CP NON-INVARANCE AND THE $K_S \to \mu^+\mu^-$ DECAY RATE

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ABSTRACT

Christ and Lee suggested CP non-invariance as an explanation of the low experimental $K_L \to \mu^+\mu^-$ decay rate. We discuss Hamiltonian realizations of this mechanism, and the lower bounds on the $K_S \to \mu^+\mu^-$ decay rate implied by them. The lower bound on the $K_S \to \mu^+\mu^-$ branching ratio varies in these models from $10 \times 10^{-7}$ (for the most economical model) to $2 \times 10^{-7}$.

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Christ and Lee \(^1\) have suggested that the disagreement between the upper bound \(^2\) on the \(K_L \rightarrow \mu^+ \mu^-\) decay rate and the corresponding theoretical lower bound \(^3\),\(^4\) can be explained by invoking CP non-invariance. The idea is that the \(K_1 \rightarrow \mu^+ \mu^-\) amplitude is so large that the small admixture of the CP even component \(K_1\) in the state \(K_L\) becomes important. The large \(K_1 \rightarrow \mu^+ \mu^-\) decay amplitude implies a large \(K_S \rightarrow \mu^+ \mu^-\) decay rate, so they obtain

\[
\frac{\Gamma(K_S \rightarrow \mu^+ \mu^-)}{\Gamma_S} \geq 5 \times 10^{-7}
\]  

(1)

where \(\Gamma_S\) is the total \(K_S\) decay rate. Gaillard \(^5\) has shown that the Christ–Lee mechanism requires only that

\[
\frac{\Gamma(K_S \rightarrow \mu^+ \mu^-)}{\Gamma_S} \geq 1.6 \times 10^{-7}
\]  

(2)

The purpose of this note is to discuss Hamiltonian realizations of the Christ–Lee mechanism, and the lower limits on the decay rate \(\Gamma(K_S \rightarrow \mu^+ \mu^-)\) implied by them.

The basic requirements of the Christ–Lee mechanism are new interactions allowing: 1) the large amplitude for \(K_1 \rightarrow \mu^+ \mu^-\) and 2) CP non-invariance for the \(K_1\) and/or \(K_2\) decay amplitude into the \(\mu^+ \mu^-\) channel. The simplest type of Hamiltonian, as discussed in class \(A\) models below, involves only a single new interaction which allows a direct CP non-invariant \(K_1 \rightarrow \mu^+ \mu^-\) transition. For class \(A\) models, we show

\[
\frac{\Gamma(K_S \rightarrow \mu^+ \mu^-)}{\Gamma_S} \geq 10 \times 10^{-7}
\]  

(3)

This result is especially interesting because there are preliminary reports \(^7\) that the experimental upper bound has been reduced to a value approaching the limit (3) and may be improved soon beyond this. Our other theoretical limits (class \(B\) and class \(C\) models) serve to point out the "interpretation" of possible experimental \(K_S \rightarrow \mu^+ \mu^-\) decay rates between the limits (2) and (3).

Our notation and assumed experimental data are the same as those of Ref. \(^5\). We assume CPT invariance and express the physical states \(K_S\) and \(K_L\) in terms of the usual CP eigenstates \(\chi_1\) and \(\chi_2\) as

\[*\)

This point has been made by us previously in unpublished manuscripts \(^6\).
\[ |K_S\rangle = |K_1\rangle + \epsilon |K_2\rangle, \]  
\[ |K_L\rangle = |K_2\rangle + \epsilon |K_1\rangle \]  
\[ \epsilon \approx 2 \times 10^{-3} e^{i\pi/4} \]  
\[ \text{(4.a)} \]
\[ \text{(4.b)} \]
\[ \text{(4.c)} \]

