SOME IMPLICATIONS OF A VIOLATION OF THE
POMERANCHUK THEOREM *)

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ABSTRACT

Implications of a possible violation of the Pomeranchuk theorem in the KN system are discussed, with emphasis on the consequences of such a violation for non-forward scattering, both for the elastic processes and for $K^0_s$ regeneration.

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1. **INTRODUCTION**

The surprising results of the first total cross-section measurements at Serpukhov \(^1\) have led to some speculation that the Pomeranchuk theorem \(^2\), which asserts the asymptotic equality of the total cross-sections for particle and anti-particle on the same target, might not be valid. I would like to discuss some properties, mainly for non-forward scattering, that follow on quite general grounds from the assumption that particle and anti-particle total cross-sections tend to different constants as the energy increases.

The near-equality of the measured \(^1\) total cross-sections for \(\pi^-p\) and \(\pi^-n\), and also for \(K^-p\) and \(K^-n\), indicates that any Pomeranchuk theorem violating amplitude which has isotopic spin one (in the \(t\), i.e., exchange, channel), if not zero, is probably quite small. Indeed, Höhler \(^3\) has argued that the difference between the \(\pi^-p\) and \(\pi^-n\) total cross-sections is not incompatible with what would be expected from ordinary Reggeized \(\rho\) exchange. However, the Serpukhov data, considered together with measurements of the total cross-section for \(K^+p\) at lower energies \(^4\), do leave room for a fairly sizable violation of the Pomeranchuk theorem with isospin zero.

Figure 1 shows the measured values of the \(K^-p\) and \(K^+p\) total cross-sections up to the highest energies at which they have been measured (\(p_{LAB} = 55\) GeV/c for \(K^-p\), \(p_{LAB} = 20\) GeV/c for \(K^+p\), from Refs. 4 and 5). The \(K^-p\) cross-section can be seen to be quite constant, at a value of about 20.5 mb, above \(p_{LAB} = 30\) GeV/c. If the difference of these cross-sections is to behave as one would expect in a simple Regge theory, that is to decrease roughly as \((p_{LAB})^{-1/2}\), then the \(K^+p\) total cross-section would have to rise by slightly more than 1 mb as \(p_{LAB}\) increased from 20 to 55 GeV/c. On the other hand, since this cross-section is such a remarkably constant function of energy below 20 GeV/c, one might expect it to continue to be constant as the energy increases further. These two possible extrapolations \(^6\) are indicated by the dashed and the solid lines, respectively, in Fig. 1. Presumably, a measurement of the \(K^+p\) total cross-section at Serpukhov which is expected to be available in the
very near future, will be able to easily distinguish between these two possibilities.

I do not want to suggest that the data which are presently available favour the interpretation that the $K^+p$ and $K^-p$ total cross-sections do asymptotically approach different constants; in fact, I shall point out below that data on $K^0_S$ regeneration seem to favour the opposite alternative. Nevertheless, it is true that if one looks at the available total cross-section data naively - that is, without theoretical prejudice - one will conclude that each total cross-section has already reached a constant value, and so it is interesting to consider the possibility that this simple behaviour persists at higher energies.

In the next section I will discuss some of the theoretical consequences of the assumption of a violation of the Pomeranchuk theorem. I will be describing some results which have been proven rigorously, but I will not give any proofs, and, in fact, I shall ruthlessly sacrifice rigour in the hope of elucidating some of the physical ideas which lie behind these results. A much more rigorous treatment of some of the ideas I shall mention, and of many that I shall not, has been given by Eden and Kaiser 7). In the last section I will discuss the extent to which the theoretically predicted effects should be expected to be observable in measurements at reasonable energies, and conversely, the extent to which an observed absence of these effects should be taken as evidence in favour of the Pomeranchuk theorem.

2. THEORETICAL CONSEQUENCES OF POMERANCHUK THEOREM VIOLATION

Let us then assume that particle and anti-particle total cross-sections approach unequal constants at high energy; as pointed out by Cornille and Simão 9), many of the results that follow from this assumption could also be obtained from somewhat more general assumptions. For purposes of notation, we will specialize to the case of $K^±p$ scattering, although the results we will describe do not depend on this specification.
Let $T_{\pm}(s,t)$ be the $s$ channel helicity non-flip amplitude for $K^\pm p$ elastic scattering, normalized so that the optical theorem reads

$$\Im m \ T_{\pm}(s,0) = \sigma_{tot}^{K^p}(s),$$

and define

$$\Delta \sigma = \lim_{s \to \infty} \left[ \sigma_{tot}^{K^p}(s) - \sigma_{tot}^{K^+ p}(s) \right].$$

The forward dispersion relations then imply that, as $s \to \infty$,

$$\Re e \ T_{\pm}(s,0) \to \pm \frac{\Delta \sigma}{\pi} \ln s.$$

One can prove the Pomeranchuk theorem if one assumes that the ratio $(\Re e T / \Im m T)$ remains bounded as the energy increases; from Eqs. (1) and (3), this assumption clearly implies that $\Delta \sigma = 0$.

