Rectifiers

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Abstract
In particle accelerators, rectifiers are used to convert the AC voltage into DC or low-frequency AC to supply loads like magnets or klystrons. Some loads require high currents, others high voltages, and others both high current and high voltage. This presentation deals with the particular class of line commutated rectifiers (the switching techniques are treated elsewhere). The basic principles of rectification are presented. The effects of real world parameters are then taken into consideration. Some aspects related to the filtering of the harmonics both on the DC side and on the AC side are presented. Some protection issues associated with the use of thyristors and diodes are also treated. An example of power converter design, referring to a currently operating magnet power supply, is included. An extended bibliography (including some internet links) ends this presentation.

1 Introduction
In particle accelerators, electrons or other charged particles are forced to move along orbits or trajectories by means of magnetic fields. The intensity of the magnetic fields needed to obtain the desired effects is related to the energy of the particles. Electromagnets, conventional hot ones or superconducting ones, are normally used. The excitation current in the magnets can range from some amperes for small orbit correction coils to some hundreds or thousands of amperes (see, for example, Refs. [1] and [2]). The power converters needed to cover such a wide current range have widely differing structures and characteristics and, for the same power requirement, several solutions are often possible.

In this paper I show the topologies and the characteristics of a particular class of rectifiers—the line commutated ones—that was and still is widely used in particle accelerator facilities. Even today, in the ‘PWM Era’, line commutated rectifiers are operating. Moreover, Switch Mode Power Supplies (SMPS) very often include in their structure ‘conventional’ rectifiers as input or output stages or both.

Since the currents in the magnets have either to be varied according to the energy (or the required changes in the orbit) of the particles or at least have to be ramped from the turn on values to their final values (this is quite important if the time constant of the load — a magnet string — is high), the rectifiers use thyristor-based structures or mixed ones (diodes and thyristors or diodes/thyristors and transistors).

The effects on the rectifier behaviour of the inductive components of the load and of the AC line will be investigated. The use of passive filters to reduce the harmonic content (ripple) of the voltage and current at the output of the rectifier will be discussed.

Even if this is not a specific topic for this lecture, some protection issues related to the components (snubber and bucket circuits) and to the converter as a whole will be briefly mentioned.

The studies to reduce the harmonics on the line-current and to improve the power factor (Refs. [3], [4]) and the use of Pulse-Width Modulation (PWM) techniques have brought forth more sophisticated rectifier designs with the absorption of a quasi-sinusoidal waveform of the line current.
Performance parameters

2.1 Definition

Before starting to examine different topologies for single-phase or multi-phase rectifiers, we should define some parameters. These parameters are needed to compare the performances among the different structures.

Let us assume we have ideal switches (diodes or thyristors) with zero commutation time (i.e., instantaneous turn on and off) and zero on-resistance (i.e., when conducting they present neither voltage drop nor losses). The load itself is an ideal resistance. The generic scheme is shown in Fig. 1. At the input of the rectifier there are one or more AC voltages from the secondary of the transformer. At the output of the rectifier, on the load, there is also a time-dependent voltage. This voltage, as will be shown, is a combination of the voltages at the input of the rectifier stage.

The DC voltage on the load is the average over the period \( T \) of the output voltage of the rectifier:

\[
V_{DC} = \frac{1}{T} \int_{0}^{T} v_L(t) \, dt. \tag{1}
\]

Similarly, it is possible to define the r.m.s. voltage on the load:

\[
V_L = \sqrt{\frac{1}{T} \int_{0}^{T} v_L^2(t) \, dt}. \tag{2}
\]

The ratio of the two voltages is the Form Factor (FF):

\[
FF = \frac{V_L}{V_{DC}}. \tag{3}
\]

This parameter is quite important since it is an index of the efficiency of the rectification process.

Having assumed the load to be purely resistive, it is possible to define the currents as

\[
i_L(t) = \frac{v_L(t)}{R_L} \tag{4}
\]

\[
i_{DC} = \frac{V_{DC}}{R_L} \tag{5}
\]
The rectification ratio ($\eta$), also known as rectification efficiency, is expressed by

$$\eta = \frac{P_{\text{DC}}}{P_L + P_D}$$  \hfill (7)

where

$$P_{\text{DC}} = V_{\text{DC}} \cdot I_{\text{DC}}$$ \hfill (8)

$$P_L = V_L \cdot I_L$$ \hfill (9)

$$P_D = R_D \cdot I_L^2.$$ \hfill (10)

