell\beta} = \gamma_{\ell\beta} R_{\gamma\beta}$$
$$R^{\ell\beta} = \gamma^{\ell\beta} R^{\gamma\beta}$$
$$R^*_{\alpha\beta} = \gamma^*_{\alpha\beta} R_{\gamma\beta}$$
$$R^{*\beta}_{\alpha\beta} = \gamma^{*\beta}_{\alpha\beta} R^{\gamma\beta}$$

In our "first order" formulation of the theory, we regard the components of the inverse vierbein $V^a$ and of the Latin connection $\Gamma^a_{b\ell}$ as independent field variables, giving 112 real superfields, for which we expect 112 independent equations of motion in superspace. Evidently the 420 equations are highly redundant and can be replaced, in many different ways, by a list of 112 equations from which the remaining 308 can be deduced with the aid of the Bianchi identities that arise from the Latin graded Jacobi identities

$$(-1)^{\ell} [\gamma_{\alpha\beta}, \gamma_{\gamma\beta}] : \text{cyclic permutations} = 0.$$ 

Nearly identical minimal lists of 112 equations have been given by MacDowell and the authors, and the full list of 420 equations derived.

We can go further and find a Lagrangian for the "first order" theory in superspace, from which a different minimal list is obtained by varying with respect to the superfields $V^a$ and $\Gamma^a_{b\ell}$ independently. Since the equations of motion are all linear algebraic relations for the $R_{\alpha\beta}$ we consider the most general Lagrangian density linear in these quantities:

$$\mathcal{L} = V(\mathcal{A} + B^A_{\ell} \gamma^{\ell}_{\alpha\beta}(\mathcal{A}) R^\alpha_{\beta\ell})$$

where $V$ is the graded determinant of the vielbein, and obtain the 112 equations of motion

$$-\delta^\alpha_{\beta}(\mathcal{A} + B^E_{\ell} R^E_{\ell\ell}) - (-1)^{\ell} \delta^\alpha_{\beta} B^E_{\ell} R^E_{\ell\ell}$$
$$+ 2(\mathcal{A} + B^E_{\ell} R^E_{\ell\ell}) + 2(-1)^{\ell+y} R^E_{\ell\ell} B^E_{\ell\ell}$$
We can then determine the coefficients $A$ and $B$ by setting $B$ equal to the most general linear function of $J_{r5}(z)$ using covariant constants, and then comparing the resulting equations with the 420 equations for $R^a$, and requiring compatibility. The same result can be obtained by substituting the special values $R_{xz}$ in Eq. (13) and solving for $A$ and $B$. The Lagrangian can thus be found from the generalized "structure constants" $f$. To within a constant of proportionality, we find the unique solution

$$A, (14)$$

Substituting back into eq. (13), we find the list of 112 explicit equations of motion given in Table II. We have not verified that with the aid of the Bianchi identities these equations imply the full list of 420 equations, but all the checks we have made are consistent with that

Table II.

Equations of motion obtained from the Lagrangian of eq. (14).

$$\sigma^{\mu}_{rs}K^{\mu}_{da} + 2\sigma^{\mu}_{rs}R^{e}_{da} = 0$$

$$\sigma^{\mu}_{rs}R^{e}_{da} = 0$$

$$-\delta^{\mu}_{rs}(\mathcal{L}/V) - 2(\sigma^{a}_{rs})^{ab}R_{rb} + 3i(\gamma^{a}_{rs})^{ab}R_{rb} - 4i(\gamma^{a}_{rs})^{ab}J_{s}^{ab} + 3i(\gamma^{a}_{rs})^{ab}R_{rb}$$

$$+ 3i(\gamma^{a}_{rs})^{ab}R_{rb} - 3i(\gamma^{a}_{rs})^{ab}R_{rb} = 0$$

$$-2\sigma^{ab}_{rs}R_{rb} - 3i(\gamma^{a}_{rs})^{ab}R_{rb} - (3i/2)\mathcal{E}(\gamma^{a}_{rs})^{ab}R_{rb}$$