where terms of relative higher order in \( \epsilon \) have been dropped (as also done hereafter, throughout). The decay amplitudes \( T_{an} \) are normalized so that the squares of their magnitudes give the relevant partial decay rates; here, \( a \) represents the \( K \) meson state (\( K_1, K_2, K_3 \) or \( K_L \)), and \( n \) stands for the decay channel. The two \( \mu^+\mu^- (\gamma \gamma) \) decay channels are denoted as \( \mu^+ (\gamma) \), where the subscript is the \( CP \) value. We assume that there are no anomalously large effective couplings *) of the states \( 3\pi, \pi\pi\gamma \), or other unknown intermediate states to \( \mu^+\mu^- \), so that the unitarity relations may be written as 5)

\[ \text{Im} \ T_{2\mu^-} = T_{2\gamma^-} \cdot \sqrt{Q_-} \]  
\[ \text{Re} \ T_{1\mu^-} = i T_{1\gamma^-} \cdot \sqrt{Q_-} \]  
\[ \text{Re} \ T_{2\mu^+} = i T_{2\gamma^+} \cdot \sqrt{Q_+} \]  
\[ \text{Im} \ T_{1\mu^+} = \frac{1}{2} \text{Re} \sum_m (T_{1m} T_{m*}) \]  
\[ \text{(5.a)} \]
\[ \text{(5.b)} \]
\[ \text{(5.c)} \]
\[ \text{(5.d)} \]

where the sum may include the \( \pi\pi \) state (including associated photons) as well as \( \gamma \gamma \); here,

*) The normal (two-photon exchange) contribution from \( 3\pi \) states has recently been shown 8) to be very small. If the \( K_L \to \pi\pi\gamma \) decay rate equals its present upper limit, there may be small corrections 9) needed to Eqs. (5.a) and (5.c). The possibility of other unknown intermediate states has been considered 10) as a possible solution of the \( K_L \to \mu^+\mu^- \) puzzle without \( CP \) non-invariance.
\[ \varphi_- \approx 1.2 \times 10^{-5} \]  
\[ \varphi_+ \approx 0.8 \varphi_- \]  
(6.a)  
(6.b)

are the $\gamma \gamma \rightarrow \mu^+ \mu^-$ amplitudes for the values of CP indicated by the subscripts. $\text{Im } \epsilon$ in Eq. (4.c) depends on the Wu-Yang phase convention; Eq. (5.c) depends, in addition, on the apparent experimental equality $\gamma_{\text{co}} = \gamma_{\text{so}}$ in $K_{L} \rightarrow 2 \pi$ decays.

**Class A Models**

The essential feature of these models is one new effective term $H'$ in the Hamiltonian

\[ H' = G' \sin \theta \cdot A_\lambda \cdot \overline{\psi_{\mu}} \gamma_{\lambda} \gamma_5 \psi_{\mu} \]  
(7)

where $A_\lambda$ is the strangeness changing component of the axial current, that is odd with respect to C. The CP violating term (7) contributes to the dispersive amplitude $\text{Im } T_{\mu_-}$, but since this provides a direct transition from $K_1$ to $\mu^+ \mu^-$, one has $\text{Re } T_{\mu_-} = 0$, and

\[ T_{L\mu_-} = i \text{ Im } T_{2\mu_-} + \text{ Re } T_{2\mu_-} + i \epsilon \text{ Im } T_{1\mu_-} \]  
(8.a)

and

\[ T_{S\mu_-} = i \text{ Im } T_{1\mu_-} \]  
(8.b)

where the term $\epsilon T_{2\mu_-}$ has been dropped in comparison with the much larger $K_{L} \rightarrow \mu^- \mu^-$ amplitude in Eq. (8.b).

For the class A, the $K \rightarrow \gamma \gamma$ amplitudes are CP invariant **), so that

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*) There exist many models which contain a term like (7). A general discussion of such models is given in Ref. 11. The application to the present problem has been discussed in Ref. 12) where it is shown that $G'$ must be about 0.01 times the usual weak coupling $G$. The numerical value of $G'$ given in Ref. 12) should be increased by a factor \( \sqrt{2} \). Such a term occurs naturally in the effective Hamiltonian in the Okubo theory [13) of CP violation.

