Top quark mass measurement in single leptonic \( t \bar{t} \) events

J. Heyninck\textsuperscript{a)}, J. D’Hondt\textsuperscript{b)}, S. Lowette\textsuperscript{b)}

\textit{Vrije Universiteit Brussel (IIHE-VUB), Pleinlaan 2, B-1050 Brussels, Belgium}

\textbf{Abstract}

The mass of the top quark is a crucial parameter both for the Standard Model and for models beyond this. Due to the large Yukawa coupling of the top quark, this mass is a key parameter in testing the electroweak and Higgs sector of the Standard Model. The precise measurement of the top quark mass is one of the main physics goals in understanding the interactions between the elementary particles. The Large Hadron Collider will collect an enormous amount of \( pp \rightarrow t\bar{t} \) events in the semi-leptonic decay mode \( t\bar{t} \rightarrow bq\ell\bar{b}\ell \ (\ell=e,\mu) \) allowing to measure the top quark mass to an unprecedented precision dominated by systematic uncertainties in the jet definition. A top quark mass estimator is constructed and its robustness versus several systematic uncertainties is optimized to reach conservatively a potential precision of better than 2 GeV/c\(^2\) which extrapolates to 1 GeV/c\(^2\) with a better understanding of the physics of proton collisions at the LHC and the CMS detector.

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1 Introduction

At the Large Hadron Collider (LHC) protons will be collided to obtain a centre-of-mass energy of 14 TeV. Relative to the Tevatron proton-anti-proton collisions at 1.96 TeV the production cross-section of $t\bar{t}$ events at the LHC is significantly increased to about 800 pb (Next-to-Leading-Order calculations in [1]), compared to about 560 pb in Leading Order. This together with the foreseen increased instantaneous luminosity, a much larger sample of $t\bar{t}$ events will be collected. During the first year of running an integrated luminosity of 10 fb$^{-1}$ will be collected and therefore result in the production of about 8 million $t\bar{t}$ events. Therefore the LHC experiment can be considered as a real top quark factory compared to the few hundreds of events collected at the Tevatron.

This increase of several orders of magnitude of selected events in the top quark sample will open new areas in the domain of top quark studies. One of the most important properties of the top quark which is at the heart of the Standard Model and strongly connected to the mechanism of electroweak symmetry breaking (EWSB), is its pole mass $M_t$. The top quark mass is measured by the CDF [2] and D0 [3] experiments at the Tevatron Collider in Fermilab with a combined result of $172.7 \pm 2.9$ GeV/c$^2$. While the Tevatron is aiming to measure $M_t$ with a precision of 2 GeV/c$^2$, the LHC experiments aim at a precision to better than 1 GeV/c$^2$. This increase in knowledge will lead to a significant improvement in our understanding of the Standard Model and the EWSB mechanism.

The note starts with a section describing the simulation samples used for the analysis, Section 2. Section 3 elaborates on the details of the event reconstruction including the event selection. The procedures for choosing the single-lepton final state and the jet combination in this final state are discussed in Sections 4 and 5. The top quark mass estimator is constructed from the result of a kinematic fit transformed into an ideogram of the event. The construction of this ideogram or likelihood ratio function is defined in Section 6, while in Section 7 the convolution method is formulated to determine the event likelihood as a function of the top quark mass. The result of an extensive study of systematic uncertainties is described in Section 8, where the total expected uncertainty on the inferred top quark mass is mentioned. Among the possible strategies for optimization or minimization of this total uncertainty, one related to the definition of jets is studied. The results are summarized in Section 9. The analysis was performed on $pp \rightarrow t\bar{t} \rightarrow bWbW \rightarrow bqq\bar{d}\mu\nu_\mu$ events, but similar results are assumed for the electron decay channel.

2 Monte Carlo simulation samples

The Monte Carlo simulated events used in this note are generated with CMKIN using the PYTHIA 6.2 event generator [4]. A full GEANT-4 detector simulation [5] was used within OSCAR [6] to simulate the detector response to the $t\bar{t}$ final state. To simulate the low-luminosity machine settings a Poissonian average of 3.5 pile-up collisions (with an instantaneous luminosity of $2 \cdot 10^{33}$ cm$^{-2}$s$^{-1}$) from minimum bias events were added and simulated with OSCAR [7]. The OSCAR output was digitized with the reconstruction software of CMS. General Data Summary Tapes or DST’s were produced with ORCA. Finally for all event reconstruction variables which were not written on the CMS standard DST’s the ORCA was used. Table 1 summarizes the absolute quantities of single-lepton $t\bar{t}$ events which were produced for the studies presented in this note. Only $t\bar{t} \rightarrow bWbW \rightarrow bqq\bar{d}\mu\nu_\mu$ events were considered as signal in this study.

<table>
<thead>
<tr>
<th>Number of events</th>
<th>Int.Luminosity fb$^{-1}$</th>
<th>Cross-section pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t\bar{t} \rightarrow bWbW \rightarrow bqq\bar{d}\mu\nu_\mu$</td>
<td>365k</td>
<td>4.39</td>
</tr>
<tr>
<td>other $t\bar{t}$</td>
<td>1962k</td>
<td>4.11</td>
</tr>
<tr>
<td>W+4jets</td>
<td>82.5k</td>
<td>0.47</td>
</tr>
<tr>
<td>Wbb+2jets</td>
<td>109.5k</td>
<td>6.44</td>
</tr>
<tr>
<td>Wbb+3jets</td>
<td>22.5k</td>
<td>3.21</td>
</tr>
</tbody>
</table>

Table 1: Overview of the number of analyzed simulated single-lepton $t\bar{t}$ and background events with their corresponding integrated luminosity calculated with the indicated Leading-Order cross-sections.

In order to study the systematic influence of pile-up collisions, event samples are generated with and without the addition of low-luminosity pile-up collisions. This was done only for the $t\bar{t} \rightarrow bWbW \rightarrow bqq\bar{d}\mu\nu_\mu$ signal events. In total more than 150k signal events are generated both with and without pile-up. The slope of the developed top quark mass estimator was estimated with event samples at three different input top quark masses: 170, 175 and 180 GeV/c$^2$ including low-luminosity pile-up collisions. All other event samples, for the study of systematic
uncertainties, are constructed using the FAMOS framework [8]. The W + jets background event samples mentioned in Table 1 are generated with AlpGen and simulated within the FAMOS framework. Due to the construction of the event selection criterion, all other background processes are negligible. It will be illustrated that the expected amount of fully hadronic $t\bar{t}$ events passing the event selection is negligible. With this observation together with the Tevatron experience [9] it is assumed that the influence of QCD events is also negligible.

3 Reconstruction of the semi-leptonic $t\bar{t}$ events

The top quark $^1$ has a branching ratio of about 100% to decay as $t \rightarrow W b$, while the W boson decays has a leptonic branching ratio of $\text{BR}(W \rightarrow l\nu_l) = \frac{4}{9}$ and a hadronic branching ratio of $\text{BR}(W \rightarrow q\bar{q}) = \frac{2}{9}$ (in the absence of QCD corrections). The generated final state topology of the semi-leptonic decay channel $pp \rightarrow t\bar{t} \rightarrow bq\bar{q}b\ell\nu_\ell$ consists of four coloured partons of which two are heavy, a muon and a neutrino. The detector final state therefore can be characterized by four hadronic jets of which two originate from a heavy quark, an isolated muon and missing transverse energy. In this paper we consider the measurement of the mass of the top quark in the semi-leptonic channel where the lepton is a muon.

Both the Level-1 and the High-Level Trigger criteria are applied on the simulated events, resulting in efficiencies shown in Table 2. The single-muon stream was used.

A muon candidate is formed when a muon track is reconstructed in the muon chambers and a matching track is found in the main tracker. Among the list of lepton candidates with their identified flavour, the lepton originating directly from the W boson decay is selected following the procedure described in [10]. This results in a unique lepton with a combined likelihood variable to be used in the event selection. In Figure 1 this likelihood variable is shown for the semi-leptonic $t\bar{t}$ signal and W+jets background processes, together with the transverse momentum, $p_T$, of the selected lepton candidate. The peak at zero for the likelihood variable is induced by events without a reconstructed lepton candidate in the final state. The observation of lepton candidates below the trigger threshold on the transverse momentum comes from events which are triggered on a lepton different than the correct one coming from the $t \rightarrow bW \rightarrow bl\ell\bar{\nu}$ decay.