$$- 3i(\gamma^{a}_{rs})^{ab}R_{rb} - 3i(\gamma^{a}_{rs})^{ab}R_{rb}$$

$$- 4i(\gamma^{a}_{rs})^{ab}J_{s}^{ab} = 0$$

$$-4i(\gamma^{a}_{rs})^{ab}J_{s}^{ab} = 0$$

$$i(\gamma^{a}_{rs})^{ab}R_{rb} - (3i/2)\mathcal{E}(\gamma^{a}_{rs})^{ab}R_{rb}$$

$$- 3i(\gamma^{a}_{rs})^{ab}R_{rb} = 0$$

The use of $N(z)$ has permitted us to avoid certain indeterminacies. As $L > 0$, the ratio of coefficients of the last two terms in eq. (14) for the Lagrangian becomes undetermined. Furthermore, when we search for the particular solution of the Bianchi identities and the Lagrangian equations of motion of Table II that is given by the "structure constants" of Table I we find that the solution is unique when $L > 0$ but that a two-fold ambiguity develops as $L < 0$.

A rather trivial indeterminacy develops as $e > 0$, namely that the scale of $R_{ab}$ becomes a free parameter, and a rescaling of the system of equations becomes possible.

Our Lagrangian density for $N=1$, which lacks completely the Einstein term $R_{ab}$, does not appear to resemble the corresponding quantity for the case $N=0$ of plain gravitation. It is therefore instructive to apply our method to the $N=0$ problem and show that the usual theory emerges. We take eq. (13) with $R=f$ and take the constant $f$ or $c$'s from Table I with spinor indices suppressed. Solving for $A$ and the $B$ coefficients and substituting back into eq. (12) for the Lagrangian density, we find the unique solution

$$\mathcal{L}/V = -\frac{1}{2}R_{ab} + (3i/2)e^2$$

that gives the correct theory.

It is very important to discover the systematics of the Lagrangian density as $N$ varies and especially to find out what happens at $N=8$. While equations of motion can always be obtained by truncation from those for higher $N$, that is not true of the Lagrangian density, which is integrated over a different number of variables as $N$ changes. For that reason the systematics of $\mathcal{L}/V$ may well be non-trivial. In particular, for $TV > 1$ we expect to contain quadratic terms in the field strengths, at least those corresponding to the internal $SO$ group. (In our present work, such terms would have shown up for the vector-spinor matter if we had treated it completely, including its equations of motion, rather than as an external source.) For $TV=0$, quadratic terms in the torsions are normally not present. Should we have constructed a Lagrangian for $N=1$ containing such quadratic terms in the torsions? We are not certain, but it appears that our linear method is
justified by the results.

It should be noted that a very simple superspace Lagrangian for $N=1$ supergravity has been given by Wess and Zumino for a kind of "second order" formalism in which there are constraints relating the vierbein and the connection. We hope that our unconstrained "first order" approach will also lead ultimately to a simple description and will be suitable for generalization to extended supergravity.

We would like very much to be able to relate our work to the important research of Arnowitt and Nath; in a recent article they have thrown new light on the relationship between the two approaches and have apparently shown that the vierbein formalism can be obtained as a special case of their metric formalism.

We are happy to acknowledge the many contributions of Lars Brink to our understanding of supergravity in superspace, useful and enlightening conversations and correspondence with Samuel MacDowell, interesting communications from Richard Arnowitt and Pran Nath, and the agreeable hospitality of the Aspen Center for Physics.

References
4. S. MacDowell: Yale preprint. We would like to thank Professor MacDowell for communicating results prior to publication.
8. R. Arnowitt and P. Nath: Northeastern Univ. preprint NUB No. 2361. We would like to thank the authors for communicating their results prior to publication.

Superspace Formulation of Local Supersymmetries

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§1. Introduction

The first realization of a local supersymmetry was achieved in superspace' in the form of Gauge supersymmetry. After the development of the supergravity theory, it was demonstrated that the $k+0$ limit of gauge supersymmetry produced supergravity in the superspace formalism. Alternate superspace formulations of supergravity using supervierbeins and without the limiting procedure have been given by Wess and Zumino and by Brink, Gell-Mann, Ramond and Schwarz. However, an equivalence of these vierbein formulations and the metric formulation can be demonstrated. We shall begin first by a brief discussion of the basic principles of gauge supersymmetry and next discuss the more recent developments in this theory which relate to quantization, vacuum symmetries with quantum effects included and the field content of the theory. The superspace formulation of supergravity and the equivalence of the vierbein and metric formulations will then be discussed.