**) Another definition of the class A models could be that only $K \rightarrow \mu^+ \mu^-$ decay amplitudes are allowed to be CP non-invariant.
\[ T_{L+} = e T_{1+} \quad ; \quad T_{S+} = T_{1+} \quad , \tag{9.a} \]
\[ T_{S-} = e T_{2-} \quad ; \quad T_{L-} = T_{2-} \quad , \tag{9.b} \]
\[ T_{1-} = T_{2+} = 0 \quad . \tag{9.c} \]

Using the experimental information
\[ \Gamma(K_S \rightarrow \gamma\gamma) \leq 1.6 \times 10^{-3} \Gamma_S \quad , \tag{10.a} \]
\[ \Gamma(K_L \rightarrow \gamma\gamma) = 5 \times 10^{-4} \Gamma_L \tag{10.b} \]

where \( \Gamma_L \) is the total \( K_L \) decay rate, one finds
\[ \Gamma(K_L \rightarrow \gamma\gamma) \geq |T_{LY-}|^2 \geq 0.99 \Gamma(K_L \rightarrow \gamma\gamma) \]

so that, to a very good approximation,
\[ |T_{2-}| = \sqrt{\Gamma(K_L \rightarrow \gamma\gamma)} \quad . \tag{11} \]

From the imaginary part of Eq. (8.a), and the unitarity relation (5.a), one gets
\[ (\text{Re} e) \sqrt{\Gamma(K_S \rightarrow \mu^+\mu^-)} \geq \sqrt{\Phi \cdot \Gamma(K_L \rightarrow \gamma\gamma) - \Gamma(K_L \rightarrow \mu^+\mu^-)} \tag{12} \]

which gives the bound [Eq. (3)]
\[ \Gamma(K_S \rightarrow \mu^+\mu^-)/\Gamma_S \geq 10 \times 10^{-7} \]
where we have used Eqs. (4.c), (6.a), (10.b) and (11), and the experimental limit
\[ \Gamma(K_L \rightarrow \mu^+\mu^-)/\Gamma_L \leq 1.8 \times 10^{-9} \quad . \tag{13} \]

The model requires a non-vanishing value of \( \text{Re} T_{2\mu^-} \) for the limit (13) to be satisfied. The required value of \( r = \text{Re} T_{2\mu^-} / \text{Im} T_{2\mu^-} \) is about \(-0.45\) for the minimum value (3) of \( \Gamma(K_S \rightarrow \mu^+\mu^-) \). Such a magnitude is not
unreasonable for the dispersive contribution from the intermediate \( \gamma \gamma \) state *).

Other new terms beside that in Eq. (7) may be present in the Hamiltonian with couplings of the order of \( G^4 \) without changing the bound (3). In fact, this bound is independent of any assumption about the decay amplitudes to the CP even state \( \mu_+^\ast \). This bound could be lowered by a factor of two if the absorptive amplitude \( \text{Re} T_{1\mu_+^+\mu_-} \) were not restricted to be zero; however, there seem to be no reasonable models for achieving this.

**Class B Models**

In contrast to the class A models where the \( \text{CP} = -1 \) states were involved, the class B ones involve the \( \text{CP} = +1 \) states, \( \mu_+ \) and \( \gamma_+ \). The essential new terms in the Hamiltonian are: 1) a CP invariant neutral current interaction allowing, in lowest order, the needed large \( K_L - \mu^+ \mu^- \) amplitude, and 2) a CP violating interaction allowing the decay \( K_2 \rightarrow \gamma_+ \); this may be either an electromagnetic \(^{15})\) or a weak-electromagnetic interaction \(^{16})\) **).