In Eq. (10), $P_D$ represents the losses in the rectifier ($R_D$ is the equivalent resistance of the rectifier). By developing Eq. (7), using Eqs. (5) and (26), we get:

$$\eta = \frac{V_{\text{DC}} \cdot I_{\text{DC}}}{V_L \cdot I_L + R_D \cdot I_L^2} = \frac{V_{\text{DC}}^2}{V_L^2} \cdot \frac{1}{1 + \left(\frac{R_D}{R_L}\right)}.$$ \hfill (11)

We have assumed ideal switches, with no losses, that is $R_D = 0$. Therefore

$$\eta = \left(\frac{V_{\text{DC}}}{V_L}\right)^2 = \left(\frac{1}{FF}\right)^2.$$ \hfill (12)

The Ripple Factor ($RF$) is another important parameter used to describe the quality of the rectification. It represents the smoothness of the voltage waveform at the output of the rectifier (we have to keep in mind that our goal is to obtain a voltage and a current in the load as steady as possible). The $RF$ is defined as the ratio of the effective AC component of the load voltage versus the DC voltage:

$$RF = \frac{\sqrt{V_L^2 - V_{\text{DC}}^2}}{V_{\text{DC}}} = \sqrt{FF^2 - 1}.$$ \hfill (13)

A transformer is most often used both to introduce a galvanic isolation between the rectifier input and the AC mains and to adjust the rectifier AC input voltage to a level suitable for the required application. One of the parameters used to define the characteristics of the transformer is the Transformer Utilization Factor ($TUF$):

$$TUF = \frac{P_{\text{DC}}}{\text{Effective Transformer VA Rating}} = \frac{P_{\text{DC}}}{\frac{VA_P + VA_S}{2}}$$ \hfill (14)

where $VA_P$ and $VA_S$ are the power ratings at the primary and secondary of the transformer.

It should be noted that some authors (e.g., Ref. [7]) use only the term $VA_S$ as 'Effective Transformer VA Rating'. Here, a more complete definition, the average of primary and secondary $VA$ ratings, has been chosen (e.g., Ref. [8] or Ref. [9]). This is why different $TUF$ values are found in the literature for those topologies—the ‘single-way’ ones—with different power ratings at primary and secondary.
In order to compare the different topologies, it is useful to also take into consideration some parameters related to the switches—diodes or thyristors—like, for example, the Peak Inverse Voltage (PIV) during the blocking state of the device or the maximum current in the load. In practice, one has to choose devices with a peak repetitive reverse voltage ($V_{RRM}$ as reported on the data sheets) and a peak repetitive forward current ($I_{FRM}$) higher than the PIV and maximum load current.

3 Basic rectifier structures

3.1 Introduction

As previously mentioned, from the particle physics point of view, the ideal power converter is the one that supplies the best direct current to the load (e.g., magnet or klystron): very low ripple, very high stability, etc. As we shall see later, this goal is achieved by using three-phase systems (on the primary winding of the transformer; at the input of the rectifier more phases can be present) and full-wave rectifiers (the stability issue is more related to the control of the converter than to its structure). Nevertheless, single-phase rectifiers are still in use both as low-power stand-alone converters (up to some kilowatts) and as output stage in Switched Mode Power Supplies (SMPS).

In this section, we shall see the main topologies for single-phase and multi-phase rectifiers. The half-wave ones are reported just for comparison.

We assume that all voltages at the input of the rectifiers have sinusoidal waveforms with period $T_{mains} = 20$ ms (corresponding to $f_{mains} = 50$ Hz). With the usual definition

$$\alpha = 2 \cdot \pi \cdot f = \frac{2 \cdot \pi}{T},$$

the generic AC voltage has the following expression:

$$v(t) = V \cdot \sin(\alpha \cdot t).$$

In this section we assume pure resistive loads and ideal switches as defined in the previous section. In Section 5 we shall see how things change in the real world.

3.2 Single-phase systems

3.2.1 Half-wave rectifier

This is the simplest structure (Fig. 2). Only one diode is placed at the secondary of the transformer.

Figure 3 shows the waveforms of the voltage at the secondary and of the current in the load. Since the load is a resistance, the voltage on the load is proportional to the current.
It is quite evident why this type of rectifier is called half-wave: the rectification process occurs only during half-periods. It is also called single-way because the load current $i_L(t)$ always circulates in the secondary winding in the same direction.