§2. Principles of Gauge Supersymmetry

The theory is based on the fundamental assumption that all the gauge fields arise from the tensor superfield $g_{\alpha}^{\mu}(z)$ which plays the role of the metric tensor in superspace, i.e., $ds^2=\sqrt{g_{\alpha}^{\mu}(z)}dz_{\alpha}$ where $z=(x^\alpha, \theta^\alpha)$ are the
supersymmetry space coordinates (x' are the Bose and \( g' = \alpha l, \alpha = 1, 2, \ldots, 4, a = 1, 2, \ldots, N \), a set of \( N \) Majorana coordinates. We shall usually suppress the internal symmetry index.) The theory has as its gauge group the general Majorana coordinate group in superspace, \( \mathcal{C}(z) \), which gives rise to the gauge change (we use right derivatives)

\[
\d g_{\mu\nu}(z) = g_{\mu\nu} \left( \xi^\mu_{\mu} + \xi^\nu_{\nu} \right), \quad (1)
\]

where /1=(0, 1) in (-1) depending on whether \( J \) is the contraction of many normally unrelated gauge groups \( \mathcal{C}(z) \). The remarkable aspect of eq. (1) is the “group theoretic unification” of many normally unrelated gauge groups such as Yang-Mills, Einstein and supersymmetry groups. The assumption of an inverse metric leads to a unique set of second order differential equations

\[
R_{\mu
u} = i, \quad (2)
\]

where \( x \) is a constant and \( R_\mu \) is the contracted curvature in superspace similar to the curvature \( R_\mu \) in Einstein theory. An invariant action can also be given which generates these field equations:

\[
I_4[g_{\mu\nu}] = \int dz \sqrt{-g} \left\{ R_{\mu\nu} + (4N - 2) R \right\}. \quad (3)
\]

Thus the principles of gauge invariance and the assumption of an inverse metric determine completely the dynamics except for the arbitrary constant \( A \). Equations (2) and (3) contain all the sources as well as Higgs interactions responsible for a possible self-contained symmetry breakdown. Symmetry breaking then must be an output of the theory rather than a contrived input through additional Higgs interactions responsible for a possible self-contained symmetry breakdown. Symmetry breaking is crucial for the extraction of the physical content of gauge supersymmetry. This is so because initially the theory has a very large gauge group with all the gauges of f(z) being preserved. It is interesting to ask if spontaneous breakdown can pluck out of the myriad gauges of \( \mathcal{C}(z) \), a set of residual preserved gauges which are relevant to physics. We have established a theorem in this regard with all quantum corrections included to the vacuum symmetry breaking conditions and we shall return to this later on.

§3. Path Integral Quantization

Gauge supersymmetry is most conveniently quantized using the path integral formulation which can also be made manifestly globally supersymmetric. The procedure is analogous to that used in quantizing globally supersymmetric gauge theories. One introduces the vacuum to vacuum transition amplitude

\[
Z = \int \mathcal{D}g_{\mu\nu} \mathcal{D}r \mathcal{D}r^* \exp \left[ i (I_\phi + I_\psi + I_\psi) \right], \quad (4)
\]

where \( r \) are the Faddeev-Popov ghost superfields and \( \psi = \psi_\psi + \psi_\psi + \psi_\psi \) is the total action with \( \psi \), the gauge fixing term and \( T_\psi \) the corresponding ghost action. As usual the gauge fixing term is chosen so that \( I = (-1/2)z g_{\mu\nu} C \cdot C \) where \( g' \) is the inverse of \( S'\mu\nu \) which is introduced as a (fixed) background field: \( g' = \frac{1}{2} g - \frac{1}{2} \). Analogous to the Einstein theory, a convenient choice of \( C \) is the linearized harmonic gauge. The corresponding Faddeev-Popov ghost action involves the (fermi-bose) ghost superfields \( r(z) \) and \( T(z) \) whose integral spin components are treated as fermions and half-integral spin components as bosons in the functional integral. This quantization scheme then incorporates global supersymmetry manifestly if one uses for the background field a globally supersymmetric \( g' \).

