THE ODD-PROTON EFFECTS ON THE POTENTIAL ENERGY SURFACES OF ODD MASS Tl, Au, Ir and Re ISOTOPES.


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Abstract

The total potential energy surfaces of thallium, gold, iridium and rhenium odd mass isotopes are calculated microscopically as functions of the quadrupole deformation, $\varepsilon_2$, when the odd protons occupy definite orbitals. The nuclear shapes and the static equilibrium deformations of these nuclei are deduced from the results of these calculations for the proton orbitals nearest to the Fermi level. The influence of the hexadecapole deformation, $\varepsilon_4$, on these results is investigated too. Finally, a few experimental data available for these odd mass nuclei are correlated to the corresponding theoretical results.

1. Introduction

The odd proton nuclei of the transition region between the rare-earth elements and the $^{197}$Au double closed shell (Tl, Au, Ir, Re) were extensively investigated experimentally during the last few years. Furthermore several theoretical descriptions of their properties were given assuming a single proton (hole, quasi-proton or proton hole cluster) coupled to vibrations or rotations of a spherical (or deformed) core [1-7]. However there is not yet available a systematic investigation of the odd proton effect on the static collective properties of these nuclei. Indeed, preliminary investigations were made in this direction earlier but they were restricted to a few odd proton nuclei near the single proton closed shells Z = 50 [8] and Z = 82 [9]. Furthermore in these calculations [8,9] only a few high and low spin proton orbitals were analysed in detail.

The purpose of the present investigation is to evaluate the total potential energy surfaces of thallium, gold, iridium and rhenium odd mass isotopes as functions of the quadrupole deformation parameter, $\varepsilon_2$, when the odd protons occupy definite orbitals in the modified oscillator model [10]. This analysis is restricted to the orbitals near to the Fermi level of each nucleus. Indeed they are expected to play a preponderant role on the description of these transitional nuclei. For each nuclide considered the total potential energy differences, $V_{00}$, between the oblate and prolate minima of the low lying orbitals as well as the quadrupole deformation parameters and possible shape transitions are discussed. Finally the influence of the hexadecapole deformation on the potential energy surfaces and other collective properties considered is also investigated.

2. Calculations

2.1. The model

The total potential energy surfaces of these odd proton nuclei are calculated microscopically with an extended version of the programme previously applied to even [11] and odd-mass $^{197}$Au nuclei. In each case, the potential energy is a sum of two terms (the liquid drop and the shell correction energies) depending on two deformation parameters (axial quadrupole $\varepsilon_2$ and hexadecapole $\varepsilon_4$) parameters. The main improvement relatively to the old programme [11] is the possibility to put the odd proton on any given orbit.

2.2. The parametrization

In the present calculations, the adopted parametrization is nearly identical to the parameter set used for the neighbouring even-even nuclei [11]. The only differences are slight changes in the $\kappa$ and $\mu$ parameters of the oscillator potential for the proton and neutron orbitals. In most calculations reported here, it is assumed no hexadecapole deformation. However, in order to investigate the influence of the $\varepsilon_4$ deformation on these results, other calculations were made with $\varepsilon_4 \neq 0$.

2.3. Single proton and Fermi levels of Tl, Au, Ir and Re

A portion of the single proton level scheme corresponding to the transition region between the rare earth elements and the lead isotopes is represented in fig. 1. In this diagram are also included the thallium, gold, iridium and rhenium Fermi levels determined with the same computing programme and parametrization. For clearness, only the Nilsson orbitals close to the Fermi levels of these nuclides are represented. Furthermore, the most important orbitals considered in the forthcoming discussion are full drawn.

2.4. Total potential energy

The projected total potential energy surfaces of a few odd mass Tl, Au, Ir and Re isotopes are represented in figs. 2-5. From their analysis, the total potential energy differences, $V_{00}$, between the oblate and prolate minima, the absolute quadrupole deformation parameter $\varepsilon_2$ and the potential energy differences, $E_{20}$, between the lowest minimum and the corresponding spherical limit ($\varepsilon_2 = 0$) for the groundstate orbital are determined (fig.6-8). The same magnitudes are deduced from the potential energy curves corresponding to the orbitals belonging to the $l_{z/2}$ and $l_{1/2}$ subshells.
Fig. 1 Partial single proton and Fermi levels of Tl, Au, Ir, and Re isotopes. Only the Nilsson orbitals close to the Fermi levels of these nuclides are represented.

(figs. 9-13). To investigate their dependence on the neutron number, the experimental and theoretical results of given isotopes are presented and discussed simultaneously. Furthermore, for greater clearness, the positive and negative parity orbitals are considered separately. Finally, the analysis of the $\varepsilon_4$ influence is illustrated in the case of negative parity orbitals (figs. 15, 16).

