Interplay between $H \rightarrow b\bar{s}$ and $b \rightarrow s\gamma$
in the MSSM with Non-Minimal Flavour Violation

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Abstract

We investigate the constraints on flavour-changing neutral heavy Higgs-boson decays $H \rightarrow b\bar{s}$ from $b \rightarrow s\gamma$ bounds on the flavour-mixing parameters of the MSSM with non-minimal flavour violation (NMFV). In our analysis we include the contributions from the SM and new physics due to general flavour mixing in the squark mass matrices. We study the case of one and two non-zero flavour-mixing parameters and find that in the latter case the interference can raise the Higgs flavour-changing branching ratios by one or two orders of magnitude with respect to previous predictions based on a single non-zero parameter and in agreement with present constraints from $B$ physics. In the course of our work we developed a new FeynArts model file for the NMFV MSSM and added the necessary code for the evaluation to FormCalc. Both extensions are publicly available.

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1 Introduction

Flavour-changing neutral current (FCNC) processes provide an extraordinarily useful tool to investigate new physics beyond the Standard Model (SM). In supersymmetry (SUSY), as in other models beyond the SM, an alternative to the direct search for new particles is to look for their radiative effects. In the SM, FCNCs are absent at tree level, but they arise at one-loop order, being strongly suppressed as a consequence of the GIM mechanism [1]. The Minimal Supersymmetric Standard Model (MSSM) [2] provides a natural framework where FCNCs are enhanced. In the scenario with minimal flavour violation (MFV) of the MSSM, squarks are assumed to be aligned with the corresponding quarks, and flavour violation originates from the Cabibbo–Kobayashi–Maskawa (CKM) matrix as the only source, proceeding via loop contributions, in analogy to the SM. Therefore, its size is expected to be very small. In more general scenarios which include misalignment between the quark and squark sectors, sizeable contributions to FCNC processes are expected to occur. This is the case of the MSSM with non-minimal flavour violation (NMFV).

The quantum contributions from supersymmetric particles in $B$-meson physics have been studied in a series of processes, including $B^0 - ar{B}^0$ mixing [3, 4], leptonic $B$-meson decays [5–8] and $B \rightarrow X_s \gamma$ [9–13], and have been found to be large. In order not to be in conflict with present experimental data, these in turn imply restrictions both on the SUSY parameters and on the parameters measuring the size of flavour mixing in the squark sector [8,13–15]. The rare decay $B \rightarrow X_s \gamma$ is at present one of the most important since its observation [16] sets stringent constraints on the parameter space of various extensions of the SM [12–14]. Furthermore, the recent data from $B \rightarrow X_s \mu^+ \mu^-$ indicate that the sign of the $b \rightarrow s \gamma$ amplitude is the same as in the SM [17].

Moreover, it is well known that FCNC processes related to Higgs-boson physics are also very sensitive to supersymmetric quantum effects [5–8, 18–23]. In particular, large rates of neutral Higgs decays into two quarks of different flavours have been predicted [18–20]. In the SM, one finds $B(H_{\text{SM}} \rightarrow bs) \approx 4 \times 10^{-8}$ for $m_{H_{\text{SM}}} = 114$ GeV. For the neutral MSSM Higgs bosons the ratios could be of $O(10^{-4}–10^{-3})$. Constraints from $b \rightarrow s \gamma$ data reduce these rates, though [19, 20].

In this paper we provide a phenomenological analysis of the general constraints on flavour-changing neutral Higgs decays $H \rightarrow b\bar{s}, s\bar{b}$, set by bounds from $b \rightarrow s \gamma$ on the flavour-mixing parameters in the squark mass matrices of the MSSM with NMFV. We include the SM and the full genuine SUSY contributions by taking into account their interference and, in particular, the influence of several flavour-changing parameters contributing simultaneously. Since the full diagrammatic approach is used, our computation is valid for all values of the characteristic parameter measuring the squark-mixing strength, beyond the mass-insertion approximation, and for all values of $\tan\beta$. Previous analyses of bounds on SUSY flavour-mixing parameters from $b \rightarrow s \gamma$ [13] have shown the importance of the interference effects between the different types of flavour violation [14]. In the present work we derive predictions for $B(H \rightarrow bs)$ compatible with present experimental $b \rightarrow s \gamma$ bounds and recent data from $B \rightarrow X_s \mu^+ \mu^-$, assuming first one and then several types of flavour mixing contributing at a time for comparison.

The paper is organized as follows. In Section 2 the squark mass matrices in the MSSM

\footnote{In the following, $B(H \rightarrow bs)$ denotes the sum of the Higgs branching ratios into $b\bar{s}$ and $\bar{b}s$. The Higgs boson $H$ stands for that of the SM, $H_{\text{SM}}$, or one of those of the MSSM, $h^0$, $H^0$, or $A^0$.}
with NMfv are described in detail and the notation is introduced. The numerical analysis
of the branching ratios $B(H \rightarrow bs)$ compatible with the observed decay rates for $B \rightarrow X_s\gamma$
and $B \rightarrow X_s\mu^+\mu^-$ is included in Section 3. In that section, we consider only one specific
flavour-mixing parameter different from zero and discuss the interference effects of the SM
and the new physics contributions. The interplay between $H \rightarrow bs$ and $b \rightarrow s \gamma$ is presented
in Section 4. The constraints on flavour-changing neutral heavy Higgs-boson decays due to
different types of flavour violation are derived there. Conclusions are given in Section 5. The
relevant Feynman rules are listed in Appendix A.