The jets are reconstructed from the combined electromagnetic and hadronic calorimeter deposits and clustered with the Iterative Cone algorithm using an opening angle of $\Delta R = 0.5$ rad. A transverse energy threshold of 0.5 GeV is applied on the input object before clustering. This results in a number of jets per event, as shown

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1) Throughout the note the charge conjugation and the inclusion of anti-matter is implicit.
for the signal semi-leptonic $t\bar{t}$ events in Figure 2 and for the background $W +$ jets events in Figure 3. Only those jets in the vicinity of the primary vertex are considered in the analyses. Jets emerging from the pile up are vetoed using a track-based method. For a jet to be associated with the primary vertex, it is demanded that 

$$\beta = \frac{\sum_{\text{tracks}} p_T}{\sum_{\text{tracks}} p_T} > 0.04$$

where the sum in the denominator is over the $p_T$ of all tracks in the jet, while the sum in the nominator runs only over those tracks in the jet associated with the primary vertex. The low value of the $\beta$-cut reflects the aim that the tracks with the highest transverse momentum should dominate.

In Figures 2 and 3 the reduction in number of jets per event is illustrated. The energy scale of the reconstructed jets is calibrated using the methods described in [11]. The resulting transverse momentum, $p_T$, after applying this calibration technique is shown in both Figures. The primary vertex requirement on the reconstructed jets reject those jets with a small transverse momentum. The bump at about 35 GeV in the transverse momentum distribution is induced by the pre-selection cuts described below.

![Figure 2: For semi-leptonic $t\bar{t}$ events passing the trigger requirements: the normalized distribution of the amount of reconstructed jets per event before and after applying the primary vertex constraint (left) and the distribution of the transverse momentum, $p_T$, of the calibrated jets before and after applying the primary vertex constraint (right).](image)

The transverse momentum components of the unobserved neutrino are estimated via the missing transverse momentum which balances the vectorial sum of the energy deposits in the calorimeter above the transverse energy threshold mentioned. The distribution for the magnitude of the reconstructed missing transverse momentum and the transverse momentum of the generated neutrino is shown in Figure 4 both for signal and background events. No direct event selection requirement is made on the missing transverse momentum, as the resolution on the missing transverse momentum is of the same order as the expected magnitude of the missing transverse momentum in QCD (or fully hadronic $t\bar{t}$) events.

The event selection consists of a series of sequential cuts on kinematic or topological variables. A first pre-selection criterion reduces the amount of events to a manageable number by requiring at least four reconstructed jets with a transverse energy, $E_T$, larger than 10 GeV and with a pseudo-rapidity in the range of the tracker, $|\eta| < 2.4$. The jets must have a flight direction through the tracker to allows for a proper performance of the b-tagging algorithm. At least one lepton is required within the tracker acceptance of $|\eta| < 2.4$ and with a combined likelihood ratio value larger than 0.01.

For the remainder of the event selection several variables are examined, resulting in a definition of some simple criteria. The event is required to have at least 4 jets after applying the primary vertex constraint with a calibrated transverse energy, $E_T$, exceeding 30 GeV. If more than four jets match this criterion, the four leading jets needed to reconstruct the partonic $t\bar{t}$ event topology, are selected as those with the highest $E_T$. Of these four jets, two have to be b-tagged according to the method applying a combined b-tag variable [12]. The distribution of the b-tag discriminant variable is shown in Figure 5 for several quark flavours. To transform the value of the combined b-
Figure 3: For W + jets events passing the trigger requirements: the normalized distribution of the amount of reconstructed jets per event before and after applying the primary vertex constraint (left) and the distribution of the transverse momentum, $p_T$, of the reconstructed and calibrated jets before and after applying the primary vertex constraint (right).

tagging discriminant into a probability only jets are considered which are clearly connected to the simulated parton direction in order to define unambiguously their flavour. A likelihood ratio is constructed as

$$L^b(x) = \frac{P_b(x)}{P_b(x) + P_c(x) + P_{\text{other}}(x)}$$

where $P_i(x)$ is the probability density function of quark flavour $i$ in the dimension of the b-tag discriminant $x$. The variable $L^b(x)$ of a jet is related to the b-tag discriminant of the jet and takes by definition values between 0 and 1. It is interpreted as the probability for the jet to originate from a b-flavoured parton. Hence the complement $(1 - L^b(x))$ is interpreted as the probability of the jet not to originate from a b-flavoured parton.

For the event to be selected, exactly two out of the four leading jets need to have a b-tag likelihood $L^b$ exceeding 0.6 and the other two need to be anti-b-tagged, hence having a b-tag likelihood $L^b$ below 0.4.

The selected lepton candidate must have a transverse momentum, $p_T$, larger than 20 GeV/c, while no selection cut is applied on the reconstructed missing transverse momentum. This is well above the trigger turn on curve in the single-muon trigger stream. An extra sequential cut on the reconstructed missing transverse momentum would not increase the signal-to-noise ratio significantly, but it would introduce possibly large systematic uncertainties. The four leading jets should not overlap in an $(\eta, \phi)$-metric to reduce ambiguities in the jet energy scale calibration procedure.

In the selected $t\bar{t}$ sample about $1/6$ is generated in a different decay channel. The selected background catalogued as other $t\bar{t}$ decay channels, breaks down according to the generated information in

- 27.6% of $t\bar{t} \rightarrow \tau\nu_\tau + X$;
- 0.10% of $t\bar{t} \rightarrow e\nu_e + X$;
- 1.75% of $t\bar{t} \rightarrow \tau\nu_\tau + \tau\nu_\tau + X$;
- 0.00% of $t\bar{t} \rightarrow e\nu_e + e\nu_e + X$;
- 8.80% of $t\bar{t} \rightarrow \mu\nu_\mu + \mu\nu_\mu + X$;
- 33.0% of $t\bar{t} \rightarrow \mu\nu_\mu + e\nu_e + X$;
Figure 4: For events passing the trigger requirements: the normalized distribution of the reconstructed missing transverse momentum for both signal and background events (left) and the distribution of the transverse momentum of the generated neutrino for signal events (right).

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Signal</th>
<th>Other tt</th>
<th>W+4j</th>
<th>Wbb+2j</th>
<th>Wbb+3j</th>
<th>S/N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before selection</td>
<td>365k</td>
<td>1962k</td>
<td>82.5k</td>
<td>109.5k</td>
<td>22.5k</td>
<td>0.032</td>
</tr>
<tr>
<td>L1+HLT Trigger</td>
<td>62.2%</td>
<td>5.30%</td>
<td>24.1%</td>
<td>8.35%</td>
<td>8.29%</td>
<td>0.74</td>
</tr>
<tr>
<td>Pre-selection</td>
<td>45.8%</td>
<td>2.68%</td>
<td>11.7%</td>
<td>3.94%</td>
<td>5.91%</td>
<td>1.10</td>
</tr>
<tr>
<td>Four jets $E_T &gt; 30$ GeV</td>
<td>25.4%</td>
<td>1.01%</td>
<td>4.1%</td>
<td>1.48%</td>
<td>3.37%</td>
<td>1.69</td>
</tr>
<tr>
<td>$p_T^{lep}$on &gt; 20 GeV/c</td>
<td>24.8%</td>
<td>0.97%</td>
<td>3.9%</td>
<td>1.41%</td>
<td>3.14%</td>
<td>1.72</td>
</tr>
<tr>
<td>b-tag criteria</td>
<td>5.5%</td>
<td>0.21%</td>
<td>0.052%</td>
<td>0.47%</td>
<td>0.70%</td>
<td>3.73</td>
</tr>
<tr>
<td>No jet overlap</td>
<td>3.0%</td>
<td>0.11%</td>
<td>0.027%</td>
<td>0.25%</td>
<td>0.44%</td>
<td>3.87</td>
</tr>
<tr>
<td>$p_T^{tau}$-cut 20%</td>
<td>1.4%</td>
<td>0.039%</td>
<td>0.0097</td>
<td>0.061</td>
<td>0.07</td>
<td>5.3</td>
</tr>
<tr>
<td>$p_T^{max}$-cut 80%</td>
<td>1.2%</td>
<td>0.025%</td>
<td>0.0085</td>
<td>0.052</td>
<td>0.05</td>
<td>6.8</td>
</tr>
<tr>
<td>$p_T^{max}$-cut 50%</td>
<td>0.7%</td>
<td>0.013%</td>
<td>0.0036</td>
<td>0.013</td>
<td>0.</td>
<td>8.2</td>
</tr>
<tr>
<td>Scaled L=1fb⁻¹</td>
<td>588</td>
<td>64</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Table 2: Overview of the selection criteria applied on the events using simulated events with pile-up collisions included. The last line indicates how many events are expected after applying the full selection criteria on a data set of 1fb⁻¹. The expected S/N values take into account the respective Leading-Order cross-sections of the processes.