§4. Symmetry Breaking Equations

Equations that determine spontaneous breaking in gauge supersymmetry may be obtained using the effective potential method. The total action involves the metric tensor \( g_\mu \), the ghost fields as well as the background metric \( g^\mu \) so that one may write \( I = I[I_\phi + r_{\psi} + g_{\psi}] \) where we have made the translation \( g_\mu = g_\mu + h_\mu \). The symmetry breaking equations arise from variations of the effective potential \( r_\mu(g_\mu, g^\mu) \) which determines \( g_\mu \). Thus one has

\[
I = \frac{1}{2} \overline{\psi} \gamma^\mu \overline{\psi} = 0. \quad (5)
\]

In eq. (5) \( r_{\mu} = I + W \), where \( I \) is the total action and \( W \) is a solution of a functional equation. Defining \( G = \overline{\psi} \gamma^\mu \overline{\psi} \), the full content of the
symmetry breaking, eq. (5), takes the form
\[ G^{M}(g^\alpha) + (Z)^{-1} \int dh_{\mu} d\bar{\eta}, d\eta \epsilon^{M} \epsilon'(G^{M}(g^\alpha + h) - G^{M}(g^\alpha)) = 0. \]  
(6)

The first term on the left hand side in eq. (6) is the tree approximation and the remainder term \( Q^* \) gives the quantum loop correction.

In looking for solutions to eq. (6), we shall require for simplicity that global supersymmetry be explicitly maintained. (More generally, there may exist additional solutions to eq. (6) that violate global supersymmetry which is of course desirable since supersymmetry must eventually be broken for a physical theory.)

The transformation functions \( \xi(z) \) that maintain global supersymmetry have the form
\[ \xi^\alpha(z) = i\Lambda^\alpha \theta; \xi^\alpha = \lambda^\alpha, \]  
(7)
where \( \lambda^\alpha \) is a constant spinor. It is then easily established that the invariance of the vacuum state requires that \( dgf \) vanish and that the most general form of such a \( g^\alpha \) is
\[ g^\alpha = (\eta_{\alpha\sigma}, -i(\bar{\theta} \Gamma_{\alpha})_{\sigma\beta}, (\gamma K)_{\alpha\beta} + (\bar{\theta} \Gamma_{\alpha})(\sigma \Gamma^{\alpha})_{\beta}. \]  
(8)

In eq. (8), \( K \) and \( F \) are matrices in the Dirac and internal symmetry space with the property that \( ijF \) is symmetric and \( ijK \) is anti-symmetric where \( y = -C \), and \( C \) is the charge conjugation matrix. Further, it is always possible to set \( K = 1 \) in eq. (8) by linear transformations in the 6 space and we shall assume this to be the case. The in eq. (8) are to be determined by the symmetry breaking equations eq. (6) which involve the full evaluation of the quantum correction term \( Q \). Since the quantization of the theory has been carried out in a globally supersymmetric fashion, \( Q \) must in general be a global tensor and we may then write \( Q \) in the form
\[ Q_M = (\lambda' \eta_{\alpha\sigma}, \lambda'(\bar{\theta} \Gamma_{\alpha})_{\sigma\beta}, (\gamma K')_{\alpha\beta} + (\bar{\theta} \Gamma_{\alpha})(\sigma \Gamma^{\alpha})_{\beta}, \]  
(9)
where \( (ijK) \) is an anti-symmetric matrix. The quantities \( \lambda' \) and \( K' \) are given by the functional integral of eq. (6) and depend on \( F \) and \( \lambda \), i.e., \( \lambda' = V(X, r, \lambda, F, K) \).