2 Non-minimal Flavour Mixing in the MSSM

In the MSSM there are two sources of flavour violation. The first one arises from different
rotations of the $d$- and $u$-quark fields from the interaction to the physical bases, and its
strength is driven by the off-diagonal CKM-matrix elements, as in the SM. The second one
consists of a possible misalignment between the rotations that diagonalize the quark and
squark sectors (NMFV). The part of the soft-SUSY-breaking Lagrangian responsible for this
non-minimal squark family mixing is given by

$$L_{\text{squark}}^{\text{soft}} = -\bar{Q}_i^j(M_2^Q)_{ij} \tilde{Q}_j - \bar{U}_i^j(M_2^U)_{ij} \tilde{U}_j - \bar{D}_i^j(M_2^D)_{ij} \tilde{D}_j + y^u_t \tilde{Q}_i H_u \tilde{U}_j + y^d_t \tilde{Q}_i H_d \tilde{D}_j,$$

where $\tilde{Q}$ is the SU(2) scalar doublet, $\tilde{U}$, $\tilde{D}$ are the up- and down-squark SU(2) singlets,
respectively, $y^u,d$ are the Yukawa couplings and $i,j$ are generation indices. The flavour-
changing effects come from the non-diagonal entries in the bilinear terms $M_2^Q$, $M_2^U$, and $M_2^D$,
and from the trilinear terms $A_u$ and $A_d$.

We assume that the non-CKM squark mixing is significant only for transitions between
the squarks of the second and third generations. They are expected to be the largest in Grand
Unified Models and are also experimentally the least constrained. The most stringent bounds
are set by $b \rightarrow s \gamma$. In contrast, there exist strong experimental bounds involving the first
squark generation, based on data from $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixing [14,15].

Our parameterization of the flavour-non-diagonal squark mass matrices for the up- and
down-type squarks, for the MSSM with real parameters, reads as follows,

$$M^2_u = \begin{pmatrix}
M^2_{L,u} & 0 & 0 \\
0 & M^2_{L,c} & \Delta^u_{LL} \\
0 & \Delta^d_{LL} & M^2_{L,b}
\end{pmatrix}
\begin{pmatrix}
m_u X_u & 0 & 0 \\
m_u X_c & 0 & \Delta^u_{LR} \\
0 & \Delta^d_{RL} & m_t X_t
\end{pmatrix},$$

$$M^2_d = \begin{pmatrix}
M^2_{L,d} & 0 & 0 \\
0 & M^2_{L,s} & \Delta^d_{LL} \\
0 & \Delta^u_{LL} & M^2_{L,b}
\end{pmatrix}
\begin{pmatrix}
m_d X_d & 0 & 0 \\
m_d X_s & 0 & \Delta^d_{LR} \\
0 & \Delta^u_{RL} & m_b X_b
\end{pmatrix},$$

where $\Delta^u,d_{LL} = M^2_{L,u,d} - M^2_{L,c,s}$, and $\Delta^u,d_{LR} = M^2_{L,u,d} - M^2_{L,c,s}$. The
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down-type squarks, for the MSSM with real parameters, reads as follows,
where

\[
\begin{align*}
M_{L,q}^2 &= M_{Q,q}^2 + m_q^2 + \cos 2\beta (T_3^q - Q_q s_W^2) m_Z^2, \\
M_{R,\{u,c\}}^2 &= M_{U,\{u,c\}}^2 + m_{u,c}^2 + \cos 2\beta Q_t s_W m_W^2, \\
M_{R,\{d,s,b\}}^2 &= M_{D,\{d,s,b\}}^2 + m_{d,s,b}^2 + \cos 2\beta Q_b s_W m_Z^2, \\
X_{u,c,t} &= A_{u,c,t} - \mu \cot \beta, \\
X_{d,s,b} &= A_{d,s,b} - \mu \tan \beta,
\end{align*}
\]

with \( m_q, T_3^q, Q_q \) the mass, isospin, and electric charge of the quark \( q \), \( m_Z \) the Z-boson mass, \( s_W \equiv \sin \theta_W, \theta_W \) the electroweak mixing angle, and \( \mu \) the Higgsino mass parameter.

We define the dimensionless flavour-changing parameters \((\delta_{ab}^{u,d})_{23} (ab = LL, LR, RL, RR)\) from the flavour-off-diagonal elements of the squark mass matrices \((2.2)\) and \((2.3)\) in the following way,

\[
\begin{align*}
\Delta_L^{u} &\equiv (\delta_{LL}^{u})_{23} M_{L,c} M_{L,\bar{t}}, & \Delta_L^{d} &\equiv (\delta_{LL}^{d})_{23} M_{L,s} M_{L,b}, \\
\Delta_{LR}^{u} &\equiv (\delta_{LR}^{u})_{23} M_{L,c} M_{R,\bar{t}}, & \Delta_{LR}^{d} &\equiv (\delta_{LR}^{d})_{23} M_{L,s} M_{R,b}, \\
\Delta_{RL}^{u} &\equiv (\delta_{RL}^{u})_{23} M_{R,c} M_{L,\bar{t}}, & \Delta_{RL}^{d} &\equiv (\delta_{RL}^{d})_{23} M_{R,s} M_{L,b}, \\
\Delta_{RR}^{u} &\equiv (\delta_{RR}^{u})_{23} M_{R,c} M_{R,\bar{t}}, & \Delta_{RR}^{d} &\equiv (\delta_{RR}^{d})_{23} M_{R,s} M_{R,b}.
\end{align*}
\] (2.5)

In our phenomenological study, they are free parameters determining the size of NMFV induced by SUSY and are analogous to those defined in the mass-insertion approximation \cite{14}.

