On Trapped Higher-order Modes in a Single-cell RF Cavity

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1. **Introduction**

The two 400 MHz superconducting accelerating structures of the LHC [1] (one for each beam of the collider) consist of eight single-cell cavities connected with large-aperture beam tubes. This geometry has a low R/Q for the fundamental mode (44.5 Ω) which reduces transient beam loading and relatively low loss factors (transverse and longitudinal) in order to reduce parasitic mode losses [2]. Most higher-order (HOM) modes can propagate into the large-aperture beam tube (diameter 300 mm) and couple to the other cavity cells. They can be damped by HOM couplers mounted on the beam tubes. This also applies to the two transverse modes (at 503 MHz and 542 MHz) which are below the cut-off frequency of the beam tube. There is, however, a mode at about three times the fundamental mode frequency (~ 1240 MHz) which is concentrated in the cavity cell and has very low fields in the beam tubes. Although this mode resonates above the cut-off frequencies of the large beam tubes (756 MHz for TM modes and 586 MHz for TE modes) it couples little or not at all to a propagating mode of the beam tubes. This phenomenon is called a trapped mode [3,4].

The weak coupling of the trapped mode to the propagating modes of the beam tubes stems obviously from the particular field patterns at the interface between the cavity cell and the beam tubes. We asked ourselves how this coupling is influenced by geometry. In the following we study the effect of the cavity cell length on the trapped mode of the LHC cavity. In addition, for a more general idea and in order to compare with previous studies [5,6] we investigate this trapped mode in a pillbox cavity with beam tubes. There are probably more trapped modes at higher frequencies. They are, however, not of interest for the frequency range under consideration given the long bunches of the LHC (The shortest bunch during storage is 1 ns long.)

2. **LHC Cavity**

2.1 **Effect of the cell length on the trapped mode**

The geometry of the LHC single-cell cavity is shown in Fig. 1 and the most important resonant modes are summarized in Table 1. The cell length ($\ell_c$) has been chosen as 320 mm which corresponds to $\lambda/\ell_c = 0.43$. This is less than $\lambda/\ell_c = 0.5$ which is usually the case for multicell cavities. By this reduction of the cell length the first higher-order mode (TE$_{111}$) is shifted up in frequency which makes the design of the fundamental mode filter for the HOM couplers easier (see 2.2). In order to investigate the influence of the cell length ($\ell_c$) on the trapped mode we performed URMEL [7] runs with the following parameters:

- total structure length $3\lambda/2 = 1122$ mm
- radius $r_1 = 104$ mm
- radius $r_2 = 25$ mm
- beam tube radius $r_B = 150$ mm
- cell length $\ell_c = 300$ mm to 360 mm

Varying the cell length obviously changes the resonant frequency of the fundamental mode. The cavity radius $r_c$ has always been chosen to be such that the fundamental frequency is within 0.1% of 400.8 MHz.

As a measure for the coupling of the trapped mode to the beam tube modes, we use the quality factor $Q_{ext}$ which characterizes the energy loss to travelling wave beam tube
• fundamental frequency = 400.8 MHz
• total structure length including both beam tubes = 3\lambda/2 = 1122 mm
• beam tube radius \( r_B = 150 \) mm
• normalized cell length \( \ell_c/\lambda = 3/8 \).

In fact, we find the trapped mode at 1376 MHz. The field patterns calculated by MAFIA are shown in Fig. 12 and field strengths on the metallic walls as function of the axial coordinate are given in Fig. 13. It is quite clear that this mode is trapped, i.e. it does not propagate into the beam tube although the cut-off frequency of the beam tube for TM modes (766 MHz) is well below 1376 MHz. With the equivalent generator method (see chapter 2.1) we find \( Q_{\text{ext}} = 30000 \).

3.1 Changing the cell length

The procedure was the following:

For a constant beam tube radius \( r_B = 150 \) mm, we selected a cell length \( (\ell_c) \) and ran URMEL; then we changed the cavity radius \( (r_c) \) until the fundamental mode was within 0.1% of 400.8 MHz, and determined \( Q_{\text{ext}} \) with the equivalent generator method [3,4]. The result is shown in Fig. 14 and the corresponding fields in Fig. 15.