**Fig. 3:** Waveforms of the single-phase, single-way, half-wave rectifier

Using the definitions reported in the previous section, we get the following results:

$$V_{DC} = \frac{1}{T} \int_{0}^{T} v_L(t) dt = \frac{1}{2\pi} \int_{0}^{\pi} V_S \sin(\phi t) dt = \frac{V_S}{\pi}.$$  \hspace{1cm} (17)

And, similarly, we can calculate the other parameters:

$$V_L = \sqrt{\frac{1}{T} \int_{0}^{T} v_L^2(t) dt} = \sqrt{\frac{1}{2\pi} \int_{0}^{\pi} V_S^2 \sin^2(\phi t) dt} = \frac{V_S}{2}.$$ \hspace{1cm} (18)

$$I_{DC} = \frac{V_{DC}}{R_L} = \frac{V_S}{\pi \cdot R_L}.$$ \hspace{1cm} (19)

$$I_L = \frac{V_{L}}{R_L} = \frac{V_S}{2 \cdot R_L} = I_S.$$ \hspace{1cm} (20)

The current in the secondary of the transformer can flow only when the diode conducts and therefore it is equal to the current in the load:

$$FF = \frac{V_L}{V_{DC}} = \frac{\pi}{2}.$$ \hspace{1cm} (21)

$$\eta = \left( \frac{1}{FF} \right)^2 = \frac{4}{\pi^2} = 0.405.$$ \hspace{1cm} (22)

$$RF = \sqrt{FF^2 - 1} = 1.21.$$ \hspace{1cm} (23)

The poor performance of this rectifier is also confirmed by the utilization of the transformer. From Eq. (14), we get

$$TUF = 0.323 \quad (or \ TUF = 0.286 \ according \ to \ some \ authors).$$ \hspace{1cm} (24)
A direct current flows in the secondary of the transformer. This may result in saturation of the core, which has to be sized accordingly.

From Fig. 3 it is clear that the inverse voltage seen by the diode in its blocking state is the negative half-wave of $v_S(t)$. Similarly, the current that flows across the diode is the same as flows in the load. For this topology, one has to choose diodes with

$$V_{RRM} > V_S \quad \text{and} \quad I_{FRM} > \frac{V_S}{R_L}.$$  \hspace{1cm} (25)

### 3.2.2 Full-wave rectifier — centre-tapped

In order to use both halves of the secondary AC voltage waveform, one can use two diodes and create a return path for the current by adding a tap at the centre of the secondary winding (Fig. 4). This is the so-called centre-tapped rectifier.

![Fig. 4: Structure of the single-phase, single-way, full-wave rectifier](image)

Diode D1 conducts during the positive half-wave of the voltage. Diode D2 conducts in the negative half. The current always flows from the common point of the diodes, through the load and back to the central tap of the transformer.

As shown in Fig. 5, the rectification occurs during the whole period of the voltage. This is a full-wave rectifier.

It has to be noted that in this case as well the current flows in the same direction through the two halves of the secondary winding. Therefore this is also a single-way structure.

![Fig. 5: Waveforms of the single-phase, single-way, full-wave rectifier](image)
Using the definitions reported in the previous section and the symmetries, we get the following results:

\[
V_{\text{DC}} = \frac{1}{T} \int_{0}^{\pi} v_L(t)dt = \frac{2}{2\pi} \int_{0}^{\pi} V_S \sin(\sigma t)dt = \frac{2 \cdot V_S}{\pi} \quad (26)
\]

\[
V_L = \sqrt{\frac{1}{T} \int_{0}^{\pi} v_L^2(t)dt} = \sqrt{\frac{1}{\pi} \int_{0}^{\pi} V_S^2 \sin^2(\sigma t)dt} = \frac{V_S}{\sqrt{2}} \quad (27)
\]

\[
I_{\text{DC}} = \frac{V_{\text{DC}}}{R_L} = \frac{2 \cdot V_S}{\pi \cdot R_L} \quad (28)
\]

\[
I_L = \frac{V_L}{R_L} = \frac{V_S}{\sqrt{2} \cdot R_L} \quad (29)
\]

\[
FF = \frac{V_L}{V_{\text{DC}}} = \frac{\pi}{2 \cdot \sqrt{2}} = 1.11 \quad (30)
\]

\[
\eta = \left( \frac{1}{FF} \right)^2 = 0.81 \quad (31)
\]

\[
RF = \sqrt{FF^2 - 1} = 0.483. \quad (32)
\]