In order to diagonalize the two 6 × 6 squark-mass matrices given above, the 6 × 6 matrices \( R^\alpha \) for the up-type squarks and \( R^\alpha \) for the down-type squarks are needed,

\[
\begin{align*}
\tilde{u}_\sigma = R_{\sigma,j}^{\tilde{u}} \begin{pmatrix} \tilde{u}_L \\ \tilde{c}_L \\ \tilde{l}_L \\ \tilde{t}_R \\ \tilde{c}_R \\ \tilde{t}_R \end{pmatrix}_j, & \quad \tilde{d}_\sigma = R_{\sigma,j}^{\tilde{d}} \begin{pmatrix} \tilde{d}_L \\ \tilde{s}_L \\ \tilde{s}_L \\ \tilde{b}_L \\ \tilde{b}_R \\ \tilde{b}_R \end{pmatrix}_j.
\end{align*}
\] (2.6)

The diagonalization yields the squark mass eigenvalues and eigenstates depending on the flavour-mixing parameters \((\delta_{ab})_{23}\), i.e. \( R^\alpha M_\alpha^{23} R^{\alpha\dagger} = \text{diag}(M_{1\alpha}^2, \ldots, M_{6\alpha}^2) \). The dependence of the squark masses on \((\delta_{LL})_{23}\) has already been studied in \cite{18}. Typically, out of the four eigenvalues involving the second and third generations, two are weakly dependent on the amount of flavour mixing and for the other two, one grows and the other decreases with \((\delta_{LL})_{23}\). In general, flavour mixing through the flavour non-diagonal entries in the squark mass matrices generates large splittings between the squark mass eigenvalues.

### 3 Flavour-changing decay processes

We focus here on the loop-induced flavour-changing decays of the MSSM heavy neutral Higgs bosons \( H = H^0, A^0 \) into second- and third-generation quarks, \( H \rightarrow b\bar{s}, s\bar{b} \) (an independent analysis for the lightest Higgs boson \( h^0 \) will be reported elsewhere \cite{24}). We include the contributions from SM particles and their superpartners (squarks, gluinos, charginos, and neutralinos), as well as those from the MSSM Higgs sector, and also their interference effects.
We use the full diagrammatic approach for arbitrary values of \( \tan \beta \) and of the flavour-mixing parameters \((\delta_{ab}^{u,d})_{23}\). Note that \( b \to s\gamma \) constrains \((\delta_{ab}^{d})_{23}\) only. For simplicity, we take the same values for the flavour-mixing parameters in the up- and down-squark sectors: \((\delta_{ab}^{s})_{23} \equiv (\delta_{ab}^{u})_{23}\). Actually, the LL blocks of the up- and down-squark mass matrices are related by the SU(2)\(_L\) gauge symmetry [14], therefore a large difference between \((\delta_{LL}^{u})_{23}\) and \((\delta_{LL}^{d})_{23}\) is not allowed. For the same reason, \( M_{\tilde{Q},u} \approx M_{\tilde{Q},d}, M_{\tilde{Q},c} \approx M_{\tilde{Q},s}, \) and \( M_{\tilde{Q},t} \approx M_{\tilde{Q},b}\).

We have taken the expression for the branching ratio \( B(\mathcal{B} \to X_s\gamma) \) to NLO from [25]. In the MSSM with NMFV, the relevant operators of the effective Hamiltonian are

\[
O_2 = \bar{s}_L \gamma \mu c \gamma^\mu b_L, \\
O_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R, \\
O_8 = \frac{g_s}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} G_{a}^{\mu\nu} t a b_R, \\
\hat{O}_7 = \frac{e}{16\pi^2} m_b \bar{s}_R \sigma_{\mu\nu} F^{\mu\nu} b_L, \\
\hat{O}_8 = \frac{g_s}{16\pi^2} m_b \bar{s}_R \sigma_{\mu\nu} G_{a}^{\mu\nu} t a b_L.
\]  

(3.1)

We have calculated the corresponding Wilson coefficients \( C_{2,7,8} \) and \( \hat{C}_{7,8} \) to one loop. The tilded operators do not contribute in the SM or in the MSSM with MFV, in the limit of massless strange quark. The data from \( B \to X_s\mu^+\mu^- \) require that the sign of the coefficient \( C_7(m_b) \) is the same as in the SM. We used FeynArts, FormCalc, and LoopTools to obtain our results. To this end, we had to modify the MSSM model file of FeynArts to include general flavour mixing, and we added 6 \( \times \) 6 squark mass and mixing matrices to the FormCalc evaluation [26, 27]. A list of the Feynman rules needed for our computation is given in Appendix A. The masses and total decay widths of the Higgs bosons were computed with FeynHiggs [28].