In terms of $J$ plane singularities of the continued $t$ channel partial wave amplitudes, the asymptotic behaviour implied by Eqs. (1) and (3), and by our assumptions that $\sigma_{tot} \to \text{const.}$, $\Delta \sigma \neq 0$, corresponds to simple poles of both even and odd signature. The real part of the amplitude, being anti-symmetric between $T_+$ and $T_-$, arises from the odd signature pole, while the imaginary part, being approximately symmetric, comes mainly from the even signature pole.

The situation for $t \neq 0$ is somewhat more complicated. The reason for the complication can be seen as follows: from Eq. (3), the high energy behaviour of the forward differential cross-section for either $K^+ p$ or $K^- p$ is given by

$$\frac{d\sigma}{dt} \bigg|_{t=0} = \frac{1}{16 \pi} \left| T \right|^2 \to \frac{(\Delta \sigma)^2}{16 \pi^3} (\ln s)^2,$$

(4)
However, we have assumed the total cross-section, a fortiori the integrated elastic cross-section, to remain bounded as the energy increases. Thus the width of the forward peak must decrease like \((\ln s)^{-2}\). This decrease is the most rapid that is allowed on general grounds \(^{10}\), and is not achieved by most simple models; for example, an ordinary Regge model predicts the width of the forward peak to decrease only like \((\ln s)^{-1}\).

Models have been constructed in which this \((\ln s)^{-2}\) decrease would be achieved, and in which analyticity requirements would be satisfied \(^{9},^{11},^{12}\). These models have suggested the following two addition properties for non-forward scattering in the case of Pomeranchuk theorem violation:

i) the \(J\) plane contains colliding complex branch points, whose position near \(t=0\) is given by

\[
\phi_\pm(t) = 1 \pm i a \sqrt{-t} + O(t) \quad (5)
\]

For \(t=0\), these branch points collapse at the point \(J=1\) to form a simple pole, as is required by Eqs. (1) and (3).

ii) the differential cross-section has zeros at values of momentum transfer \(t_1\) which approach \(t=0\) as the energy increases:

\[
\frac{d\sigma}{dt}(s,t) = 0 \quad \text{for} \quad t_1(s) = \frac{(\text{const})i}{(\ln s)^2} \quad (6)
\]

Subsequently, it was shown that property i) follows from general requirements of analyticity and unitarity, independently of any model \(^{13}\). A weaker form of property ii) has also been proven: it has been shown that the elastic scattering amplitude, considered as a function of complex \(t\), must have zeros either on or very close to the negative real (i.e., physical) \(t\) axis, and that these zeros approach the point \(t=0\) as the energy increases \(^{7},^{14},^{15}\).

One might expect such zeros, even if not quite on the real \(t\) axis, to have a profound effect on the differential cross-section. Unfortunately, specific counter-examples have demonstrated
that, without further assumptions, one cannot prove that these zeros lead to any observable structure at all \(^{16}\). On the other hand, Roy has shown \(^{17}\) that if the violation of the Pomeranchuk theorem is severe enough - in a sense which I shall discuss below - then the zeros do have to lie in the real \(t\) axis, and so would have to affect the differential cross-section.

It has been widely felt that the necessity for such complicated behaviour constitutes evidence, on aesthetic grounds, against the assumption of a violation of the Pomeranchuk theorem. However, it may be that the complication is only apparent; perhaps we are looking at a simple phenomenon in a complicated way. In fact, properties i) and ii) are closely analogous, at least mathematically, to the simplest picture we can have of high energy scattering.