As it is a single-way topology, there is a direct current in both the secondary windings; this results in a low \(TUF\) (compared to the bridge solutions, see next section).

\[TUF = 0.671 \text{ (or } TUF = 0.572 \text{ according to some authors).} \quad (33)\]

Even though this solution is much better than the previous one, there are some drawbacks. As can be seen from Fig. 4, when a diode is conducting, the other, which is in the blocking state, sees the inverse voltage of both windings of the secondary. The \(PIV\) of the diodes is higher. From the diode current point of view, this topology is equivalent to two half-waves acting alternately. For this topology, one has to choose diodes with

\[
V_{\text{RMM}} > 2 \cdot V_S \quad \text{and} \quad I_{\text{FRM}} > \frac{V_S}{R_L}. \quad (34)
\]

### 3.2.3 Full-wave rectifier — bridge

The bridge structure is the best single-phase rectifier (Figs. 6 and 7). At the cost of two more diodes, several advantages are obtained. This is a full-wave rectifier, but compared with the centre-tapped solution it uses a simpler transformer, with a single secondary and no additional taps.
The rectification takes place by the conduction of couples of diodes. Diodes D1 and D4 are conducting during the positive half-wave of the voltage. Diode D2 and D3 are conducting during the negative half. This is a double-way topology. In each half-cycle the current flows in both directions in the secondary winding but always in the same direction in the load. There is no DC component in the winding and the core can be smaller than that for a centre-tapped rectifier with the same DC power rating.

Since this is a full-wave topology, Eqs. (28) to (32) are still valid but the transformer utilization factor is different. A sinusoidal current flows in both the primary and secondary windings, therefore \( V_{A_P} = V_{A_S} \). From the definition (14), using (26) and (28) and considering that \( i_S(t) = i_L(t) \) we get

\[
TUF = \frac{V_{DC} \cdot I_{DC}}{\sqrt{2} \cdot \sqrt{2}} = 0.813. \tag{35}
\]

This is considerably higher than the \( TUF \) of the centre-tapped structure shown in (33).

Looking at the \( PIV \) of the diodes, \( V_S \) is the highest voltage seen by each diode in its blocking state. Therefore the diodes must have
Summing up: at the cost of two more diodes with reduced voltage ratings, we have a full-wave rectifier, which, compared to the centre-tapped case of Section 3.2.2, for the same $V_{DC}$ and $P_{DC}$ requires a simpler and smaller transformer (23% oversized instead of 75%).

### 3.2.4 Summary

Table 1 (taken from Ref. [7]) summarizes the main performance parameters defined in Section 2 for the three configurations presented above.

<table>
<thead>
<tr>
<th>Table 1: Performance parameters for single-phase topologies</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td><strong>Half-wave</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Peak repetitive reverse voltage $V_{RRM}$</td>
</tr>
<tr>
<td>r.m.s. input voltage per transformer leg $V_{Sm}$</td>
</tr>
<tr>
<td>Diode average current $I_{F(AV)}$</td>
</tr>
<tr>
<td>Diode peak repetitive forward current $I_{FRM}$</td>
</tr>
<tr>
<td>Diode r.m.s. current $I_{F(m)}$</td>
</tr>
<tr>
<td>Form factor of diode current – $I_{F(rms)}/I_{F(AV)}$</td>
</tr>
<tr>
<td>Form factor – $FF$</td>
</tr>
<tr>
<td>Rectification ratio – $\eta$</td>
</tr>
<tr>
<td>Ripple factor – $RF$</td>
</tr>
<tr>
<td>Transformer rating primary $VA$</td>
</tr>
<tr>
<td>Transformer rating secondary $VA$</td>
</tr>
<tr>
<td>Transformer utilization factor – $TUF$</td>
</tr>
<tr>
<td>Output ripple frequency $f_{R}$ ($f_{mains} = 50$ Hz)</td>
</tr>
</tbody>
</table>

The values reported have been reorganized in terms of $V_{DC}$ (designer view): to achieve a given DC output voltage one has to find the other design parameters going backwards from the output to the AC mains.

Single-phase diode rectifiers, in the bridge configuration as well, require a high transformer $VA$ rating for a given DC output power. This type of rectifier is suitable for low power applications, up to some kilowatts.

### 3.3 Multi-phase systems

#### 3.3.1 Single-way structures (also known as star-connected rectifiers)

The use of single-way configurations—one diode per phase, each diode is conducting while the others are blocked—becomes more convenient as the number of phases increases.