- 25.7% of $t\bar{t} \to \mu\nu_{\tau} + \tau_{\nu_{\tau}} + X$;
- 2.19% of $t\bar{t} \to e\nu_{e} + \tau_{\nu_{\tau}} + X$;
- 0.92% of $t\bar{t} \to X$ or the fully hadronic decay channel;

where X denotes colored partons. The first two decay channels contain on the hadronic side a fully hadronic decay of a top quark, and hence contain the same information about its mass as the signal $t\bar{t} \to bQ\bar{b}f\bar{f}\nu_{\mu}$ channel. All others give by construction a biased top quark mass information. About $\frac{4}{5}$ of the selected events in the other $t\bar{t}$ channels reflect a di-leptonic topology with one of the leptons being a muon, while only about $\frac{1}{5}$ reflect a semi-leptonic decay with a $\tau$ lepton wrongly identified as a muon candidate. The fraction of fully hadronic $t\bar{t}$ events selected is negligible, illustrating that the influence of QCD produced jet events is minor. This fraction is further reduced by the topological cuts described in the next Sections.
Figure 5: Distribution of the combined b-tag discriminant for jets in semi-leptonic $t\bar{t}$ events originating from different flavoured quarks, this becomes $P_{b}(x)$ after normalization (left). On the right the b-tag probability $L_{b}(x)$ (or likelihood ratio $S/(S+B)$) as a function of the combined b-tag discriminant.

4 Selection of the correct $t\bar{t}$ decay channel

To reduce the amount of events produced via a different $t\bar{t}$ decay channel in the selected event sample, three observables are constructed:

- the transverse momentum of the selected muon candidate;
- the transverse momentum of the lepton candidate with the second largest likelihood ratio variable in the list of reconstructed lepton candidates in the event;
- the smallest transverse energy among the four leading jets.

The distribution of these observables are shown in Figure 6 together with the likelihood ratio defined as $S/(S+B)$ where $S$ are the $t\bar{t}$ semi-leptonic muon decays and $B$ all other $t\bar{t}$ decay channels. The dependency of $S/(S+B)$ is fitted as a function of the three observables defined above. The information of the three likelihood ratios is combined into a combined likelihood by multiplication. The distribution of the combined likelihood $L_{\text{sign}}$ is shown in Figure 7. From these distributions a probability is determined for the selected event to be a semi-leptonic muon event. In the event selection an extra sequential cut is applied by requiring this probability $P_{\text{sign}}$ to exceed 0.8. The efficiency of this extra cut is shown in Table 2. In the selected event sample 14% of the semi-leptonic muon events are rejected, while 36% of the other $t\bar{t}$ events are removed.

5 Jet combinations

When the event has been selected as described above, it will have 4 reconstructed jets with a transverse energy exceeding 30 GeV and a muon with a transverse momentum exceeding 20 GeV/c.

Among the four reconstructed jets, three have to be choosen to form the hadronic decaying top quark. The efficiency or purity of this selection was largely enhanced by applying a likelihood ratio method combining the information from several sensitive variables. The event selection requires exactly two jets with a b-tag probability above 0.6 and two below 0.4. This classification is used to define the two light quark jets from the hadronic W boson decay, and reduces the possible combinations to two.

For each jet combination a constraint kinematic fit was applied as described in [14] forcing the correct W boson mass, $M_{W}$, for the hadronic decaying W boson in the event. Before applying the kinematic fit the energy scale
Figure 6: Distributions of the observables used to select the correct $\tau\tau$ decay channel (left) and the corresponding likelihood ratios (right).
of the light quark jets are corrected for an overall bias in the reconstructed W boson mass. Following the method described in [15] after the event selection mentioned above, an inclusive jet energy scale correction of -9.7% was obtained and applied to light quark jet candidates. The reconstructed hadronic W boson mass spectrum before and after applying this correction is shown in Figure 10. The energy scale of the jets defined with a b-quark flavour is unchanged.

The jet objects are parametrized as \((E_\tau, \eta, \phi)\) objects with a fixed mass equal to zero. The resolution of the jet kinematic variables are differentiated as a function of their transverse energy. In Figure 11 the dependency is shown. In general the angular resolution and the relative resolution on the transverse energy improves when the jet has a larger transverse energy.

Within an event only jet combinations are considered with a probability of the kinematic fit calculated from its \(\chi^2/\text{ndf}\) exceeding 0.2. For some events none of the jet combinations fulfill this criterium, therefore reducing the total event selection efficiency.

Additional the information of several observables is used to select which of the two b-quark jets combines with the hadronic W boson to a top quark

- \(\Delta R(\text{b}_{\text{lep}}, l)\) : angle in an \((\eta, \phi)\)-metric between the b-quark jet from the leptonic top quark decay and the lepton;
- \(\Delta Q\) : combined electric charge observable defined as the charge of the lepton multiplied by the difference in jet; charge of the b-quark jets from respectively the hadronic and leptonic top quark decay;
- \(p_T^{\text{top}}/ <p_T^{\text{top}}>\) : magnitude of the reconstructed transverse momentum of the hadronic top quark in this jet combination relative to the average of the transverse momentum of the top quarks over all possible jet combinations in the event;
- \(\Delta R(\text{b}_{\text{had}}, W_{\text{had}})\) : angle in an \((\eta, \phi)\)-metric between the b-quark jet and the reconstructed W boson direction both from the hadronic top quark decay.

For the signal \(\tau\tau\) events the distributions of these variables are shown in Figures 8 and 9. The information of the observables is combined via a likelihood ratio method similar to the one described in [13].

The distribution of the combined likelihood ratio \(L_{\text{comb}}\) is shown in Figure 12 for both the correct and wrong jet combinations in agreement with the above criteria for events containing a correct combination. In Figure 13 this
Figure 8: Distributions of the observables used to select the correct jet combination from the signal $t\bar{t}$ events (left) and the corresponding likelihood ratios (right).
Figure 9: Distributions of the observables used to select the correct jet combination from the signal $t\bar{t}$ events (left) and the corresponding likelihood ratios (right).
Figure 10: For the selected events the distribution of the reconstructed hadronic W boson mass before (left) and after (right) applying the post-calibration corrections on the energy scale of the light quark jets.

Figure 11: As a function of the transverse momentum, the applied Gaussian resolution on the variables (E_T, η, φ) of the parametrized jet objects.

is plotted for all jet combinations. If no jet combination in the event matches the generated true combination, all possible jet combinations of the event were treated as combinatorial background. The jet combination with the largest L_comb value was taken as the best pairing. The L_comb value is transformed into a probability P_comb for the chosen combination to be the correct one, as shown in Figure 12. In a window of 25 GeV/c^2 around the expected top quark mass of about 175 GeV/c^2, the purity was above 76.6%. The event probability P_comb is used in the event selection where events are selected if their value for P_comb exceeds P_comb threshold. Relative to the events remaining after the event selection the efficiency and purity of this sequential cut is shown in Figure 13, where a scan is made over the full range of P_comb. When considering only events for which the best pairing has a probability P_comb larger than P_comb threshold = 60%, the purity of the selected jet pairings is increased to 81.6%. The impact of this selection cut is illustrated in Figure 14 and mentioned in Table 2.

The jet combination procedure described results in a unique value for the hadronic top quark mass for each selected event. This is shown for true jet combinations in Figure 15 both before and after applying the kinematic fit. When estimating the top quark mass, M_t, from the selected event sample via the estimator m, from a simple Gaussian fit, G(m | m_0), in a range of 20 GeV/c^2 in both directions around the modal bin, a value of m before = 176.5 ± 0.65 GeV/c^2 is obtained before applying the kinematic fit and m after = 172.2 ± 0.48 GeV/c^2 after applying the
Figure 12: Distribution of the combined likelihood value for the chosen correct and wrong jet combinations of the signal $t\bar{t}$ events containing a correct combination (left), and for the same events the relation between the combined likelihood variable $L_{\text{comb}}$ and the probability $P_{\text{comb}} (= S/(S+B))$ for the chosen combination to be the correct one (right).

kinematic fit. The estimator $\hat{m}_t^{\text{before}}$ would obtain the same statistical precision as $\hat{m}_t^{\text{after}}$ when it is applied on a data sample with an increased number of events by a factor 2, hence collecting twice as much data. The width of the top quark mass distribution is reduced from 15.0 GeV/c$^2$ to 13.0 GeV/c$^2$ when applying the kinematic fit. The top quark mass after the kinematic fit is shown in Figure 16 for all relevant processes contributing to the selected event topology.