Use of eqs. (8) and (9) in eq. (6), reduce the symmetry breaking equations to the set
\[ \Gamma_{\mu}^{\gamma} = -\frac{1}{2}(\lambda - \lambda'); \text{tr}(\Gamma_{\mu}^{\gamma} \Gamma^{\nu}) = -4(\lambda - \lambda'); \]  
(10)
\( \Gamma_{\mu}^{\gamma} \) and \( K' \) may be decomposed so that \( \Gamma_{\mu}^{\gamma} = \gamma_{z}(\gamma^{\mu} + i\gamma_{5} I_{(\alpha)}) \) and \( K' = K_{6} + i\gamma_{5} K'_{4}. \) \((I_{(\alpha}), I_{(\beta)}, \ldots \) are real symmetric and anti-symmetric matrices and \( K_{\alpha} \) are real symmetric matrices in the internal symmetry space.)

§5. Residual Symmetries after Spontaneous Breaking

To determine the residual symmetries present in the spontaneously broken theory, we examine the content of the symmetry breaking equations eq. (10). One may proceed by expanding \( F_{\mu} \) in terms of the symmetric and anti-symmetric matrices of the internal symmetry space; \( T_{\tau}, r_{\alpha, l} \) and \( r_{\alpha} = r_{l} r_{\tau}\). The \( (\alpha, /\beta) \) constitute a set of \( N \) vacuum expectation values of the Higgs fields for the theory. Note, however, that eq. (10) represent a set of \( (N(N+1)+1 \) equations which is an impossible situation unless some of the equations are redundant as a consequence of an internal symmetry group. Thus one has the following fundamental result:

**Theorem (i):** The globally supersymmetric solutions of the symmetry breaking eqs. (10) require a rigorously preserved internal symmetry subgroup.

Thus the spontaneous breaking of gauge supersymmetry with globally supersymmetric vacuum solutions contains in an automatic fashion a set of preserved gauges and the theory thus contains in it the possibility of a natural explanation of why some gauges such as the Maxwell and QCD gauges are exactly preserved in nature. We discuss now the question of precisely what gauges are preserved and what are broken in \( \% \). This problem is equivalent to determining which \( f(z) \) leave \( gf \) invariant so that \( df \neq 0 \). Recently, we have obtained a general solution to this problem and the results are
\[ \xi^\alpha(z) = (\varepsilon^{\alpha} + \varepsilon^{\alpha} \lambda^{\alpha} + \lambda'(\bar{\theta} \Gamma_{\alpha})(\sigma \Gamma^{\alpha})_{\beta}, (ij2)\epsilon_{\mu\nu}(\sigma^{\mu\nu} \theta)^{\alpha} \]  
\[ + \lambda'^{(\Gamma_{\alpha})(\sigma \Gamma^{\alpha})_{\beta}, \]  
(12)
where \( M = M_{0} + i\gamma_{5} M_{1} \) are anti-symmetric internal symmetry matrices which obey the condition
\[ [I_{\mu}, M] = 0, M_{0,1} = -M_{0,1}. \]  
(12)

Thus the preserved internal symmetry transformations are subgroups of \( 0(A_{0} \) which com-
The above analysis then leads us to the second fundamental result given below:

**THEOREM (ii):** The vacuum symmetries of the theory with full quantum corrections included consist of the Poincaré transformations (simultaneously on \(x^a\) and \(\theta^i\)), global supersymmetry transformations generated by \(X^a\) and linear orthogonal internal symmetry transformations in \(d\) space generated by \(M\) obeying eq. (12).

§6. Field Content

For each of the preserved global symmetries after spontaneous breaking there corresponds a local symmetry and a preserved gauge field. Thus one has as the preserved gauges the Einstein gauge and the corresponding field \(g^{\mu\nu}(x)\), the supergravity gauge and the corresponding gravitino field \(\xi^a = \lambda^a(x)\); the supergravity gauge and the corresponding gravitino field \(\xi^a = \lambda^a(x)\). Equation (1) is then integrated order by order in \(\theta\). Calculations to the order needed to generate supergravity field equations yield:

\[
\begin{align*}
\hat{\xi}^a &= i\lambda^a\theta + \frac{1}{2}(\tilde{\xi}_\mu^a\theta)(\tilde{\lambda}_\mu^a) + \Delta\xi^a(z) + O(\theta^3), \\
\hat{g}^{\mu\nu} &= g^{\mu\nu}(x) + i\tilde{g}_\mu^a\theta - i\hat{\theta}^\mu_{\nu}\lambda^{\nu}\theta - (\tilde{\xi}_\mu^a\theta)(\tilde{\xi}_\nu^a) + \Delta g^{\mu\nu}(z) + O(\theta^3), \\
g_{\alpha\beta} &= g_{\alpha\beta}(z) = -i(\tilde{\theta}^\mu_{\nu}\lambda^{\nu}\theta + (\tilde{\xi}_\mu^a\theta)(\tilde{\xi}_\nu^a)) + \Delta g_{\alpha\beta}(z) + O(\theta^3)
\end{align*}
\]

where \(A\) and \(A_\alpha\) are terms of order \(k\) or higher. Further, the analysis determines the vierbein affinity \(r^\mu\) to be correctly that of supergravity; \(r^\mu = (il4)_{\alpha\beta}g^{\alpha\beta}\), where \(co^{TM}\) correctly includes the supergravity torsion. Thus the process of gauge completing the metric naturally brings the torsion into the metric without having to modify the Riemannian nature of the superspace. Insertion of the gauge completed metric in eq. (2) then leads to the supergravity field equations in the \(k\rightarrow0\) limit.

Recently, alternate superspace formulations of supergravity have been given which use the super vierbein \(V(z)\) (the Latin indices represent local quantities) and do not require the limiting procedure. The equivalence of these vierbein approaches to the previous metric formulation of supergravity can now be established for physical (mass-shell) spaces. The analysis uses the fact that one may always form an \(N=1\) global supersymmetry multiplet and constructed a \(g_{\alpha\beta} = g_{\alpha\beta}^{TM}\) satisfying eq. (1) and the supergravity transformation law for \(e^{TM}\) and \(p^\alpha\). The metric \(g_{\alpha\beta}(z)\) would then be made gauge complete with respect to the supergravity gauge group generated by a single spinor \(Z(x)\). This process should then lead to a superspace formulation of supergravity.

To implement the program one expands \(g_{\alpha\beta}(z)\) and \(C(z)\) in powers of \(0\) with the lowest non-vanishing terms given by

\[
g_{\rho\sigma}(z) = g_{\rho\sigma}(x); \quad g^{\rho\sigma}(z) = (k/2)\tilde{g}_\rho^a(z); \quad g_{\alpha\beta} = k\eta_{\alpha\beta}
\]

and \(\xi^a = i\lambda(x)\gamma^a\theta; \quad \tilde{\xi}^a = \lambda^a(x)\).
construct a Riemannian metric given a set of supervierbein:
\[ g_{AB}(z) = V_A^B(z) \eta_{AB}(-1)^{(1+n)/2} V_B^A(z); \]
\[ \eta_{AB} = \begin{pmatrix} \eta_{mn} & 0 \\ 0 & k\eta_{ab} \end{pmatrix}, \tag{15} \]
where \( \eta_{ab} \) is the tangent space metric with \( \eta_{mn} \) and \( \eta_{ab} \) being the bose and fermi matrices.

The vierbein approach of ref. (6) uses the technique of auxiliary Breitenlohner fields in superspace. If one eliminates these Breitenlohner fields and uses the supergravity field equations as required for consistency, one finds that the supervierbein of ref. (6) so reduced when inserted in eq. (15) produces precisely the metric of eq. (14) when mass-shell conditions are also imposed there. This result holds for arbitrary values of \( k \). Further, the transformation functions of ref. (6) are seen to be identical to those of eq. (14) since \( d\tilde{a} \); all vanish on the mass-shell. Thus the vierbein and the metric approaches are equivalent for physical mass-shell manifolds.