For a concrete evaluation, we choose the following six flavour-diagonal MSSM parameters as input parameters: \( m_A, \tan \beta, \mu, M_2, M_{\text{SUSY}}, A \), where \( m_A \) denotes the mass of the CP-odd Higgs boson \( A^0 \). For simplicity, we have taken a common value for the soft SUSY-breaking squark mass parameters \( M_{\text{SUSY}} \equiv \{ M_{\tilde{u},\{u,t,c\}} = M_{\tilde{d},\{d,b,s\}} \} \), and all the various trilinear parameters to be universal, \( A \equiv A_t = A_b = A_c = A_s \). These parameters and the \( \delta \)'s will be varied over a wide range, subject only to the requirements that all the squark masses be heavier than 100 GeV, \( |\mu| > 90 \) GeV and \( M_2 > 46 \) GeV [29].

The following MSSM parameters have been chosen as a default set (if not specified differently),

\[
M_{\text{SUSY}} = 800 \text{ GeV}, \quad M_2 = 300 \text{ GeV}, \quad M_1 = \frac{5 s_W^2}{3 c_W^2} M_2, \\
A = 500 \text{ GeV}, \quad m_A = 400 \text{ GeV}, \quad \tan \beta = 35, \quad \mu = -700 \text{ GeV}.
\]  

(3.2)

A detailed study of the values of the Higgs-boson decay widths in a wider range of these parameters and for \((\delta_{LL}^{\mu})_{23} \neq 0\) has been done in [19].

Let us remark that the \( \mu \)-parameter plays an important role in our analysis. The \( H^0 \) decay width is approximately symmetric under \( \mu \to -\mu \), depending on the \( m_A \) values, and increases with \( \mu \) up to a certain value around 600 GeV; then it reaches a maximum value, and finally decreases [18, 19]. Moreover, the \( \mu \)-parameter enters in the gluino and chargino contributions to \( b \to s\gamma \), producing significant cancellations at large \( \tan \beta \).

For illustration, we give in this section an overview over the various contributions to both \( H^0 \to bs \) and \( B \to X_s\gamma \), keeping only a single flavour-off-diagonal element, \( (\delta_{LL}^{\mu})_{23} \). For the Higgs decays, we agree with the results of previous studies on the subject [18–20].
Figure 1: $B(H^0 \rightarrow bs)$ [left] and $B(B \rightarrow X_s\gamma)$ [right] as a function of $(\delta_{LL})_{23}$. In the upper row $M_{\tilde{g}}$ is determined from $M_2$ through the GUT relation, in the lower row $M_{\tilde{g}} = 300$ GeV is chosen. The other input parameters are given in Eq. (3.2). The SM and the MSSM Higgs sector contributions (sH) are included (horizontal lines in $B(B \rightarrow X_s\gamma)$ [right], but invisible in $B(H^0 \rightarrow bs)$ [left]).
Fig. 1 shows on the left $\mathcal{B}(H^0 \rightarrow bs)$ and on the right $\mathcal{B}(B \rightarrow X_s\gamma)$ as a function of $(\delta_{LL})_{23}$. In the upper row the GUT relation $M_3 = \alpha_s/(\alpha s_W^2)M_2$ is assumed, yielding the rather large gluino mass of $M_{\tilde{g}} \approx 1$ TeV. In the lower row $M_{\tilde{g}} = 300$ GeV is chosen.

Our purpose is to show the size of the radiative corrections separately for the different sectors and their interference effects. The SUSY electroweak (SUSY-EW) contribution includes charginos, neutralinos, and the MSSM Higgs sector, the latter being negligible in $H^0 \rightarrow bs$ but important in $b \rightarrow s\gamma$ (horizontal lines in $\mathcal{B}(b \rightarrow s\gamma)$ because they do not depend on $(\delta_{LL})_{23}$). The chargino and neutralino contributions are plotted independently to show that in both cases the charginos dominate. Gluino–squark loops constitute the SUSY-QCD contribution.

$\mathcal{B}(H^0 \rightarrow bs)$ increases with $(\delta_{LL})_{23}$, as already known [18–20]. We can see that the SUSY-QCD contribution is dominant, at least one order of magnitude larger than that of SUSY-EW. Both interfere with opposite signs, as discussed in [19]. The SM value for the Higgs branching ratio is several orders of magnitude smaller than the corresponding MSSM value, of $\mathcal{O}(10^{-8})$.

In $b \rightarrow s\gamma$, the SUSY-QCD contribution is also dominant in most of the flavour parameter space, but becomes subdominant for large gluino masses (see the upper row in Fig. 1). This result, however, strongly depends on the choice of parameters and changes if the GUT relation is relaxed. The interference effects of the various MSSM sectors in $b \rightarrow s\gamma$ must therefore be carefully considered.

## 4 Compatibility between $H^0 \rightarrow bs$ and $b \rightarrow s\gamma$

Next we derive the maximum values of $\mathcal{B}(H^0 \rightarrow bs)$ compatible with $\mathcal{B}(B \rightarrow X_s\gamma)_{\text{exp}} = (3.3 \pm 0.4) \times 10^{-4}$ [16] within three standard deviations by varying the flavour-changing parameters of the squark mass matrices. The results for the $A^0$ boson are very similar and we do not show them separately.