6 Construction of the events ideogram

Rather than developing top quark mass estimators on samples of events, an event-by-event likelihood approach is pursued. The fitted kinematics of the three jets connected to the hadronic decaying top quark are used to determine the top quark mass. From the covariance matrices of the kinematics of these three fitted jets the uncertainty on the top quark mass can be determined for each event via error propagation. The result can be written as

$$\chi^2(\{p_j\}|m_t) = \left(\frac{m_t - m_t^{\text{fit}}}{\sigma_{m_t}^{\text{fit}}}\right)^2$$

(2)

for the measured event kinematics $\{p_j\}$ of the reconstructed event to agree with a reconstructed top quark mass $m_t$, given the result from the kinematic fit as $m_t^{\text{fit}}$ and the uncertainty $\sigma_{m_t}^{\text{fit}}$. This $\chi^2$ variable can be transformed into a probability as

$$P(\{p_j\}|m_t) \, dm_t \sim \exp\left(-\frac{1}{2} \cdot \chi^2(\{p_j\}|m_t)\right)$$

(3)

where $P(\{p_j\}|m_t)$ represents the resolution function or likelihood ratio mapping of the event in the space of the reconstructed top quark mass $m_t$. It is often called an ideogram of the event [16]. It reflects the relative compatibility of the reconstructed kinematics of the event with the hypothesis that one heavy object with mass $m_t$ decays into three jets of which two originate from the W boson.

This probability scan $P(\{p_j\}|m_t)$ can also be determined explicitly by forcing a reconstructed top quark mass to the event in the kinematic fit. The hypothesis of a Gaussian resolution function on the fitted top quark mass is not needed in this approach, but the computing time is increase by a large factor. In Figure 17 the one-dimensional
ideogram is shown for several selected events, and as an illustration a comparison is made between both approaches. For the results below both the parametrized and the full probability scan are used. In the discussion of the systematic uncertainties the advantage of each is illustrated.

When applying the full probability scan for the ideogram $P\left(\{p_{ij}\}|m_t\right)$, the event is only selected if the largest probability provided by the kinematic fit exceeds 0.2 in the relevant mass range $125 < m_t < 225$ GeV/c$^2$.

Only for the comparison of the parametrized and the full ideogram, the maximum of the function $P\left(\{p_{ij}\}|m_t\right)$ is fixed at unity. The width of the full ideogram is narrower compared to the parametrized ideogram. The covariance matrix of the fitted b-quark jet was used to determine the full ideogram, while its covariance matrix at the reconstruction level was used for the parametrized ideogram. This could induce the observed difference between the parametrized and full ideograms.

7 Top quark mass estimator

As the amount of $W+$jet events in the selected event sample is negligible, these are not considered in the estimation of the top quark mass.

To obtain information about the true value of the top quark mass $M_t$ we convolute the reconstructed ideogram with the theoretical expected probability density function $P(m_t|M_t)$ in the reconstruction space

$$L_i(M_t) = \int P\left(\{p_{ij}\}|m_t\right) \cdot P(m_t|M_t) \, dm_t$$  \hspace{1cm} (4)

where one integrates over the kinematic relevant range of $m_t$ to obtain a likelihood function $L_i(M_t)$ for each event $i$ in the true top quark mass dimension. Several contributions should be added in the expected density $P(m_t|M_t)$: a Breit-Wigner shape for the correct jet combinations $S(m_t|M_t)$, a parametrized combinatorial background contribution $B_{\text{comb}}(m_t)$ and a parametrized process background contribution $B_{\text{proc}}(m_t)$. This results in a function

$$P(m_t|M_t) = P_{\text{sign}} \cdot \left[ P_{\text{comb}} \cdot S(m_t|M_t) + (1 - P_{\text{comb}}) \cdot B_{\text{comb}}(m_t) \right] + (1 - P_{\text{sign}}) \cdot B_{\text{proc}}(m_t)$$  \hspace{1cm} (5)

where each contribution is weighted according to the probabilities extracted from the observed event. The probability $P_{\text{comb}}$ is determined for each selected event as the likelihood ratio value provided by $L_{\text{comb}}$, see Figure 12.
Figure 14: For signal $t\bar{t}$ events only, the distribution of the mass of the hadronic decaying top quark before (left) and after (right) applying the 60% cut on the best pairing probability $P_{\text{comb}}$.

The probability $P_{\text{sign}}$ is determined according to the combined likelihood ratio shown in Figure 7. The two background contributions do not depend (in first order) on the value of $M_t$. The possible effect of an $M_t$ dependency of $B_{\text{proc}}(m_t)$ for process background from other $t\bar{t}$ final states is included as a systematic uncertainty.

After combining the likelihoods $L_i(M_t)$ from all selected events, a maximum likelihood method is applied to obtain the best value for the estimator $M_t$. The linearity of the estimator has been checked and illustrated in Figure 18. The slope of the curve of the measured versus the generator top quark mass is found to be $0.86 \pm 0.18$ for the $\tilde{M}_t^{\text{fit}}$ estimator fitting a Gaussian function on the reconstructed $m_t$ distribution in the range of 30 GeV/c$^2$ around the modal bin. This becomes $1.01 \pm 0.16$ for the $\tilde{M}_t^{\text{ParIdeo}}$ estimator using the parametrized ideogram and $1.01 \pm 0.13$ for the $\tilde{M}_t^{\text{FullIdeo}}$ estimator using the full ideogram. It is observed that these slopes are compatible with unity.

The width of the pull distribution of the top quark mass estimators $\tilde{M}_t$, shown in Figure 19, are found to be 0.82 for $\tilde{M}_t^{\text{fit}}$, 1.04 for $\tilde{M}_t^{\text{ParIdeo}}$ and 1.02 for $\tilde{M}_t^{\text{FullIdeo}}$. The resulting statistical uncertainty on the estimators $\tilde{M}_t$ is rescaled with this number. The resulting top quark mass for the estimator $\tilde{M}_t^{\text{fit}}$ applied on the simulated events samples with a generated top quark mass of 175 GeV/c$^2$ is $174.16 \pm 0.59$ GeV/c$^2$, hence reflecting a bias of -0.84 GeV/c$^2$. For the convolution method this is $170.65 \pm 0.54$ GeV/c$^2$ and $172.42 \pm 0.31$ GeV/c$^2$ for respectively the $\tilde{M}_t^{\text{ParIdeo}}$ and the $\tilde{M}_t^{\text{FullIdeo}}$ estimator, reflecting respectively biases of -4.35 GeV/c$^2$ and -2.58 GeV/c$^2$. The statistical uncertainties mentioned are corrected to obtain a pull distribution with a width equal to unity.

These numbers together with the expected statistical uncertainty on the top quark mass estimators with semi-leptonic $t\bar{t}$ (the lepton being a muon) for both 1 and 10 fb$^{-1}$ of integrated luminosity are shown in Table 3. Figure 20 shows the distribution for the simple Gaussian fit top quark mass estimator together with the combined likelihood curves for the ideogram based estimators. The $\Delta \chi^2(M_t)$ curves are fitted with a parabol in the region of $\pm 20$ GeV/c$^2$ around the minimum.

<table>
<thead>
<tr>
<th></th>
<th>Gaussian Fit</th>
<th>Gaussian Ideogram</th>
<th>Full Scan Ideogram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bias (GeV/c$^2$)</td>
<td>-0.84±0.59</td>
<td>-4.35±0.54</td>
<td>-2.58±0.31</td>
</tr>
<tr>
<td>Pull</td>
<td>0.82</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Expected uncertainty for 1fb$^{-1}$ (GeV/c$^2$)</td>
<td>1.01</td>
<td>1.14</td>
<td>0.66</td>
</tr>
<tr>
<td>Expected uncertainty for 10fb$^{-1}$ (GeV/c$^2$)</td>
<td>0.32</td>
<td>0.36</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 3: Overview of the statistical properties of the three top quark mass estimators defined in the text. The expected uncertainty quoted is rescaled for a non-unity pull.
In the convolution method described only one event reconstruction hypothesis is taken into account. For example only one definition is taken into account of the jets, the lepton and the missing transverse energy, and the information of only one jet combination is used. As an extension of the convolution method several event reconstruction hypotheses $\mathcal{H}_k$ can be included. The ideogram $P'((\mathcal{P}_1') | m_{\text{fit}}, \mathcal{H}_k)$ can be constructed for several jet combinations, or for several jet clustering algorithms. The combined event ideogram $P'((\mathcal{P}_1') | m_{\text{fit}}, \{\mathcal{H}_k\})$ would be a sum or weighted sum of the individual ideograms of each element $k$ in the set of event reconstruction hypotheses $\{\mathcal{H}_k\}$. This strategy is to be followed in the future when optimizing the analysis results for systematic uncertainties.