The neutral current yields the dispersive amplitude \( \text{Re} T_{1\mu_+^+\mu_-} \) but the absorptive part ***) \( \text{Im} T_{1\mu_+^+\mu_-} = 0 \), except possibly for a small contribution from the \( \gamma \gamma \) state discussed below. Of course, the absorptive part \( \text{Re} T_{2\mu_+^+\mu_-} \) is associated with the \( \gamma \gamma \) intermediate state and is non-zero, Eq. (5.c). On the basis of known interactions, one expects some non-zero CP conserving amplitudes \( T_{2\mu_+^+\mu_-} \) and \( T_{2\gamma_+} \); however, it can be shown \(^1,5)\) that in order to get the lowest bound on \( \text{Im}(K_2 \rightarrow \mu^+ \mu^-) \), one can neglect all amplitudes

*\) The apparent disagreement between the required sign of \( r \) and that in an explicit model calculation \(^3)\) need not be considered serious because of the uncertainty in the form factor for the \( K_2 \rightarrow \gamma \gamma \) vertex. On the other hand, there exist class A models \(^{13}\) for example, that of Okubo \(^{13}\) which allow the CP violating decay amplitude \( K_2 \rightarrow 2\pi \) in an \( I=0 \) state. For such models, the quantity \( \text{Re} T_{2\mu_+^+\mu_-} \) would have, in addition to the contribution from the \( \gamma \gamma \) state, a term \( = \alpha \langle \text{Im} T_{1\mu_+^+\mu_-} \rangle \) where the factor

\[
\alpha = \frac{\langle \pi^+ \pi^-, I=0 | H' | K_2 >}{\langle \pi^+ \pi^-, I=0 | H | K_1 >}
\]

takes into account \(^{14})\) the transformation to the Wu-Yang phase convention.

**) The model discussed by Barshay \(^{17})\) is, effectively, a class B model.

***) For the possibility \( \text{Im} T_{1\mu_+^+\mu_-} \neq 0 \), see the class C models below.
to CP odd final states. In that case, if the branching ratio for $K_S \to \gamma \gamma$ decay is sufficiently small ($<10^{-4}$) so that the contribution of the amplitude $T_L \mu_+ \gamma_+$ to $T_1 \pi_+ \mu_+$ can be ignored, the analysis is similar to that for class A models. The relation

$$\text{Re} T_L \mu_+ = (\text{Re} \epsilon) \cdot \text{Re} T_1 \mu_+ + \text{Re} T_2 \mu_+$$

and Eq. (5.c) leads to the bound (12) with $\varphi_-$ replaced by $\varphi_+$, yielding

$$\frac{\Gamma (K_S \to \mu^+ \mu^-)}{\Gamma_S} \geq 6 \times 10^{-7}.$$  (15)

This result is the Christ–Lee lower bound with our input data.

The bound (15) holds also if the $K_S \to \gamma \gamma$ branching ratio is not negligibly small. Since the $\gamma \gamma$ intermediate state determines the absorptive amplitudes $\text{Re} T_2 \mu_+$ and $\text{Im} T_1 \mu_+$, this bound does not depend on the $K_S \to \gamma \gamma$ decay rate.

Class C Models

To obtain a still lower bound, we continue to consider the CP = +1 final states, but now allow the amplitude $T_1 \mu_+$ to have a large absorptive part. Since the main product of $K_1$ decay is the $\pi \pi$ state, the reasonable way to achieve this is to replace assumption 1) of the class B models by the introduction of a direct CP conserving $\pi \pi \to \mu \mu$ interaction with a coupling of the order of electromagnetism — much stronger than that provided by a reasonable calculation of the $2 \gamma$ exchange contribution. The sequence $K_1 \to \pi^+ \pi^+ \pi^- \to \mu^+ \mu^+ \mu^-$, then, provides the amplitudes $\text{Re} T_2 \mu_+$ and $\text{Im} T_1 \mu_+$; Barishay has given an explicit discussion of such an interaction.