To see this, consider the elastic amplitude \(T(s,t)\) written in the \(s\) channel impact parameter representation:

\[
T(s,t) = 8\pi \int_0^\infty b db J_0(b\sqrt{t}) a(s,b).
\]  

(7)

Imagine that we somehow knew that \(a(s,b)\) was zero for \(b\) greater than some number \(R\), and that we also knew that the total cross-section, which from Eqs. (1) and (7) is given by

\[
\sigma_{\text{tot}} = 8\pi \int_0^\infty b db \text{Im} a(s,b),
\]  

(8)

had the value \(\sigma_{\text{tot}} = 4\pi R^2\). Since unitarity requires

\[
|a(s,b)| \leq 1
\]  

(9)

we would then know that \(a(s,b) = i\) for \(b < R\) is necessary in order that the right-hand side of (8) attain its maximum value of \(4\pi R^2\). This in turn would enable us to know the scattering amplitude away from the forward direction; from (7), \(T(s,t)\) would be given by
\[ T(s, t) = \frac{8\pi i R}{\sqrt{-t}} J_1(R\sqrt{-t}) \]

and so would have zeros coming from the zeros of the Bessel function \( J_1 \). The point of this example is to show that, when we know that we saturate a unitarity bound, we are in good position to prove things about the angular distributions.

Now, Martin has shown \(^{18}\) that there is at each energy a finite radius \( R \), in the sense that in the case in which we are interested we can get the right answer if we imagine that

\[ a(s, b) = 0 \quad \text{for} \quad b > R \equiv \frac{\ln s}{2m_\pi} \]

the pion mass appears because the two-pion state is the lightest system which can be exchanged in the \( t \) channel. According to Eqs. (1) and (3), the amplitude \( T \) is, at high energy, predominantly real; this means that the real part of \( a(s, b) \) will be, although small, quite close to its unitarity limit, since unitarity for a predominantly real amplitude is much more restrictive than is implied by Eq. (9). We saw this before when we noticed that unitarity required the width of the forward peak to decrease with \( (\ln s)^{-2} \), which is the maximum decrease allowed by Eq. (11).

It is the fact that the real part of \( T \) comes close to its unitarity limit which, together with the analyticity and crossing properties that one expects from local quantum field theory, enables one to prove results at non-zero values of \( t \), such as the position of Regge singularities given by Eq. (5) or the existence of zeros on or close to the negative \( t \) axis. The actual proofs are of course quite complicated, but we may obtain these same results in a simple way if we imagine that the unitarity limit is approached so closely, that the real part of \( a(s, b) \) is forced to be a constant for \( b < R \).
We would then have, in analogy to Eq. (10),

$$\text{Re } T_\pm (s,t) = \frac{\pm y m_n}{\pi \sqrt{-t}} \Delta \sigma J_1 (R \sqrt{-t})$$

(12)

where $R$ is defined in Eq. (11). But notice that the amplitude in Eq. (10) is imaginary, while in the case in which we are interested it is predominantly real. Equation (12) is in fact the model amplitude which has been proposed by Anselm et al. $^{11}$ We can see from Eq. (12), and from the knowledge that the real part of the amplitude dominates, that there will be structure in the angular distribution given by

$$\frac{d\sigma}{dt} (s,t) \approx 0 \text{ when } -t = \frac{y m_n^2 C_i^2}{(\ln s)^2}$$

(13)

where $C_i$ is the $i^{th}$ zero of $J_1$. Also, if we choose to analyze Eq. (12) in the $t$ channel angular momentum plane, we can find complex branch points whose position is given in Eq. (5) (with $a = 1/2 m_n$), but this is merely a complicated way of looking at a simple result.

Another consequence of having a finite radius $R$ is that one expects a minimum value, $\Delta t$, to the width of the forward peak, proportional to $R^{-2}$ [roughly, the distance to the first zero in Eq. (13)]. That is, we expect the integrated elastic cross-section $\sigma_{el}$ to obey

$$\sigma_{el} \geq (\Delta t) \left( \frac{d\sigma}{dt} \right)_{t=0}, \text{ when } \Delta t \sim \frac{1}{R^2} \sim \frac{1}{(\ln s)^2}$$

(14)

From Eqs. (4) and (14), we obtain a lower bound on $\sigma_{el}$ in terms of $\Delta \sigma$. More precisely, it can be proven $^{7,10}$ that, at sufficiently large energy
\[ \sigma_{el}(K^\pm p) \geq \frac{m_n^2}{\pi^2}(\Delta \sigma)^2 \]  \hspace{1cm} (15)

Another result can be obtained from the observation, from Eqs. (1) and (3), that the real part of the amplitude dominates the imaginary part, and that the magnitude of \( \text{Re} T_+ \) is the same as the magnitude of \( \text{Re} T_- \). This circumstance enabled Kinoshita\(^{19}\) to prove that, at least very close to \( t = 0 \), the ratio of the \( K^+ p \) and \( K^- p \) differential cross-sections must approach unity as the energy increases.