## 8 Systematic uncertainties

Several systematic effects could induce an uncertainty on the top quark mass estimator. They originate from our understanding of the detector performance, the robustness of the reconstructed objects, for example jets, and the general description of the proton collisions in the simulation. For the theoretical or phenomenological uncertainties the prescription of [17] was used.

As mentioned below the effect of the background processes on the top quark mass estimator is small. Therefore the systematic effects described are only introduced in the signal events as the same effect in the background processes would only be a second order effect.

To enhance the correlation between the sample with and the sample without the systematic effect, the same generated events are used in the simulation of both samples. The Jackknife method is applied to take this correlation into account when estimating the uncertainty on the difference in top quark masses from both samples being $\Delta m_1$. Starting from a sample of $N$ measurements $\mathcal{X}$, the Jackknife begins by throwing out the first measurement $x_1$, leaving a Jackknived data set of $N - 1$ values. The statistical analysis is performed on the reduced sample, giving a measured value of a parameter $\Delta m_1^{\text{jack}}$. The process is repeated for each measurement $i$ in the sample, resulting in a set of parameter values $\{\Delta m_i^{\text{jack}}, i \in \{1, \ldots, n\}\}$. The standard uncertainty on the parameter, being $\sqrt{\text{Var}[\Delta m_i]}$, estimated on the full sample of $N$ measurements is given by the formula:

$$\text{Var}[\Delta m_i] = \frac{N - 1}{N} \cdot \sum_{i=1}^{n} \left( \Delta m_i^{\text{jack}, i} - \Delta m_i \right)^2$$

where $\Delta m_i$ is the result of inferring the parameter on the full sample. The advantage of using this method compared to other bootstrap techniques is that no knowledge is assumed about the underlying probability density function.
Figure 16: Distribution of the mass of the hadronic decaying top quark for the selected events after applying the kinematic fit. The contribution of all relevant background processes is shown.
Figure 17: For some typical selected signal $t\bar{t}$ events the reconstructed ideogram from both a kinematic fit where $M_t$ is free (full line) and a complete scan over several top quark mass hypotheses (dashed line).
The systematic effects are determined on the three estimators described above: a simple Gaussian fit on the reconstructed top quark mass spectrum, a convolution technique with a Gaussian parametrization of the ideogram and a convolution technique with a full probability scan. The advantages of the choice for the latter are demonstrated.

If the systematic effect on the top quark mass is compatible with zero within its statistical uncertainty, then for this systematic effect as summarized in Table 12 a number is quoted which corresponds to the statistical precision of the test. The components which are dominated by their precision are denoted by * in Table 12.

8.1 Pile-Up description

For the signal events simulation samples of about 150k events are produced with and without the superposition of low-luminosity pile-up collisions (with an instantaneous luminosity of $2 \cdot 10^{33} \text{cm}^{-2}\text{s}^{-1}$). Only in-time inelastic pile-up collisions are taken into account, while the out-of-time contribution could be as large as the in-time contribution. The difference in the estimated top quark mass is determined and the values obtained for the three estimators are shown in Table 12.

It is shown that the convolution methods with the parametrized or the full ideogram are less sensitive to the present of pile-up collisions. The effect is reduced with a factor 2 compared with the simple to quark mass estimator $M_{\text{fit}}$. The systematic uncertainty is defined as 30% of the observed shift when neglecting pile-up collisions completely.

8.2 Underlying event

Apart from the multiple proton collisions detected simultaneous, the fragmented remnant of the protons of beam remnant in a single collision is also observed in the detector. The phenomenology of this so-called underlying
Figure 20: Distribution of the mass of the hadronic decaying top quark before the kinematic fit used for the $\hat{M}_t^\text{fit}$ estimator (left) and the combined $\Delta \chi^2(M_t)\) function over all events for both ideogram based estimators $\hat{M}_t^{\text{ParIdeo}}$ and $\hat{M}_t^{\text{FullIdeo}}$ (right).

Table 4: Effect of the primary vertex constrain on the systematic uncertainty due to pile-up collisions.

<table>
<thead>
<tr>
<th></th>
<th>Gaussian Fit</th>
<th>Gaussian Ideogram</th>
<th>Full Scan Ideogram</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30% \Delta m_t (\text{no pile up} - \text{with pile up})$</td>
<td>-1.9±0.4</td>
<td>-1.4±0.4</td>
<td>-1.2±0.3</td>
</tr>
</tbody>
</table>

Table 5: Systematic uncertainty due to the underlying event.

<table>
<thead>
<tr>
<th></th>
<th>Gaussian Fit</th>
<th>Gaussian Ideogram</th>
<th>Full Scan Ideogram</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2} \Delta m_t (\text{PARP}(82) = 2.4 - \text{PARP}(82) = 3.4)$</td>
<td>-1.0±0.2</td>
<td>-0.7±0.2</td>
<td>-0.5±0.1</td>
</tr>
</tbody>
</table>

In the PYTHIA model for the underlying event the main parameter is the color screening $p_T$ cut-off value (PARP(82)). When tuned to CDF and UA5 data, its value is equal to 2.9 GeV, with a $3\sigma$ confidence interval of [2.4, 3.4] GeV. With FAMOS in total 200k signal event are produced with both color screening $p_T$ cut-off values 2.4 GeV and 3.4 GeV. The result for each top quark mass estimator are shown in Table 5.

Half of the difference on the top quark mass between both samples is taken as a systematic uncertainty. This estimation is conservative as the $3\sigma$ confidence interval for the color screening $p_T$ cut-off value is used to obtain a $1\sigma$ systematic uncertainty of the top quark mass estimator.
8.3 Jet energy scale

The top quark is reconstructed from its three jets in the decay. In determining the mass of the top quark the reconstructed angles of the jets have a much better resolution compared to the energy scale of these jets. To study the effect of systematic shifts on the inclusive energy scale, the 4-momenta of the jets are scaled by a factor

\[ p_{\text{scaled}}^{jet} = (1 \pm \alpha) E, (1 \pm \alpha) p_x, (1 \pm \alpha) p_y, (1 \pm \alpha) p_z \]  

(7)

where a difference is made between jet origination from light (u,d,s,c) or heavy (b) quarks. A non-zero value of \( \alpha \) was applied before the event selection and before the kinematic fit, resulting in a systematic shift on the value of the inferred top quark mass shown in Figure 21 and Figure 22, respectively for a light and heavy quark jet energy scale shift. The effect is shown for the three top quark mass estimators described above.

Figure 21: Estimated shift in the top quark mass versus a shift \( \alpha \) applied on the inclusive light quark jet energy scale. This is shown for the three different top quark mass estimators: gaussian fit (left), gaussian ideogram (middle) and full scan ideogram (right).

Figure 22: Estimated shift in the top quark mass versus a shift \( \alpha \) applied on the inclusive heavy quark jet energy scale. This is shown for the three different top quark mass estimators: gaussian fit (left), gaussian ideogram (middle) and full scan ideogram (right).

In the reconstruction of the top quark both light and heavy quark jets are included, hence the effect of including both systematic shifts must be combined. The effect of applying the same value of \( \alpha \) for both light and heavy quark jets is shown in Table 6. The effect due to the light quark jet energy scale is only present in the case of the simple Gaussian fit estimator as the \( m_W \) inclusive calibration was not applied for this estimator. As the \( m_W \) inclusive calibration only influences the light quark jet energy scale, this large differentiation between the estimators is not present for the heavy quark jet energy scale systematics.