### 4.1 One flavour-mixing parameter

As a first step, we select one possible type of flavour violation in the squark sector, assuming that all the others vanish. The interference between different types of flavour mixing is thus ignored. Previous analyses on bounds coming from $b \rightarrow s\gamma$, by using the mass-insertion approximation and by neglecting any kind of interference effects, led to bounds on the single off-diagonal element for the down sector of $|\langle \delta_{LL,RR} \rangle_{23}| < \mathcal{O}(1)$, $|\langle \delta_{LR,RL} \rangle_{23}| < \mathcal{O}(10^{-2})$ (see [13, 14] and references therein). The MSSM inputs are those in Eq. (3.2) with GUT relations.

In Fig. 2 we show $\mathcal{B}(B \rightarrow X_s\gamma)$ as a function of the flavour parameters $(\delta_{ab})_{23}$ with $ab = LL, LR, RL, RR$. The results are expressed in standard deviations (s.d.) through

$$\frac{\Delta \mathcal{B}(B \rightarrow X_s\gamma)}{1 \text{ s.d.}} = \frac{\mathcal{B}(B \rightarrow X_s\gamma) - \mathcal{B}(B \rightarrow X_s\gamma)_{\text{exp}}}{\Delta \mathcal{B}_{\text{exp}}}.$$ (4.1)

We can see in Fig. 2 that the flavour-off-diagonal elements are independently constrained to be at most $(\delta_{ab})_{23} \sim 10^{-3}–10^{-1}$. As expected [13, 14], the bounds on $(\delta_{LR})_{23}$ are the strongest, $(\delta_{LR})_{23} \sim 10^{-3}–10^{-2}$. The data from $B \rightarrow X_s\mu^+\mu^-$ further constrain the parameters $(\delta_{LL})_{23}$ and $(\delta_{LR})_{23}$, the others remaining untouched.

The allowed intervals for the corresponding flavour-mixing parameters thus obtained are indicated in Fig. 3, in which we present the results for $\mathcal{B}(H^0 \rightarrow bs)$ as a function of $(\delta_{ab})_{23}$. The predictions compatible with the $b \rightarrow s\gamma$ constraints can be read off directly from there.
4.2 Two flavour-mixing parameters

In this second part of our analysis, we investigate whether the maximum values reachable by the $H^0$ branching ratios remain stable when several off-diagonal elements of the squark mass matrix contribute simultaneously. It is known that the participation of several types of flavour-changing parameters weaken the bounds, imposed by $b \to s\gamma$, on the off-diagonal elements of the squark-mass matrix by at least one order of magnitude [13]. We derive bounds on these flavour-mixing parameters by switching on simultaneously two of those parameters, with all the others vanishing. Indeed, we performed the analysis for all possible combinations of two of the four dimensionless parameters (2.5).

Fig. 4 displays the results for our parameter set (3.2). Drawn are contours of constant $\Gamma(H^0 \to bs) \equiv \Gamma(H^0 \to b\bar{s}) + \Gamma(H^0 \to s\bar{b})$ for various combinations $(\delta_{ab})_{23}-(\delta_{cd})_{23}$ of flavour-mixing parameters, which we shall refer to as “ab–cd planes” for short in the following. The coloured bands represent regions experimentally allowed by $B \to X_s\mu^+\mu^-$. The red bands are regions disfavoured by $B \to X_s\mu^+\mu^-$. For our reference point (3.2) we find that the largest allowed value of $B(H^0 \to bs)$, of $\mathcal{O}(10^{-3})$ or $\mathcal{O}(10^{-5})$, is induced by $(\delta_{RR})_{23}$ or $(\delta_{LL})_{23}$, respectively. These are the flavour-changing parameters least stringently constrained by the $b \to s\gamma$ data. $B(H^0 \to bs)$ can reach $\mathcal{O}(10^{-6})$ if induced by $(\delta_{LR})_{23}$ or by $(\delta_{RL})_{23}$, the most stringently constrained flavour-changing parameter. We remark that, because of the restrictions imposed by $b \to s\gamma$, $B(H^0 \to bs)$ depends very little on $(\delta_{LR})_{23}$ and $(\delta_{RL})_{23}$.
Figure 3: $\mathcal{B}(H^0 \to bs)$ as a function of $(\delta_{LL,RR,LR,RL})_{23}$. The allowed intervals of these parameters determined from $b \to s\gamma$ (see Fig. 2) are indicated by coloured areas. The red areas (b) are disfavoured by $B \to X_s\mu^+\mu^-$. 
Figure 4: Contours of constant $\Gamma(H^0 \rightarrow bs)$ in various planes of the flavour-mixing parameters $(\delta_{ab})_{23}$. The coloured bands indicate regions experimentally allowed by $B \rightarrow X_s \gamma$. The red bands show regions disfavoured by $B \rightarrow X_s \mu^+ \mu^-$. 
Contributions in the LR–RL and LR–RR planes lead to maximal values

\[ \Gamma(H^0 \to bs)_{\text{max}} = 3 \times 10^{-4} \text{ GeV} \quad \text{for } (\delta_{LR})_{23} = 0.04, \quad (\delta_{RL})_{23} = 0.04, \quad (4.2) \]
\[ \Gamma(H^0 \to bs)_{\text{max}} = 0.15 \text{ GeV} \quad \text{for } (\delta_{LR})_{23} = 0.035, \quad (\delta_{RR})_{23} = \pm 0.7. \quad (4.3) \]

This translates to branching ratios compatible with experimental data of \( B(H^0 \to bs)_{\text{max}} \sim 10^{-5} \) in the first, and \( O(10^{-3}) \) in the second case. Here we have used the total width of \( \Gamma(H \to X) \approx 26 \text{ GeV}, H = H^0, A^0 \), for the point \( \mathbf{B2} \) in the MSSM with MFV.