Finally, consider the amplitude for the regeneration of \( K^0_s \) from neutrons\(^{20}\):

\[ T(k_s^0 n \rightarrow K^0_s n) = \frac{i}{2} \left[ T(k_s^0 n \rightarrow K^0_n) - T(k_s^0 \rightarrow K^0_s n) \right] \]  \hspace{1cm} (16)

which by isospin invariance can be rewritten

\[ T(k_s^0 n \rightarrow K^0_s n) = \frac{i}{2} \left[ T_+ - T_- \right] \]  \hspace{1cm} (17)

We have seen that the difference of \( T_+ \) and \( T_- \) is much larger than the sum; thus, at large energy the regeneration amplitude is essentially the same as \( T_+ \). This means that the properties we have described for the elastic process, such as the need for rapid shrinkage, the probable existence of structure in the angular distribution and the bound on the integrated cross-section given in Eq. (15), hold for the regeneration process also. Similarly, the amplitude for \( K^0_s \) regeneration from protons is the difference of the amplitudes for \( K^+ n \) and \( K^- n \) scattering, and so again should have the same properties. Let us define

\[ \sigma_{Reg} = \int dt \left( \frac{d\sigma}{dt} \right)_{K_s^0 p \rightarrow K_s^0 p} \]  \hspace{1cm} (18)
and record for discussion in the next section the bound \(^{21}\) analogous to Eq. (15):

\[
\lim_{s \to \infty} \sigma_{\text{reg}} (s) \geq \frac{m_n^2}{n^3} (\Delta \sigma_n)^2
\]  

(19)

where

\[
\Delta \sigma_n \equiv \lim_{s \to \infty} \left[ \sigma_{\text{tot}}^n (s) - \sigma_{\text{tot}}^{n+} (s) \right]
\]  

(20)

3. PHENOMENOLOGICAL CONSEQUENCES OF POMERANCHUK THEOREM VIOLATION

Suppose that we may write the elastic amplitude \( T \) as the sum of an abnormal part, \( T_A \), which violates the Pomeranchuk theorem, and a normal part, \( T_N \), which does not:

\[
T = T_A + T_N
\]  

(21)

Roughly speaking, we may identify \( T_A \) with the real part of \( T \), and \( T_N \) with the imaginary part, or \( T_A \) with the odd signature part and \( T_N \) with the even signature part. From Eqs. (1) and (3),

\[
\frac{|T_A|}{|T_N|} \sim \frac{\text{Re} T}{\text{Im} T} = \frac{\Delta \sigma}{\sigma_{\text{tot}}} \frac{\ln s}{\pi}
\]  

(22)

Thus, at sufficiently large energies the abnormal part wins, and the effects we have been discussing must show up, unless \( \Delta \sigma = 0 \). However, the experimental indication is that \( \Delta \sigma / \sigma_{\text{tot}} \) is at most about 15\%, and even at Batavia \( \ln s \) (in GeV\(^2\)) \( \gtrsim 2 \pi \); hence \( T_A \) constitutes at most 30\% of the amplitude, or 10\% of the cross-section, and so there is no reason to expect the predicted effects to be observable. Of course, it could be that the estimate in Eq. (22) is
erroneous, and that Pomeranchuk theorem violating effects do constitute
a significant portion of the elastic cross-section at attainable
energies, but we should not be surprised if these effects are not
observed.

When we consider an inelastic reaction, such as that
for $K_s^0$ regeneration, the situation is quite different. Here $T_A$
has to compete not with the ordinary Pomeron, which it beats only
by a factor of log $s$, but with ordinary odd signature Regge poles,
which it beats by a power of $s$. If the difference of the $K^+p$ and
$K^-p$ total cross-sections is sensibly constant above $p_{LAB} = 30$ GeV/c,
then $T_A$ must already be the dominant part of the odd signature
amplitude at this energy, and we would guess, although we certainly
cannot prove, that the effects we have been describing would have
begun to appear by that energy.