3. Analysis of the results

3.1. Thallium isotopes

According to the position of the Thallium Fermi level, the $1/2^+[400]$, $3/2^+[402]$ and $1/2^+[411]$ orbitals should be the most relevant configurations for the description of the positive parity states. Similarly, the $9/2^-[505]$ (or $1/2^-[541]$), $11/2^-[505]$ (or $1/2^-[550]$) orbitals should be the preponderant configurations for the description of the negative parity states.

3.1.1. Positive parity orbitals

The ground state, $1/2^+$, of all odd mass Thallium isotopes is identified to the $1/2^+[400]$ single proton configuration. Indeed, this configuration is the closest to the Fermi level for small quadrupole deformations $|\varepsilon_2| \leq 0.15$. The corresponding potential energy curves (fig. 2) show two flat minima giving no clear indication of static quadrupole deformations. However, the $V_{pp}$ diagram (fig. 6) predicts a small quadrupole oblate deformation for the light and medium Tl isotopes in their ground states. Indeed, the calculated absolute quadrupole deformation parameter (fig. 7) is small ($\varepsilon_2 \leq 0.06$-0.07) and nearly independent of the neutron number. Nevertheless, the small energy difference between the lowest oblate minimum $1/2^+[400]$ and the spherical limit, $3s_{1/2}$ of the potential energy curve (fig. 8) compared to the zero point energy ($E_{zp} \leq 1.09$ MeV) calculated for the heavier Tl isotopes [13] suggests a nuclear shape instability.

The first excited state with positive parity, $3/2^+$, in the odd mass Tl isotopes should have a large contribution of the orbital $3/2^+[402]$, according to the analysis of the single proton and Fermi levels (fig. 1). The corresponding potential energy curves are slightly more asymmetric than those of the orbital $1/2^+[400]$. However, this asymmetry is small and the corresponding minimum rather flat. Consequently, the Tl isotopes in their ground and first excited states are expected to behave like soft vibrators.

3.1.2. Negative parity orbitals

All the potential energy curves corresponding to the high spin $9/2^+[505]$ orbitals of the $1h_{9/2}$ proton subshell (fig. 2) have a minimum at a finite deformation on the oblate side ($\varepsilon_2 \leq -0.10$). This minimum is deeper than the corresponding one of the $1h_{9/2}$ low spin $1/2^+[541]$ orbital appearing at $\varepsilon_2 > 0.08$. The same conclusion can be drawn from the analysis of the $V_{pp}$ diagram (fig. 9). Indeed, $V_{pp}$ is negative and nearly constant ($V_{pp} \leq 0.50$ MeV) for all Tl isotopes. However, for the heavier thallium isotopes ($N > 120$), the $V_{pp}$ curve decreases rapidly. The quadrupole deformation parameter has a similar behaviour. Moreover, the
Fig. 2 Total potential energy dependence on the nuclear quadrupole deformation, $\varepsilon_2$, of a few odd mass $\text{Tl}$ isotopes in their lowest positive and negative parity orbitals.

Fig. 3 Total potential energy dependence on the nuclear quadrupole deformation, $\varepsilon_2$, of a few odd mass $\text{Au}$ isotopes in their lowest positive and negative parity orbitals.

Fig. 4 Total potential energy dependence on the nuclear quadrupole deformation, $\varepsilon_2$, of a few odd mass $\text{Ir}$ isotopes in their lowest positive and negative parity orbitals.

Fig. 5 Total potential energy dependence on the nuclear quadrupole deformation, $\varepsilon_2$, of a few odd mass $\text{Re}$ isotopes in their lowest negative parity orbitals.
Fig. 6 Total potential energy difference between the oblate and prolate minima of Ti, Au, Ir and Re in the groundstate proton orbital.

Fig. 7 Absolute quadrupole deformation parameters corresponding to Ti, Au, Ir and Re groundstate orbitals. For the semi-empirical data see text.

Fig. 8 Total potential energy differences between the lowest minima of Ti, Au, Ir and Re in the groundstate orbitals and the corresponding spherical limit.
Fig. 9 Total potential-energy difference between the oblate and prolate minima of the $9/2[505]$ and $1/2[541]$ proton orbitals.

Fig. 10 Total potential energy difference between the oblate and prolate minima of the $1/2[550]$ and $11/2[505]$ proton orbitals.

Fig. 11a Absolute quadrupole deformation corresponding to the lowest minimum of $9/2[505]$ or $1/2[541]$ proton orbital.

Fig. 11b Absolute quadrupole deformation corresponding to the lowest minimum of $1/2[550]$ or $11/2[505]$ proton orbital.
broad and small $E_{\text{def}}$ curve ($E_{\text{def}} < 1$ MeV for $N > 120$ fig. 12) suggests a spherical shape for the heavier Tl isotopes which should behave like a spherical vibrator.