Further, it is seen possible to gauge complete the vierbein even off-shell without the use of Breitenlohner fields by relaxing the tangent space assumptions of refs. (5) and (6) while maintaining correctly the supergravity transformation group. In general the supervierbein obey the transformation law
\[ \delta V_A^B = V_B^D \xi_D^B + (-1)^{(1+n)/2} \eta_{AB} \xi_A^B, \]
where we have included the tangent space transformations generated by \( e^\alpha \). Commonly, the tangent space transformations are chosen to be super-Lorentzian, \( \mathbb{GL}(n) \), and \( e^\alpha \) generate Lorentz transformations. However, it is not possible to gauge complete this vierbein off-shell using only the fields \( e^\alpha \) and \( (p^\alpha \chi) \). In ref. (6) Breitenlohner fields were introduced to overcome this difficulty. However, this difficulty may also be circumvented by an expansion of the tangent coefficients \( e^\alpha \) while still preserving the supergravity group. We thus abandon the super-Lorentzian condition and utilize the full \( e^\alpha \) in the process of gauge completing the vierbein with the only restriction that the invariance of \( S \) be preserved. This restriction requires that
\[ \xi_A^B \eta_{CB} + (-1)^{(1+n)/2} \xi_B^E \eta_{EC} \eta_{CA} = 0. \tag{17} \]

Using the restrictions of eq. (17) (rather than the super-Lorentzian condition), it has been possible to gauge complete the vierbein to the same order as the metric. Further, using this gauge completed supervierbein one may generate supergravity field equations. This can be done either by use of the gauge supergravity field equations
\[ R_{AB} = (X_{rs})^A_B V^r_s V^s_B R_{rs} = 0 \tag{18} \]
in the limit \( k \to 0 \) or using the superspace equations
\[ \tilde{R}_{AB} = (X_{rs})^A_B V^r_s V^s_B R_{rs} = 0 \]
without taking the limit \( & \to 0 \). [Here the \( X \) are Lorentz rotation matrices and \( R_{rs} \) are given in ref. (6).] Thus once again one finds the equivalence of the vierbein and metric formalisms.

\section*{§8. Quantum Loop Corrections}
Calculations of quantum loops in gauge supersymmetry are currently underway. We have established the remarkable fact that the one-loop corrections to the effective potential are finite. This arises due to the global supersymmetry which sets to zero the divergent parts of loop integrals. Calculations of propagator corrections are more complicated but preliminary analyses show that here again global supersymmetry acts to eliminate most (and perhaps all) divergences. Since quantum calculations are now technically feasible, a more realistic investigation of the scope of gauge supersymmetry theory can now be undertaken.

\textbf{References and Footnotes}
to be published in Phys. Rev.


12. $ \mathcal{C}_A = g_{\alpha\beta} \Sigma^{(\alpha)\Sigma \beta} - (1/2)(-1)^{\hat{\alpha} + \hat{\beta}}(g_{\alpha\beta} \Sigma^{(\alpha)\Sigma \beta})_{A}$

13. $I = \int d^d z \, b^A \wedge \cdots \wedge (A^A - (i/2) (I + I^-) \wedge \cdots \wedge g^{\alpha = 2n} I)$ where "\wedge" means covariant differentiation with respect to the full metric $g_{\alpha\beta}(z)$ and $V(A; ID = Y_{\alpha} n \rightarrow (I/2)^{-1} y_{\alpha} A)$.


15. $W[g_{\alpha\beta}] = i \ln \delta I_{d^d \Sigma} / 7d^{d+ \epsilon} e^{I}$ with $I$ given by $\delta I_{d^d \Sigma} / 7d^{d+ \epsilon} e^{I}$ with $V_{\alpha \mu \nu}(S' = A'; \Sigma - K S A \Sigma^{-1} \Sigma^{-1})$ and $g^{\alpha = 2n} I$.

16. $det \equiv s P(t;x)^{\alpha(3\Sigma)} (1/2) \Sigma = (3^\alpha + \Sigma = 1)^{\alpha}.$

Session C7: Quantum Electrodynamics

Chairman: Y. OHNUKI
Organizer: T. KINOSHITA
Scientific Secretaries: A. UKAWA
                        N. NAKAGAWA

1. Recent Development of Quantum Electrodynamics
   T. KINOSHITA

2. Muonium Hyperfine Structure
   V. W. HUGHES

(Friday, August 25, 1978; 15:40-17:00)