The bounds on \( (\delta_{LR})_{23} \), the best constrained for only one non-zero flavour-off-diagonal element, are dramatically relaxed when other flavour-changing parameters contribute simultaneously. Values of \( (\delta_{LR})_{23} \sim 10^{-1} \) are allowed. In particular, large although fine-tuned values of \( (\delta_{LL})_{23} \) and \( (\delta_{LR})_{23} \) combined are not excluded by \( b \to s\gamma \), yielding e.g.

\[ \Gamma(H^0 \to bs)_{\text{max}} = 0.25 \text{ GeV} \quad \text{for } (\delta_{LR})_{23} = -0.22, \quad (\delta_{LL})_{23} = -0.8, \quad (4.4) \]
\[ \Gamma(H^0 \to bs)_{\text{max}} = 0.35 \text{ GeV} \quad \text{for } (\delta_{LR})_{23} = \pm 0.04, \quad (\delta_{RR})_{23} = -0.15. \quad (4.5) \]

The branching ratio is \( B(H^0 \to bs)_{\text{max}} \sim 10^{-2} \) in both cases.

Finally, we studied the combined effects of RL–LL and RR–LL. We obtain

\[ \Gamma(H^0 \to bs)_{\text{max}} = 2.5 \times 10^{-3} \text{ GeV} \quad \text{for } (\delta_{RL})_{23} = 0.006, \quad (\delta_{LL})_{23} = -0.128, \quad (4.6) \]
\[ \Gamma(H^0 \to bs)_{\text{max}} = 0.12 \text{ GeV} \quad \text{for } (\delta_{RR})_{23} = 0.65, \quad (\delta_{LL})_{23} = \pm 0.14. \quad (4.7) \]

This means \( B(H^0 \to bs)_{\text{max}} \sim 10^{-4} \) for the RL–LL and \( B(H^0 \to bs)_{\text{max}} \sim 10^{-2} \) for the RR–LL case.

Therefore, we conclude that the predictions on \( B(H^0 \to bs) \) induced by \( (\delta_{RR})_{23} \) or \( (\delta_{LL})_{23} \) only, of \( O(10^{-3}) \) or \( O(10^{-5}) \), are greatly exceeded by the combination of \( (\delta_{LL})_{23} \) and \( (\delta_{LR})_{23} \) or \( (\delta_{RR})_{23} \) and \( (\delta_{RL})_{23} \) or \( (\delta_{LL})_{23} \) and \( (\delta_{RR})_{23} \), which are of \( O(10^{-2}) \). Values of \( B(H^0 \to bs) \sim O(10^{-3}) \) emerge when considering the LR–RR. Thus, the destructive interference of the combined set of flavour-mixing parameters leads to large allowed values for the \( H^0 \to bs \) branching ratio, which in general can be two orders of magnitude larger than if induced by just one flavour-mixing parameter.

5 Conclusions

In this work we carried out a phenomenological analysis of the predictions for flavour-changing neutral heavy Higgs decays \( H \to b\bar{s}, s\bar{b} \), taking into account the constraints from \( b \to s\gamma \) on the flavour-mixing parameters \( (\delta_{LL,LR,RL,RR})_{23} \) of the MSSM with NMFV. We followed a fully diagrammatic approach at one loop, valid for arbitrary values of \( \tan \beta \) and the \( \delta \)’s, as long as they are kinematically allowed. The calculations were performed with \textit{FeynArts}, \textit{FormCalc}, and \textit{LoopTools}, which were extended to include the MSSM with NMFV in a new model file and the corresponding \( 6 \times 6 \) diagonalization, both of which are now available in the public distributions of \textit{FeynArts} and \textit{FormCalc}. The contributions and interfering effects of the SM and the various sectors of the MSSM were fully accounted for and the interplay between the different types of non-minimal flavour mixing explored, both individually and in combinations.

We found that \( B(H \to bs) \), whose value is at most of \( O(10^{-6}) \) when induced by \( (\delta_{LR})_{23} \) or \( (\delta_{RR})_{23} \), can be of \( O(10^{-5}) \) or \( O(10^{-3}) \) if induced by \( (\delta_{LR})_{23} \) or \( (\delta_{LL})_{23} \). By imposing the constraints from \( b \to s\gamma \) over the flavour-changing parameters, the \( H \to bs \) branching ratio
depends only weakly on $(\delta_{RL})_{23}$ and $(\delta_{LR})_{23}$. The combined effect of two of the flavour-mixing parameters may raise the predictions for $\mathcal{B}(H \rightarrow bs)$ by one or two orders of magnitude, typically $\mathcal{O}(10^{-3} - 10^{-2})$.

Overall, the predictions for $H \rightarrow bs$ are more optimistic when the bounds from $b \rightarrow s\gamma$ are accounted for in the more realistic case of several flavour-mixing parameters contributing simultaneously.