A similar analysis of the 1/2[550] and 11/2[505] potential energy curves (fig. 2) and the $V_{\text{po}}$ diagrams indicate prolate equilibrium deformations for all the Tl isotopes. However, for $N > 114$, the $V_{\text{po}}$ differences and the $\varepsilon_2$ deformations are small ($V_{\text{po}} < 0.20$ MeV fig. 10; $\varepsilon_2 < 0.06$ fig. 7). Finally, no significant departure from spherical shape is predicted for the heavier isotopes from the $E_{\text{def}}$ values ($E_{\text{def}} < 0.5$ MeV for $N > 114$ fig. 13) on account of the neglected zero point energy contribution.

3.2. Gold isotopes

According to fig. 1, the 3/2[402] (or 1/2[411]), 1/2[400], 9/2[505] (or 1/2[541]), and 11/2[505] (or 1/2[540]) orbitals are the preponderant configurations for the description of the groundstate and the low and medium excited levels in gold.

3.2.1 Positive parity orbitals

The potential energy curves (fig. 3) and the $V_{\text{po}}$ diagrams (fig. 6) corresponding to the 3/2[402] (or 1/2[411]) and 1/2[400] orbitals suggest prolate deformations for the light gold in their 1/2$^+$ or 3/2$^+$ groundstate. The greatest deformation is attained in the 1/2[400] orbital where $0.10 < \varepsilon_2 < 0.17$. Moreover, the $V_{\text{po}}$, $\varepsilon_1$, and $E_{\text{def}}$ diagrams give indications for a possible shape transition for the 3/2$^+$ groundstate of the heavier gold isotopes (fig. 6-8).

For these isotopes, there is a satisfactory agreement between the static theoretical $\varepsilon_2$ and the dynamic semi-empirical quadrupole deformations deduced from the relation:

$$\varepsilon_2 = \frac{\sqrt{5}}{16 \pi} \beta - 15/16 \pi \beta$$

where $\beta$ is determined from the experimental B(E2) values of the 5/2$^+ - 3/2^+$ transitions as described in ref. 15.

3.2.2 Negative parity orbitals

All the gold potential energy curves of the 9/2[505] and 1/2[541] orbitals have a minimum on the oblate (prolate) side at $\varepsilon_2 \approx -0.10 (+0.10)$ like those of the Tl isotopes (fig. 1). The nearly symmetric $V_{\text{po}}$ diagrams of these orbitals (fig. 9) predict prolate shapes for the light gold isotopes and oblate shapes for the heavier ones ($A > 189$). Consequently, a well defined shape transition is expected at $N = 110$ (or near by that neutron number) on account of the contribution of the zero point energy. Finally, the absolute quadrupole deforma-
tion |c₂| and the potential energy differences, Edef, corresponding to these 1h₁₂/₂ orbital are considerably higher in the gold isotopes than in the thallium ones (figs. 11a and 12).

A similar analysis of the potential energy curves of the 11/2[505] and 1/2[550] orbitals shows smaller asymmetry than in the 1h₁₂/₂ lowest orbitals (fig. 1). Furthermore, the Vpo diagrams predict prolate deformations for the light gold isotopes (fig. 10). A shape transition may occur for the heavier ones (N > 114) but it is less clear than for the 1h₁₂/₂ lowest orbitals due to the smallness of the oblate Vpo minimum (Vpo < 0.20 MeV). Similarly, the corresponding |c₂| and Edef curves (fig. 11b, 13) do not give any clear indication of a permanent oblate deformation.

3.3. Iridium isotopes

The Fermi levels of the iridium and gold isotopes are close and parallel for |c₂| < 0.3 (fig. 1). Consequently, one expects the same preponderent effects of the same odd proton configurations in both elements apart the different influence of three and five proton hole clusters in the lead shell.

3.3.1. Positive parity orbitals

The potential energy curves corresponding to the positive parity configurations 1/2[400] and 3/2[402] of the heavier iridium isotopes (185', 199'Ir) have a similar shape (fig. 4) to the curves of the light gold isotopes (187 ', 191'Au). Furthermore, the potential energy differences Vpo predict prolate quadrupole deformations for the positive parity states of the light iridium isotopes and a possible shape transition at N = 114 for the 3/2⁺ ground-state (fig. 6).

A better agreement between the semi-empirical and the static c₂ values is obtained for the heavier (N > 114) than for the lighter iridium isotopes (fig. 7). However, the experimental B(E2) results [16] are too much scattered in order to derive any definite conclusion from this comparison.

Finally, the total potential energy difference, Edef, between the lowest minimum of the Ir groundstate orbital and the corresponding 2d₁₂/₂ spherical limit is larger in Th and Au isotopes (fig. 8). Consequently, these calculations predict a considerable permanent prolate deformation for the Ir isotopes in their ground states.