If the $K_S \to \gamma \gamma$ branching ratio is very small ($<10^{-4}$) and we assume $\gamma_+ = \gamma_0$ for $K_L \to \pi \pi$ decay [as done in Eq. (5.c)], the introduction of a non-vanishing $\text{Im} T_1 \mu_+$ gives

$$|\epsilon| \sqrt{\frac{\Gamma (K_S \to \mu^+ \mu^-)}{\Gamma_S}} \geq \sqrt{\varphi_+ \cdot \Gamma (K_L \to \gamma \gamma)} - \sqrt{\Gamma (K_L \to \mu^+ \mu^-)}$$

which lowers the previous limit by a factor of two:

$$\frac{\Gamma (K_S \to \mu^+ \mu^-)}{\Gamma_S} \geq 3 \times 10^{-7}.$$  (17)
It seems strange that the result (16) involves \(|\epsilon|\), a quantity having a value depending on the \(K^\pm\bar{K}\) relative phase, rather than \(\text{Re}\epsilon\), as in (12). However, in the present case, because of the essential role of the \(\pi\pi\) intermediate state, \(|\epsilon|\) is correctly evaluated as \(|\eta_{++}| = |\eta_{oo}|\). The bound (17) can be lowered to some extent if one allows \(^+\) a small non-zero \((\eta_{++} - \eta_{oo})\).

If the \(K_S \to \gamma\gamma\) branching ratio is higher, the bound (17) gets lowered. If the present limit (10.a) is saturated, one gets, assuming \(\text{Im}T_{1\gamma} = 0\),

\[
\Gamma(K_s \to \mu^+\mu^-)/\Gamma_S \geq 2 \times 10^{-7}.
\]

Gaillard \(^5\) finds a slightly lower value (2) by allowing a large absorptive part in \(T_{1\gamma}\), which seems difficult to obtain dynamically. The points is that this absorptive part should arise predominantly from the \(\pi\pi\) intermediate state, and any reasonable estimate \(^4\) of this gives a contribution to the rate much smaller than the limit (10.a).

In conclusion, the Table shows a summary of our results. In using our numerical results, or similar ones of others \(^1\), \(^5\), it should be remembered that there may be some \(^**\) correction due to the \(\pi\pi\gamma\) intermediate state, and that no attempt has been made to include the experimental errors of the input data. We consider model A as the only "economical" realization of the Christ–Lee mechanism. It should be noted that model C (needed to go from the Christ–Lee number to that of Gaillard) requires an anomalous muon–hadron interaction. As emphasized by Barshay \(^18\), such anomalous interactions can be constructed so as to solve the \(K_L \to \mu^+\mu^-\) puzzle without \(\text{CP}\) violation.

\(^*)\) Present data \(^19\) allow such a small number: \(|\eta_{oo}/\eta_{++}| = 1.00 \pm 0.06\), \(\delta_{oo} \approx \delta_{++} \pm 20^\circ\). There is, of course, no reason to rule out \(\eta_{++} \neq \eta_{oo}\) in these models since there is an electromagnetic origin of \(\text{CP}\) violation.

\(^**)\) The estimate of a 11\% correction \(^9\) is somewhat too high \(^20\).
<table>
<thead>
<tr>
<th>Class of Model</th>
<th>Essential Channel</th>
<th>New Interaction</th>
<th>New Amplitude</th>
<th>Assumed Lower Bound on Branching Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$^1S_0$</td>
<td>centiweak neutral current</td>
<td>none</td>
<td>$\text{Im} \ T_{\mu^-}$</td>
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<tr>
<td>B</td>
<td>$^3P_0$</td>
<td>centiweak neutral current</td>
<td>electromagnetic or weak electromagnetic</td>
<td>$\text{Re} \ T_{\mu^+}$ and $\text{Re} \ T_{\mu^+}$</td>
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<tr>
<td>C</td>
<td>$^3P_0$</td>
<td>$\pi \pi - \mu \bar{\mu}$ of electromagnetic strength</td>
<td>$\text{Re} \ T_{\mu^+}$ and $\text{Im} \ T_{\mu^+}$</td>
<td>$\text{negligible (} &lt; 10^{-4}$</td>
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</tbody>
</table>

**TABLE**: Summary of various classes of models and the lower bounds on the $K_S \rightarrow \mu^+\mu^-$ branching ratio in those models.
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