The potential uncertainty on the inclusive jet energy scale for light quark jets is estimated to be around 3% with 1
fb$^{-1}$ of accumulated data and could reach 2% with 10 fb$^{-1}$ [15]. As the analysis described in this paper focusses on data sets with a larger integrated luminosity a value of 2% systematic uncertainty on the inclusive jet energy scale was taken as a benchmark. The estimated systematic uncertainties on the top quark mass for both the 1 and 10 fb$^{-1}$ jet energy scale benchmarks are shown in Table 6. The systematic uncertainties of 2% on both the light and the heavy quark jet energy scale are combined linearly.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Gaussian Fit $\Delta m_t$ (GeV/c$^2$)</th>
<th>Gaussian Ideogram $\Delta m_t$ (GeV/c$^2$)</th>
<th>Full Scan Ideogram $\Delta m_t$ (GeV/c$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3% light quarks JES shift (1 fb$^{-1}$)</td>
<td>3.6±0.6</td>
<td>0.32±0.23</td>
<td>0.10±0.20</td>
</tr>
<tr>
<td>2% light quarks JES shift (10 fb$^{-1}$)</td>
<td>2.4±0.2</td>
<td>0.22±0.08</td>
<td>0.08±0.07</td>
</tr>
<tr>
<td>3% heavy quarks JES shift (1 fb$^{-1}$)</td>
<td>2.0±0.5</td>
<td>2.0±0.2</td>
<td>1.8±0.2</td>
</tr>
<tr>
<td>2% heavy quarks JES shift (10 fb$^{-1}$)</td>
<td>1.4±0.2</td>
<td>1.3±0.07</td>
<td>1.2±0.07</td>
</tr>
</tbody>
</table>

Table 6: Systematic uncertainties due to a 3% and 2% systematic uncertainty on the inclusive jet energy scale of both light and heavy quark jets.

### 8.4 Perturbative QCD Radiation

Both initial and final state radiation (ISR and FSR) are produced according to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [18] in the PYTHIA showering algorithm where color coherence in the parton shower is accounted for. The main parameters in this model is the general QCD scale parameter $\Lambda_{QCD}$ used in the DGLAP evolution (PARP(61), PARP(72), PARJ(81)) and the virtuality cut-off scale $Q^2_{\text{max}}$ which defines the allowed phase-space for initial state radiation (PARP(67)) and indicates where the final state radiation takes over (PARP(71)). The central values of these parameters are taken according to the tuning of the model to the CDF data as mention in [17] and used to simulate a FAMOS event sample for the signal. The value of $\Lambda_{QCD}$ is changed from 0.25 GeV to respectively 0.15 and 0.35 GeV, while the value of $Q^2_{\text{max}}$ is shifted by changing PARP(67) to get $Q^2_{\text{max}} = 0.25Q^2_{\text{hard}}$ and $Q^2_{\text{max}} = 4Q^2_{\text{hard}}$ for the initial state and accordingly changing PARP(71) to get $Q^2_{\text{max}} = 1Q^2_{\text{hard}}$ and $Q^2_{\text{max}} = 16Q^2_{\text{hard}}$ for the final state perturbative radiation. The resulting shifts in the three top quark mass estimators is shown in Table 7.

For both parameters in the perturbative radiation the extreme shift in the top quark mass is determined, hence $\Delta m_t(\Lambda_{QCD} = 0.35 \text{GeV} - \Lambda_{QCD} = 0.15 \text{GeV})$ and $\Delta m_t(Q^2_{\text{max}} = 0.25Q^2_{\text{hard}} - Q^2_{\text{max}} = 4Q^2_{\text{hard}})$ where the $Q^2_{\text{max}}$ value for the initial state radiation is changed (with corresponding changes in the $Q^2_{\text{max}}$ value for the final state radiation). The systematic uncertainty is defined as the maximum of both shifts divided by four (a factor of two as both the negative and positive side around the central value are accounted for, and another factor of two to reduce this extreme shift to a more realistic interval).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Gaussian Fit $\Delta m_t$ (GeV/c$^2$)</th>
<th>Gaussian Ideogram $\Delta m_t$ (GeV/c$^2$)</th>
<th>Full Scan Ideogram $\Delta m_t$ (GeV/c$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{4} \Delta m_t(\Lambda_{QCD} = 0.35 \text{GeV} - \Lambda_{QCD} = 0.15 \text{GeV})$</td>
<td>-0.8±0.1</td>
<td>-0.27±0.08</td>
<td>-0.22±0.07</td>
</tr>
<tr>
<td>$\frac{1}{4} \Delta m_t(Q^2_{\text{max}} = 0.25Q^2_{\text{hard}} - Q^2_{\text{max}} = 4Q^2_{\text{hard}})$</td>
<td>0.6±0.1</td>
<td>0.06±0.09</td>
<td>-0.03±0.08</td>
</tr>
</tbody>
</table>

Table 7: Systematic uncertainties due to QCD radiation effects as described in the text. The values of $Q^2_{\text{max}}$ are those for the initial state radiation, while those for the final state radiation are changed simultaneously according to the description in the text.

### 8.5 Hadronization

When the perturbative DGLAP evolution equation break down, the non-pertubative fragmentation or hadronization takes over providing the observed hadrons. The phenomenology of this process happens in the confined phase-space of low-momentum transfer and as the process is not yet understood from first principles it is modelled. At this stage no new jets are formed with a high transverse momentum with respect to the parent parton. The local parton-hadron duality supposes that the flow of momentum and quantum numbers at the hadron level tends
to follow that flow established at the parton level. All of the present models are formulated in a probabilistic language to allow them to be simulated by Monte Carlo techniques. In PYTHIA the so-called string hadronization is performed [19].

The parameters of the fragmentation models are tuned to for example LEP and SLD data introducing a large correlation between the parameter values. Two main parameters can be identified, the others being strongly correlated to these. The first is Lund $b$ (PARJ(42)), strongly anti-correlated to Lund $a$ (PARJ(41)), as both arise in the same Lund fragmentation functions for light quarks. These functions express the probability that a hadron consumes a given fraction of the available longitudinal energy-momentum. For the heavy quark fragmentation the Peterson function is used instead. The transverse momenta $p_T$ of the hadrons are generated according to a flavour independent Gaussian probability density function of width $\sigma_q$ (PARJ(21)), the second of two main parameters. It is predicted to be of the order of 0.3 GeV.

The values of both Lund $b$ and $\sigma_q$ tuned to the OPAL data are respectively equal to $0.52 \pm 0.04$ and $0.40 \pm 0.03$ GeV. These uncertainties are however only indications of the resolution of these parameters within the Lund string model in PYTHIA. The fit of the model parameters is performed on several data distributions, but even the best fit results in a $\chi^2$/NDF which is significantly deviating from unity, as the phenomenology and/or implementation of the model does not reflect the true physics.

When changing the parameters Lund $b$ and $\sigma_q$ from their central values to $2\sigma$ deviations, a shift is obtained on the inferred top quark mass. Simulating with FAMOS event samples in these different configurations results in shifts shown in Table 8. For both parameters the average shift from both directions is calculated. The systematic uncertainty is defined as the maximum of the averages of both parameters.

<table>
<thead>
<tr>
<th>$\Delta m_1 \left(Lund b = 0.60 - Lund b = 0.44\right)$</th>
<th>Gaussian Fit $\Delta m_1$ (GeV/c$^2$)</th>
<th>Gaussian Ideogram $\Delta m_1$ (GeV/c$^2$)</th>
<th>Full Scan Ideogram $\Delta m_1$ (GeV/c$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pm 0.2$</td>
<td>$0.2 \pm 0.2$</td>
<td>$0.1 \pm 0.2$</td>
<td>$0.0 \pm 0.2$</td>
</tr>
<tr>
<td>$\pm 0.4$</td>
<td>$0.4 \pm 0.2$</td>
<td>$0.4 \pm 0.2$</td>
<td>$0.3 \pm 0.1$</td>
</tr>
</tbody>
</table>

Table 8: Systematic uncertainties due to our knowledge of the fragmentation as described by the Lund string model implemented in PYTHIA.

### 8.6 Algorithms for b-tagging

In [13] a method is described to measure the b-tagging efficiency of the combined b-tag algorithm from real data. A potential relative uncertainty on this efficiency is demonstrated of about 4% (in the barrel region) and 5% (in the endcap region) applying the method on 10 fb$^{-1}$ of data. The measurement method is however dominated by systematic uncertainties, therefore these potential uncertainties on the b-tag efficiency could be considered as conservative. This would result in a change in the signal-to-noise ratio of the event selection.

To illustrate the effect on the estimated top quark mass, the b-tag criterium in the event selection was changed. Nominally exactly two jets are required to have a b-tag probability $L^b$ above $L^b_{up}=60\%$ and two below $L^b_{low}=40\%$. These limits are changed relative the the nominal ones and the effect on the top quark mass estimators is shown in Table 9.