Acknowledgements

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A NMFV Feynman Rules

The following table lists Feynman rules needed for our computation in the MSSM with NMFV. These vertices were generated automatically from the new FeynArts model file FVMSSM.mod, available from www.feynarts.de.

The upper and lower entries of the FFS couplings are the prefactors of the left- and right-handed chirality projectors. The matrices $T^g_{ab}$ are the SU(3) generators with gluon index $g$ and colour indices $a,b$. The matrices $U, V (Z)$ are the chargino (neutralino) mixing matrices and CKM is the Cabibbo–Kobayashi–Maskawa quark mixing matrix, with $i,j$ the fermion and $\rho, \sigma$ the squark generation indices. The angle $\alpha$ is the mixing angle in the neutral Higgs sector.

<table>
<thead>
<tr>
<th>[FFS] Gluino – Quark – Squark</th>
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<tbody>
<tr>
<td>$C(\tilde{g}, \tilde{u}, \tilde{u}) = \begin{bmatrix} \sqrt{2} i g_s R^{\tilde{u}}<em>{\sigma,i} T^g</em>{\tilde{u},a} \ -\sqrt{2} i g_s R^{\tilde{u}}<em>{\sigma,i} T^g</em>{\tilde{u},b} \end{bmatrix}$</td>
</tr>
<tr>
<td>$C(\tilde{g}, \tilde{d}, \tilde{d}) = \begin{bmatrix} \sqrt{2} i g_s R^{\tilde{d}}<em>{\sigma,i} T^g</em>{\tilde{d},a} \ -\sqrt{2} i g_s R^{\tilde{d}}<em>{\sigma,i} T^g</em>{\tilde{d},b} \end{bmatrix}$</td>
</tr>
<tr>
<td>$C(\tilde{g}, \tilde{u}, \tilde{u}^\dagger) = \begin{bmatrix} \sqrt{2} i g_s R^{\tilde{u}}<em>{\sigma,i} T^g</em>{\tilde{u},a} \ \sqrt{2} i g_s R^{\tilde{u}}<em>{\sigma,i} T^g</em>{\tilde{u},b} \end{bmatrix}$</td>
</tr>
<tr>
<td>$C(\tilde{g}, \tilde{d}, \tilde{d}^\dagger) = \begin{bmatrix} \sqrt{2} i g_s R^{\tilde{d}}<em>{\sigma,i} T^g</em>{\tilde{d},a} \ \sqrt{2} i g_s R^{\tilde{d}}<em>{\sigma,i} T^g</em>{\tilde{d},b} \end{bmatrix}$</td>
</tr>
</tbody>
</table>
[FFS] Chargino – Quark – Squark

\[ C(\tilde{\chi}_c^-, \tilde{d}_j, \tilde{u}_\sigma) = \begin{bmatrix} \sum_{i=1}^{3} \frac{i e m_d \cdot \text{CKM}_{ij} \cdot R_{\sigma,i}^{\tilde{u}} \cdot U_{\sigma,i}}{\sqrt{2} c_\beta M_W s_W} \\ - \sum_{i=1}^{3} \frac{i e \text{CKM}_{ij}^{*}}{2 M_W s_\beta s_W} \left( 2 M_W R_{\sigma,i}^{\tilde{u}} s_\beta V_{c1} - \sqrt{2} m_u R_{\sigma,3+i}^{\tilde{u}} V_{c2} \right) \end{bmatrix} \]

\[ C(\tilde{\chi}_c^+, \tilde{u}_i, \tilde{d}_\sigma) = \begin{bmatrix} \sum_{j=1}^{3} \frac{i e m_u \cdot \text{CKM}_{ij} \cdot R_{\sigma,j}^{\tilde{d}} \cdot V_{\sigma,j}^{*}}{\sqrt{2} M_W s_\beta s_W} \\ - \sum_{j=1}^{3} \frac{i e \text{CKM}_{ij}^{*}}{2 c_\beta M_W s_W} \left( 2 c_\beta M_W R_{\sigma,j}^{\tilde{d}} U_{c1} - \sqrt{2} m_d R_{\sigma,3+j}^{\tilde{d}} U_{c2} \right) \end{bmatrix} \]

\[ C(d_j, \tilde{\chi}_c^+, \tilde{u}_\sigma) = \begin{bmatrix} - \sum_{i=1}^{3} \frac{i e \text{CKM}_{ij}^{*}}{2 M_W s_\beta s_W} \left( 2 M_W R_{\sigma,i}^{\tilde{u}} s_\beta V_{c1}^{*} - \sqrt{2} m_u R_{\sigma,3+i}^{\tilde{u}} V_{c2}^{*} \right) \\ \sum_{i=1}^{3} \frac{i e m_d \cdot \text{CKM}_{ij} \cdot R_{\sigma,i}^{\tilde{d}} \cdot U_{c2}}{\sqrt{2} c_\beta M_W s_W} \end{bmatrix} \]

\[ C(u_i, \tilde{\chi}_c^-, \tilde{d}_\sigma) = \begin{bmatrix} - \sum_{j=1}^{3} \frac{i e \text{CKM}_{ij}^{*}}{2 c_\beta M_W s_W} \left( 2 c_\beta M_W R_{\sigma,j}^{\tilde{d}} U_{c1}^{*} - \sqrt{2} m_d R_{\sigma,3+j}^{\tilde{d}} U_{c2}^{*} \right) \\ \sum_{j=1}^{3} \frac{i e m_u \cdot \text{CKM}_{ij} \cdot R_{\sigma,j}^{\tilde{u}} \cdot V_{c2}}{\sqrt{2} M_W s_\beta s_W} \end{bmatrix} \]