3.3.2. Negative parity orbitals

The projected total potential energy surfaces corresponding to the lowest orbitals of the 1h₁₂/₂ subshell in all the Ir isotopes (fig. 4) have deep minima on the oblate or prolate sides for 0.12 ≤ |c₂| ≤ 0.17. Furthermore, similar shape transitions as observed for these orbitals in gold, occur also in iridium (fig. 9). However, the Vpo iridium curve is more asymmetric than the gold one. Such fact is stressed by the Edef curve which shows a larger energy difference for the prolate (Edef = 6 MeV for N=104) than for oblate (Edef = 1.0 MeV for N=120) lowest 1h₁₂/₂ iridium orbitals (fig. 12). Finally, the absolute quadrupole deformations, |c₂|, are nearly identical in Ir and Au isotons (fig. 11a).

The same analysis of the potential energy curves corresponding to the highest 1h₁₂/₂ iridium orbitals (fig. 10) leads to similar conclusions previously obtained for the gold isotopes (sect. 3.2.2). In particular, the prolate part of the Vpo curve is steeper in iridium than in gold. On the contrary, no clear evidence exists for permanent oblate shapes in the 1/2 [550] orbital. Furthermore, the |c₂| deformations of the isotons of both elements are nearly identical (fig. 12b).

However, the maximum value of Edef in the Ir isotopes is nearly proportional to the corresponding values of Th and Au isotons (fig. 13).

3.4. Rhenium isotopes

The ground state and low lying excited states of the odd mass Re isotopes are based essentially on 5/2[402] and 9/2[514] proton orbitals due to their lower Fermi level and greater quadrupole prolate deformation.

3.4.1. Positive parity orbitals

The total potential energy dependence on the nuclear quadrupole deformation, c₂, of a few odd mass rhenium isotopes in their groundstate is represented in fig. 14.

![Fig. 14 Total potential energy dependence on the quadrupole deformation, c₂, of a few odd mass Re isotopes in the 5/2[402] orbital.](image-url)
3.4.2. Negative parity orbitals

The total potential energy dependence on the nuclear quadrupole deformation of a few odd mass rhenium isotopes in their lowest $h_1/2$ and highest $l_{1/2}$ proton orbitals is represented in fig. 5. Following the general trend of this systematic study, the $v_{po}$, $|e_2|$ and $E_{def}$ diagrams (figs. 9-13) corresponding to the same orbitals were deduced from the analysis of the potential energy curves. However, there is not yet any experimental evidence for rhenium excited states built on the proton (oblate) $1/2[514]$ or $1/2[505]$ orbitals. Indeed, the low and medium excited negative parity levels of the odd mass Re isotopes are built on the $9/2[514]$ proton orbital. For that reason, this orbital was also included in the total potential energy curves of T1, Au and Ir isotopes (figs. 2-4).

3.5. Influence of the $e_2$ deformation

The $v_{po}$ diagram given in figure 15 is relative to the lowest $h_1/2$ and the highest $l_{1/2}$ orbitals. Moreover, the $e_2$ dependence is represented graphically in the inset of the same figure. From the comparison between the figures 9, 10 and 15, one sees a relatively small increase of the $v_{po}$ values of the different isotopes. The biggest changes are for the $l_{1/2}$, lowest orbitals of thallium. Similar differences are not observed for the other elements. In particular, the shape transitions are independent of these small $e_2$ deformations. The $E_{def}$ diagrams represented in figure 16 correspond to the same $l_{1/2}$ and $l_{1/2}$ orbitals and $e_2$ dependence previously quoted. The comparison between this figure and the corresponding ones where $e_2$ deformation was neglected (figs. 12, 13) reveals a small influence of hexadecapole deformation for the elements considered.

4. Conclusions

This systematic investigation of the odd proton effects in T1, Au, Ir and Re isotopes leads to the following conclusions:

- The positive parity states based on the low spin $1/2[400]$, $3/2[402]$ or $5/2[402]$ proton configurations correspond to small oblate (T1) or medium prolate (Au, Ir, Re) quadrupole deformations.
- The negative parity states, built on proton hole states in the highest Nilsson orbitals of the $l_{1/2}$ subshell, are connected with relatively stable prolate quadrupole deformations.
- The negative parity states, arising from proton particle states in the lowest Nilsson orbitals of the $l_{1/2}$ subshell, correspond to prolate $e_2$ defor-
mations in Au, Ir and Re Isotopes. Such sharp transitions of the shape are not observed in Tl isotopes where the lowest \( l_{\pi/2} \) particle orbitals have smaller influence (\( \langle \epsilon_{\beta} \rangle = -0.10 \)) for all these isotopes.

References