For an uncertainty of 5% on the b-tag efficiency a corresponding shift in the applied b-tagging cuts in the dimension of the combined b-tag discriminant is determined. The inclusive distributions of the b-tagging discriminant are shown for the simulated $t\bar{t}$ events in Figure 5 for different quark flavours. In the event selection the criteria $L^b_{up}=70\%$ corresponds to a combined b-tag discriminant value of about 1.0. In order to select 5% more b-quarks in a jet sample, this cut-value of the discriminant should shift from 1.0 to 0.74, corresponding according to Figure 5 (right) to a value of $L^b_{up}$ equal to 65%. Assuming a linear dependency between $L^b_{up}$ and the b-tagging discriminant in the relevant region around the value $L^b_{up}=60\%$, it is therefore concluded that the 5% uncertainty in the estimate of the b-tagging efficiency transforms into a 5% change in the value of $L^b_{up}$. The systematic uncertainty due to the b-tagging algorithms is defined as the average of the relative shifts on the top quark mass estimators when changing $L^b_{up}$ from 60% to both 55% and 65%.
To estimate the effect of the other $t\bar{t}$ background processes, the individual event weight for these background events is rescaled from unity to 1.2 or 0.8 reflecting a change of $\pm$ 20% in the cross-section of the process involved. The effect on the top quark mass estimators is shown in Table 10. It is found to be negligible for each of the three top quark mass estimators. As it is expected that $W + jets$ and QCD processes have a negligible contribution after the event selection, it is assumed that their influence on the top quark mass estimators is negligible.

<table>
<thead>
<tr>
<th>Other $t\bar{t}$ decays</th>
<th>Gaussian Fit $\Delta m_t$ (GeV/c$^2$)</th>
<th>Gaussian Ideogram $\Delta m_t$ (GeV/c$^2$)</th>
<th>Full Scan Ideogram $\Delta m_t$ (GeV/c$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20%</td>
<td>0.1±0.5</td>
<td>-0.1±0.4</td>
<td>-0.2±0.4</td>
</tr>
<tr>
<td>+20%</td>
<td>0.2±0.4</td>
<td>-0.4±0.4</td>
<td>-0.4±0.4</td>
</tr>
</tbody>
</table>

Table 10: Systematic uncertainties due to the scale of the background processes.

### 8.7 Background processes

The reference method is taken to be the CTEQ6M which includes Next-to-Leading Order QCD corrections. The central PDF’s can be reweighted according to the estimated uncertainties (both to the positive ($j+$) and negative ($j-$) side of the central fitted value ($c$)) on the 20 parameters used in the CTEQ fit method. The event weight depends on the value of $x$, the flavour of the partons in the hard process and the $Q^2$ value of the event taken to be equal to $m_t^2$. This procedure results in 40 shifts $\Delta m_t^j$ from the central value of $m_t^2$, as shown in Figure 23. The typical uncertainty on the difference between the top quark mass inferred from the central set and the reweighted set is 0.02 GeV/c$^2$, hence significantly smaller than the shift on the mass itself. The actual effect due to the uncertainties arising from the CTEQ fits is determined via the following procedure:

$$ (\Delta_{PDG} m_t^j)^2 = \sum_{j=1}^{20} (\max [m_{t+j} - m_t^c, m_{t-j} - m_t^c, 0])^2 $$

and
\[
(\Delta_{PDFl}^m)^2 = \sum_{j=1}^{20} \left( m^{\text{central}} - m^{j_+}, m^{\text{central}} - m^{j_-}, 0 \right)^2
\]  

(9)

where \( m^{\text{central}} \) is the central top quark mass inferred from the \texttt{FAMOS} simulated event sample applying the central \texttt{CTEQ6M} fit. This procedure takes into account the sign of the correlation between the PDF parameters and the inferred observable of interest, in this case the top quark mass. The resulting systematic uncertainty, \( \Delta_{PDFl}^m \), and \( \Delta_{PDFl}^m \) are quoted in Table 11. The largest of the two is taken as a systematic uncertainty.

Figure 23: The 40 values of \( \Delta m \) obtained by applying the three top quark mass estimators on the reweighted samples rather than the central sample as described in the text: simple Gaussian fit (left), Gaussian parametrized ideogram (middle) and full scanned ideogram (right).

<table>
<thead>
<tr>
<th>( \Delta_{PDFl}^{m_+} ) (GeV/c(^2))</th>
<th>Gaussian Fit</th>
<th>Gaussian Ideogram</th>
<th>Full Scan Ideogram</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta_{PDFl}^{m_-} ) (GeV/c(^2))</td>
<td>0.09 ± 0.02</td>
<td>0.08 ± 0.01</td>
<td>0.06 ± 0.01</td>
</tr>
</tbody>
</table>

Table 11: Systematic uncertainties due to the estimated uncertainties on the fitted parameters of the \texttt{CTEQ6M} parton distribution fits as described in the text.

8.9 Reducing systematic uncertainties

The largest systematic uncertainty originates from the definition of the 4-momenta of hadronical jets. In the analysis described only the Iterative Cone (IC) algorithm was applied, although several alternative clustering algorithms exist. The top quark mass estimator could be made more robust against systematic uncertainties when a set of different clustering algorithms result in the same direction of the four leading jets in the final state. The Midpoint Cone (MC) and the \( k_T \) (KT) algorithms are applied to the events, and angular deviations between the four selected leading jets are studied. These four jets are selected from those connected to the primary vertex and with the highest transverse momentum. All possible jet-to-jet pairings are made between the three different clustering algorithms, and the sum of the angular difference in an \((\eta, \phi)\)-metric is determined and denoted as \( \alpha_{a,b} \) for the differences between jet clustering algorithm \( a \) and \( b \). Among all jet-to-jet pairings within an event, the one resulting in the smallest value of \( \alpha_{a,b} \) is chosen. For that jet-to-jet pairing between events clustered with different algorithms, a single jet pair results in the largest distance in an \((\eta, \phi)\)-metric among the four pairs in the event, this distance is denoted as \( \alpha_{a,b}^{\text{max}} \). In Figure 24 the values for \( \alpha_{MC,IC}^{\text{max}} \), \( \alpha_{KT,IC}^{\text{max}} \), and \( \alpha_{MC,KT}^{\text{max}} \) are shown for the selected signal events. The four leading jets rather than the three from the hadronic top quark decay are considered to be independent of the jet combination method within an event.

For 85.7%, 87.5% and 78.1% of the selected signal events, the value of respectively \( \alpha_{MC,IC}^{\text{max}}, \alpha_{KT,IC}^{\text{max}}, \) and \( \alpha_{MC,KT}^{\text{max}} \) is larger than 0.3. On the basis of this observation an alternative selection is proposed including an extra event selection cut which rejects those events where one of the values \( \alpha_{MC,IC}^{\text{max}}, \alpha_{KT,IC}^{\text{max}}, \) or \( \alpha_{MC,KT}^{\text{max}} \) exceeds 0.3. The efficiency of this cut for the selected signal events is 76.1%. Applying the full scanned ideogram based top quark mass estimator on this sample of events, results in systematic uncertainties shown in Table 12.
Figure 24: Values of $\alpha^{\text{IC,MC}}_{\text{max}}$, $\alpha^{\text{KT,MC}}_{\text{max}}$, $\alpha^{\text{IC,K}T}_{\text{max}}$ (middle) and $\alpha^{\text{KT,M}C}_{\text{max}}$ (right) as described in the text for the selected signal events.

Table 12: Overview of all uncertainty components for the top quark mass estimators described in the text. When the component is marked with ($*$) the statistical uncertainty dominated the systematic shift of the effect.

<table>
<thead>
<tr>
<th></th>
<th>Standard Selection</th>
<th>Alternative Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian Fit</td>
<td>$\Delta m_t$</td>
<td>$\Delta m_t$</td>
</tr>
<tr>
<td></td>
<td>(GeV/c$^2$)</td>
<td>(GeV/c$^2$)</td>
</tr>
<tr>
<td>Pile-Up</td>
<td>1.9</td>
<td>1.2</td>
</tr>
<tr>
<td>Underlying Event</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Jet Energy Scale (light)</td>
<td>2.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Jet Energy Scale (heavy)</td>
<td>1.4</td>
<td>1.2</td>
</tr>
<tr>
<td>Radiation (pQCD)</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>Fragmentation</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>b-tagging</td>
<td>2.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Background ($*$)</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Parton Density Functions</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Total Systematical uncertainty</td>
<td>4.9</td>
<td>1.9</td>
</tr>
<tr>
<td>Statistical Uncertainty (10fb$^{-1}$)</td>
<td>0.32</td>
<td>0.31</td>
</tr>
<tr>
<td>Total Uncertainty</td>
<td>4.9</td>
<td>1.9</td>
</tr>
</tbody>
</table>

The total systematic uncertainty on the top quark mass estimator is not significantly reduced from 1.9 GeV/c$^2$ for the standard event selection. When including the extra event selection cut on $\alpha^{a,b}_{\text{max}}$ where $a$ and $b$ are the Iterative Cone, Midpoint Cone or $k_T$ clustering algorithm only a small improvement is observed on the systematic uncertainty from the knowledge of the jet energy scale. This improvement in not significant, but should be checked with larger simulated event samples.