In fact, presently available data on $K_s^0$ regeneration
is almost good enough to rule out the possibility of any significant
violation of the Pomeranchuk theorem. Figure 2, which is taken from
Ref. 21), shows the values of $\sigma_{Reg}$ measured in the experiment of
Brody et al. at SLAC 22) up to $p_{LAB} = 7$ GeV/c. The one open data
point is a guess at an upper limit for $\sigma_{Reg}$ at $p_{LAB} = 36$ GeV/c,
based on the forward regeneration measurement of Ref. 23). The
horizontal line in the Figure indicates the lower bound of 8 $\mu$b for
$\sigma_{Reg}$ obtained from Eq. (19) using the value $\Delta \sigma_n = 2.25$ mb, and
the dashed line is a fit by hand to the data.

Since the limit in (19) need not be approached from above,
it could be that the value of $\sigma_{Reg}$ at some energy were slightly
below 8 $\mu$b, and that the bound (19) was still obeyed. However, if
the value of $\sigma_{Reg}$ falls very far below 8 $\mu$b, as the trend of
the SLAC data indicates that it will, then the eventuality that
$\sigma_{Reg}$ would ultimately rise to obey the bound (19), while not
completely impossible, must be regarded as an added burden of im-
plausibility for the hypothesis of Pomeranchuk theorem violation to
bear. The data presented in Fig. 2 can be considered to be evidence
against a violation of the Pomeranchuk theorem in the kaon-nucleon system.
On the other hand, this Figure indicates that if the Pomeranchuk theorem is indeed violated, then it is violated in a particularly interesting way. Suppose we define

\[
\lambda = \frac{\pi r^2 \lim_{s \to \infty} \sigma_{\text{Reg}}}{m^2 (\Delta \sigma_n)^2}
\]

the bound (19) implies (for \( \Delta \sigma_n \neq 0 \)) that \( \lambda \geq 1 \). This parameter \( \lambda \) may be taken to be a measure of the strength of the violation of the Pomeranchuk theorem, because Roy has shown \(^{17}\) that if \( \lambda \) is sufficiently close to 1, then the scattering amplitude cannot be too different from that given in Eq. (12), which in turn implies that zeros of the amplitude must indeed lie on the real \( t \) axis, not too far from the positions given in Eq. (13). In general, \( \lambda \) need not be anywhere near one; however, it appears reasonable from Fig. 2 that, if the bound (19) is to be respected at all, then \( \lambda \) will be of the order of one.

A very exciting, if extremely unlikely, possibility, would be for the observed trends both of the total cross-sections and of the regeneration cross-section to continue to higher energy. Suppose that the difference of the \( K^+ n \) and \( K^- n \) total cross-sections was observed to be extremely constant throughout the Serpukhov, or even the Batavia range, and that the value of \( \sigma_{\text{Reg}} \) continued to fall rapidly as the energy increased. In this circumstance we could still say that we had not yet reached the asymptotic region - the total cross-section difference, no matter how constant it seemed, could ultimately disappear, or the regeneration cross-section, no matter how small it had become, could ultimately rise - but I think we would also be wise to seriously question some of the assumptions, such as isospin invariance or even the analyticity which can be derived from a local field theory, which have been used in the derivation of the bound (19). Certainly nothing in the present situation indicates that this must be the case. I would like to suggest, however, that if the \( K^+ p \) total cross-section is seen not to rise above 17.5 mb when it is measured at Serpukhov, then a measurement of \( \sigma_{\text{Reg}} \) above \( p_{\text{LAB}} = 10 \text{ GeV/c} \) would be extremely interesting.
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FOOTNOTES AND REFERENCES

3) G. Höhler, Report to the Sixth Rencontre de Moriond (1971).
6) Numerical fits corresponding to these two possibilities have been given by R.J.N. Phillips, Fifth Rencontre de Moriond (1970), and by V. Barger, Sixth Rencontre de Moriond (1971).
16) L.B. Okun and V.S. Popov, private communication, and G. Auberson, rumour.
20) Since Eq. (16) will get us in trouble later on, it should be pointed out that its validity depends on the correctness of our understanding of the quantum mechanics of the neutral kaon system. This equation is valid in the usual theories of CP violation.


22) A.D. Brody et al., Contribution to the XVth International Conference on High-Energy Physics, Kiev (1970).

FIGURE CAPTIONS

Figure 1 The $K^+p$ and $K^-p$ total cross-sections. Data points (solid line) or that it falls like $(p_{LAB})^{-1/2}$ (dashed line).

Figure 2 The $K_L^0 \rightarrow K_S^0$ integrated cross-section, compared with its lower bound of $8 \mu b$ from Ref. 19). Data points (solid line) are from Ref. 22).