[FFS] Neutralino – Quark – Squark

\[ C(\tilde{\chi}_n^0, \tilde{u}_i, \tilde{d}_\sigma) = \begin{bmatrix} \frac{i e}{3 \sqrt{2} c_W M_W s_\beta s_W} \left( 4 M_W R_{\sigma,3+i}^{\tilde{u}} s_\beta s_W Z_{n1}^{*} - 3 c_W m_u R_{\sigma,i}^{\tilde{u}} Z_{n4}^{*} \right) \\ - \frac{i e}{3 \sqrt{2} c_W M_W s_\beta s_W} \left( 3 c_W m_u R_{\sigma,3+i}^{\tilde{u}} Z_{n4} + M_W R_{\sigma,i}^{\tilde{u}} s_\beta (s_W Z_{n1} + 3 c_W Z_{n2}) \right) \end{bmatrix} \]

\[ C(\tilde{\chi}_n^0, \tilde{d}_j, \tilde{\tilde{d}}_\sigma) = \begin{bmatrix} - \frac{i e}{3 \sqrt{2} c_\beta c_W M_W s_W} \left( 2 c_\beta M_W R_{\sigma,3+i}^{\tilde{d}} s_W Z_{n1}^{*} + 3 c_W m_d R_{\sigma,i}^{\tilde{d}} Z_{n3}^{*} \right) \\ - \frac{i e}{3 \sqrt{2} c_\beta c_W M_W s_W} \left( 3 c_W m_d R_{\sigma,3+i}^{\tilde{d}} Z_{n3} + c_\beta M_W R_{\sigma,i}^{\tilde{d}} (s_W Z_{n1} + 3 c_W Z_{n2}) \right) \end{bmatrix} \]
\[
C(u_i, \tilde{\chi}_u^0, \tilde{u}_i^1) = \\
\left[ -\frac{ie}{3\sqrt{2}c_W M_W s_e s_W} \left( M_W R_{\sigma,1}^{u,i} s_{\beta} s_{\beta} Z_{n1}^* + 3c_W \left( M_W R_{\sigma,1}^{u,i} s_{\beta} Z_{n2}^* + m_u, R_{\sigma,3+i}^{u,i} Z_{n4}^* \right) \right) \right] \\
\left[ -\frac{ie}{3\sqrt{2}c_W M_W s_e s_W} \left( 4M_W R_{\sigma,3+i}^{u,i} s_{\beta} s_{\beta} Z_{n1} - 3c_W m_u, R_{\sigma,3+i}^{u,i} Z_{n4} \right) \right].
\]

\[
C(d_i, \tilde{\chi}_d^0, \tilde{d}_i^1) = \\
\left[ -\frac{ie}{3\sqrt{2}c_B c_W M_W s_e s_W} \left( c_{\beta} M_W R_{\sigma,1}^{d,i} s_{\beta} s_{\beta} Z_{n1}^* - 3c_W \left( c_{\beta} M_W R_{\sigma,1}^{d,i} s_{\beta} Z_{n2}^* - m_d, R_{\sigma,3+i}^{d,i} Z_{n3}^* \right) \right) \right] \\
\left[ -\frac{ie}{3\sqrt{2}c_B c_W M_W s_e s_W} \left( 2c_{\beta} M_W R_{\sigma,3+i}^{d,i} s_{\beta} s_{\beta} Z_{n1} + 3c_W m_d, R_{\sigma,3+i}^{d,i} Z_{n3} \right) \right].
\]

[SSS] Higgs – 2 Squarks

\[
C(H^0, \tilde{u}_i^*, \tilde{d}_i^*) = \\
-\sum_{i,j=1}^{3} \frac{ie}{6c_W M_W s_e s_W} \left\{ R_{\rho,3+i}^{\tilde{u}_i^*} \frac{4c_{\alpha+\beta} \delta_{ij} M_W M_Z R_{\sigma,3+j}^{d,i} s_{\beta} s_{W}^2}{-3c_W R_{\sigma,j}^{d,i} (c_{\alpha} \delta_{ij} m_u, \mu - m_u, s_{\alpha} A_{ji}^u) + 6c_W \delta_{ij} m_u, R_{\sigma,3+j}^{d,i} s_{\alpha}} \right\} \\
-\frac{ie}{3c_W M_W s_e s_W} \left\{ R_{\rho,3+i}^{\tilde{d}_i^*} \frac{3c_W R_{\sigma,3+j}^{d,i} (A_{ij}^d c_{\alpha} m_d, \mu - \delta_{ij} m_d, s_{\alpha}) + 6c_W \delta_{ij} m_d, R_{\sigma,3+j}^{d,i} s_{\alpha}}{2c_{\alpha+\beta} c_{\beta} \delta_{ij} M_W M_Z R_{\sigma,3+j}^{d,i} s_{W}^2 + 3c_W R_{\sigma,j}^{d,i} (\delta_{ij} m_d, \mu s_{\alpha} - c_{\alpha} m_d, A_{ji}^d)} \right\}.
\]

References