8.10 Combining systematic uncertainties

The systematic uncertainties as described above are summarized in Table 12. Conservatively a total precision on the top quark mass of 1.9 GeV/c$^2$ can be reached. The uncertainty is dominated by systematic effects like pile-up collisions and the knowledge of the jet energy scale of b-quark jets. Upon a better understanding of the accelerator settings and the detector performance however this total uncertainty will reduce. When both in-time and out-of-time pile-up collisions will be monitored the residual uncertainty is provided by the uncertainties in the description of the underlying event and to a smaller extend due to systematic fluctuations in the pile-up. The main effect of the pile-up collisions is on the energy scale of the reconstructed jets, which will be measured with dedicated analyses performed on data. Therefore the largest part of this systematic uncertainty on the top quark mass overlaps with the uncertainty quoted from the jet energy scale. The effect of the description of the underlying event on each of the top quark estimators is mentioned in Table 12 and is small but not negligible. In Table 12 a 3$\sigma$ effect on the most important parameter in the tuning of the underlying event description is accounted for, hence conservative.
Our understanding of the underlying event model can be considered (and certainly in the future when new tuning data becomes available) as being better with a factor of two. In Table 13 the systematic uncertainty on the top quark mass due to the underlying event description is therefore reduced with a factor of two.

It is believed that the magnitude of pile-up collisions can be monitored to the level of 10%. This reduces the uncertainties quoted in Table 12 with a factor of 3. For example for the $M_{t}^{\text{fullidego}}$ estimator the pile-up effect can be extrapolated to 0.42 GeV/c$^2$ which overlaps with the uncertainty due to the jet energy scale knowledge. To take into account this overlap, the systematic shift in the top quark mass estimators due to a 10% variation in the pile-up collisions is divided by two, hence 0.21 GeV/c$^2$ for the $M_{t}^{\text{fullidego}}$ estimator.

The uncertainty on the energy scale of b-quark jets is taken to be 2% in Table 12. This energy scale can be calculated either from independent event samples like $Z\ell\ell$ or can be determined as a ratio with respect to the energy scale of light quark jets. This number can be extrapolated to about 1.5% upon a better understanding of the detector performance and with the application of advanced tools like energy flow algorithms. Also the worse understood regions in the detector could rejected for the measurement of the top quark mass. For example for the $M_{t}^{\text{fullidego}}$ estimator the effect of a 1.5% uncertainty on the jet energy scale is 0.96 GeV/c$^2$ which is a linear combination of the effect on light and heavy quark jets.

In Table 12 for the b-tagging performance a 5% uncertainty is taken on the b-tag efficiency dominated by systematic uncertainties of radiation effects. The experience at the Tevatron collider [20] illustrates that an uncertainty of 2% could be reached. Therefore the uncertainties on the top quark mass estimators can be rescaled to match this precision. For example for the $M_{t}^{\text{fullidego}}$ estimator the effect of the b-tagging uncertainty becomes 0.18 GeV/c$^2$.

The systematic effect determined on the top quark mass estimators due to the remaining background (20% variation) is dominated by its statistical precision. All of the 6 shifts in Table 10 deviate from zero by no more than 1 standard deviation. It is therefore assumed that the real effect, extrapolated to larger simulated event samples, is half of this statistical precision. For example for the $M_{t}^{\text{fullidego}}$ estimator the effect of the background becomes 0.25 GeV/c$^2$.

<table>
<thead>
<tr>
<th></th>
<th>Gaussian Fit $\Delta m_t$ (GeV/c$^2$)</th>
<th>Standard Selection $\Delta m_t$ (GeV/c$^2$)</th>
<th>Full Scan Ideogram $\Delta m_t$ (GeV/c$^2$)</th>
<th>Alternative Selection $\Delta m_t$ (GeV/c$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pile-Up</td>
<td>0.32</td>
<td>0.23</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>Underlying Event</td>
<td>0.50</td>
<td>0.35</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Jet Energy Scale (light)</td>
<td>1.80</td>
<td>0.15</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Jet Energy Scale (heavy)</td>
<td>1.05</td>
<td>0.98</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>Radiation (pQCD)</td>
<td>0.80</td>
<td>0.27</td>
<td>0.22</td>
<td>0.20</td>
</tr>
<tr>
<td>Fragmentation</td>
<td>0.40</td>
<td>0.40</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>b-tagging</td>
<td>0.80</td>
<td>0.20</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>Background</td>
<td>0.30</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Parton Density Functions</td>
<td>0.12</td>
<td>0.10</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>Total Systematical uncertainty</td>
<td>3.21</td>
<td>1.27</td>
<td>1.13</td>
<td>1.07</td>
</tr>
<tr>
<td>Statistical Uncertainty (10fb$^{-1}$)</td>
<td>0.32</td>
<td>0.36</td>
<td>0.21</td>
<td>0.31</td>
</tr>
<tr>
<td>Total Uncertainty</td>
<td>3.23</td>
<td>1.32</td>
<td>1.15</td>
<td>1.11</td>
</tr>
</tbody>
</table>

Table 13 summarizes and combines the extrapolated systematic uncertainties on each of the top quark mass estimators. The uncertainty on the inferred top quark mass of about 1 GeV/c$^2$ is dominated by the uncertainty on the energy scale of the b-quark jets. This relative uncertainty is taken to be 1.5% which is feasible by selecting only events which have their leading jets in a detector region which is better understood, usually the central or barrel region of the detector. Also in this central region the contributions from underlying event and pile-up are smaller compared to the more forward regions. The inclusive jet energy scale and its resolution can be improved by applying more advanced reconstruction tools as for example energy or particle flow algorithms connecting the calorimeter information with the information provided by the central tracker device. An uncertainty of 1.5% on the b-quark jet energy scale can therefore be set as a goal for the performance of jet calibration methods.
9 Conclusion

The reconstruction and selection of semi-leptonic $t\bar{t}$ events is described for the decay channel where the lepton is a muon. The event selection reaches a high signal-to-noise ratio and the background from non-$t\bar{t}$ processes is negligible. A kinematic fit is applied to force the reconstructed W boson mass into the event to its precise measured value, resulting in improved resolutions of the four-momentum of the jets. Remaining $t\bar{t}$ events from other decay channels are reduced by a likelihood ratio method combining the information of several topological observables which can differentiate the correct from the wrong combinations. Three different top quark mass estimators are constructed: a simple fit on the reconstructed top quark mass spectrum and two event-by-event likelihood methods which convolute the resolution function of the event or so-called ideogram with the expected theoretical template. In the theoretical template used in the convolution several event weights are applied according to the likelihood of having the correct $t\bar{t}$ decay channel and the correct jet combination. The properties of each of the three estimators are studied. The results indicate a slope of unity between the generated and estimated top quark mass, a unit width of the pull distribution and a small bias. The improvement of the convolution techniques with respect to the fit on the reconstructed top quark mass spectrum is shown to be significant.

The effect on the estimated top quark mass from all relevant systematic uncertainties is estimated for each of the three estimators. Again a clear improvement is demonstrated by applying the event-by-event convolution methods including a kinematic fit with respect to the simple approach of fitting the reconstructed top quark mass spectrum. Measuring the top quark mass at the LHC with an uncertainty below 2 GeV/c$^2$ is feasible. Aiming for an uncertainty below 1 GeV/c$^2$ remains challenging, but after a better understanding of mostly the jet energy scale of b-quark jets and the invent of more advanced analysis tools still feasible. Benchmarks or goals for the performance of jet calibration tools are set. An uncertainty of 1.5% on the jet energy scale of b-quark jets should be obtained in part of the detector range to measure the top quark mass with a precision of about 1 GeV/c$^2$.

Apart from the theoretical uncertainties which are conservatively estimated in this note, the most important systematical uncertainties are the uncertainty on the b-jet energy scale and the influence of pile-up collisions. With the reconstructed tools and statistical inference methods applied in this note a significant reduction of the uncertainty on the top quark mass is obtained compared to simple top quark mass estimators.

Constraining the analysis to events for which the three main jet clustering algorithms give comparable results for the direction of the four leading jets, a small but not significant improvement is observed on the total uncertainty on the inferred top quark mass.

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References