The trapped mode appears from 1371 to 1384 MHz. It is well trapped in the cavity cell for \( \ell_c = 262 \) to 290 mm. The maximum value \( Q_{\text{ext}} \) is \( 1.9 \times 10^5 \) at \( \ell_c = 278 \) mm. Outside this range \( Q_{\text{ext}} \) drops by about three orders of magnitude. Below 260 mm and above 300 mm, the mode is no longer trapped. But another trapped mode appears from 1443 to 1509 MHz when the cell length is between 310 and 320 mm (Fig. 16).

3.2 Changing the beam tube radius

For this test we have chosen \( \ell_c = 300 \) mm which gives a relatively low value of \( Q_{\text{ext}} = 270 \) at 1372 MHz. Then we varied \( r_B \) from 120 to 150 mm. Again, the fundamental frequency is adjusted with the cavity radius \( r_c \). The results are shown in Figs. 17 and 18.

The \( Q_{\text{ext}} \) of the mode at 1372 MHz increases and that of the mode at 1508 MHz decreases when the beam tube radius varies from 120 to 150 mm.

3.3 Asymmetric beam tubes

Following the above results, we assume that symmetric structures favour the appearance of trapped modes. For the following test we take the case of the highly trapped mode found in 3.1. We choose \( \ell_c = 280.5 \) mm with two beam tubes of radius 150 mm, obtaining \( Q_{\text{ext}} = 3 \times 10^4 \) at 1371 MHz. This symmetric situation is shown on the bottom plot of Fig. 20. As the radius of one beam tube is progressively reduced down to 110 mm, keeping the other beam tube at a radius of 150 mm, the mode becomes less and less trapped (see Figs. 19 and 20). \( Q_{\text{ext}} \) is reduced by about a factor of 100. This also happens for the case of a relatively low \( Q_{\text{ext}} \), for example at \( \ell_c = 300 \) mm. The results are shown in Figs. 21 and 22. Again the trapped mode reaches more into the beam with increasing asymmetry of the beam tube radii.
Table 1. Most important resonant modes of the LHC single-cell cavity

<table>
<thead>
<tr>
<th>Mode type</th>
<th>Frequency (MHz)</th>
<th>$R/Q(\Omega)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TM_{010}$</td>
<td>400.8</td>
<td>44.5</td>
</tr>
<tr>
<td>$TE_{111}$</td>
<td>503</td>
<td>3.2</td>
</tr>
<tr>
<td>$TM_{110}$</td>
<td>542</td>
<td>11.0</td>
</tr>
<tr>
<td>$TM_{011}$</td>
<td>768</td>
<td>26.5</td>
</tr>
</tbody>
</table>

Figure 1: Geometry of the LHC single-cell cavity
Figure 3: Field plots of the trapped mode of the LHC cavity:
Upper plot: electric field
Lower plot: magnetic field
The cell length is 320 mm. $Q_{\text{ext}} = 290$. 
Figure 5: URMEL contour plot of the electric field of the trapped mode for different cell lengths from 300 to 360 mm (0.4 < $\ell_c/\lambda$ < 0.48). The LHC cavity has $\ell_c = 320$ mm. The highest $Q_{ext}$ is for 332 mm.
Figure 7: Characteristic impedance $R/Q$ of the fundamental mode versus the cell length ($0.4 < \ell_c/\lambda < 0.48$).

Figure 8: Normalized peak electric field of the fundamental mode versus the cell length ($0.4 < \ell_c/\lambda < 0.48$).
Figure 11: Geometry of the pillbox cavity with beam tubes
Figure 13: Surface field strengths of the trapped mode (1376 MHz) versus the axial coordinate of the pillbox cavity with cell length 280.5 mm ($\ell_c/\lambda = 0.375$).
Figure 15: URMEL contour plots of the electric field of the trapped mode for different cell lengths ($0.33 < \ell_c/\lambda < 0.45$).
Figure 17: $Q_{ext}$ of matched beam tube loads for the mode at 1372 MHz versus the beam tube radius of the pillbox cavity ($0.16 < r_B/\lambda < 0.2$). Cell length = 300 mm.

Figure 18: $Q_{ext}$ of matched beam tube loads for the trapped mode at 1508 MHz versus the beam tube radius of the pillbox cavity ($0.16 < r_B/\lambda < 0.2$). Cell length = 300 mm.
Figure 20: Asymmetric beam tubes.
URMEL contour plots of the electric field of the trapped mode for different beam tube radii of one beam. The radius of the second beam tube is constant (150 mm). Cell length = 280.5 mm.