The circuit shown in Fig. 2 is single-phase. The circuit in Fig. 4 could be called bi-phase. By extension, the circuit in Fig. 8 is $m$-phase.
Figure 9 shows the waveforms of the phase voltages (in this example $m = 3$) and of the current in the load. Each phase voltage has the same amplitude ($V_S$) and the same frequency. There is a phase displacement of $2\pi/m$ electrical radians between one voltage and the next. In one period there is a specific number of peaks (usually called pulses), depending on the number of phases and on the structure of the rectifier. The number of pulses in a period is indicated by $p$.

For single-way topologies, the number of pulses is equal to the number of phases, i.e., $p = m$.

Each diode is conducting for $2\pi/m$ electrical radians and the rectified voltage can be expressed by [8]

$$V_{DC} = \frac{V_S}{2 \cdot \pi} \int_{\frac{\pi}{m}}^{\frac{2\pi}{m}} \cos(\phi t) dt = V_S \cdot \frac{\sin \left( \frac{\pi}{m} \right)}{\frac{\pi}{m}}, \quad \text{(37)}$$
From the definition of ripple factor (13), it is possible to write

$$m \to \infty \Rightarrow FF \to 1 \Rightarrow RF \to 0.$$  

This means that by increasing the number of phases in a multi-phase, single-way rectifier, the result of the rectification is improved, i.e., the output voltage is smoother.

Connecting to a conventional three-phase mains distribution, it is possible to increase the number of ‘phases’ by using transformers with $m$ separated secondary coils. The secondary coils can be connected in a great number of combinations, sometimes quite exotic, as can be found in the literature (see for example Ref. [10]).

**Fig. 10:** Six-phase star-connected rectifier

The $m$-phase single-way connections are also known as star-connected rectifiers. Looking at the configuration of the secondary windings of the rectifier presented in Fig. 10 (taken from Ref. [7]), the origin of the name is quite clear.

The values for $V_{DC}$ and some other parameters have been calculated for $m = 6$, $m = 12$ and $m = 24$ and are reported in Table 2.

As can be seen, passing from 6 to 12 pulses one gets a 3.5% improvement in the rectified voltage while passing from 12 to 24 pulses this improvement is less than 1%. This is also shown by the figures of the form factor for the three cases.
Table 2: Performance parameter comparison for multi-phase, single-way topologies

<table>
<thead>
<tr>
<th></th>
<th>m = 6</th>
<th>m = 12</th>
<th>m = 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{DC}/V_S$</td>
<td>0.955</td>
<td>0.989</td>
<td>0.997</td>
</tr>
<tr>
<td>$V_L/V_S$</td>
<td>0.956</td>
<td>0.989</td>
<td>0.997</td>
</tr>
<tr>
<td>Form factor ($V_L/V_{DC}$)</td>
<td>1.001</td>
<td>1.0001</td>
<td>1.0000</td>
</tr>
<tr>
<td>Ripple factor</td>
<td>0.042</td>
<td>0.0103</td>
<td>0.0026</td>
</tr>
<tr>
<td>Ripple frequency</td>
<td>$6f_{mains}$</td>
<td>$12f_{mains}$</td>
<td>$24f_{mains}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>12 vs. 6</th>
<th>24 vs. 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{DC}$ vs. $V_{DC}$</td>
<td>1.035</td>
<td>1.009</td>
</tr>
<tr>
<td>$V_L$ vs. $V_L$</td>
<td>1.034</td>
<td>1.009</td>
</tr>
<tr>
<td>FF vs. FF</td>
<td>0.999</td>
<td>1</td>
</tr>
<tr>
<td>RF vs. RF</td>
<td>0.245</td>
<td>0.249</td>
</tr>
</tbody>
</table>

From Fig. 9 and Table 2 it is clear that the frequency of the ripple on the output is $p$ times the mains frequency $f_{mains}$. This means that by increasing the number of phases (as stated before, in single-way topologies, $m = p$), the ripple frequency increases and its amplitude decreases. This fact eases the making of filters to reduce the ripple in the load.

The advantages of the reduced amplitude and increased frequency of the voltage ripple for the 12 and 24 pulses structures are counterbalanced by the growing complexity of the connections of the transformer’s secondary windings. In practice, for single-way connections, the maximum number of pulses is normally 12. As will be shown later, a higher number of pulses can be achieved by using combinations of bridge structures.

In addition to the major complexity of the connections at the transformer’s secondary, in single-way structures the current always flows in the same direction in each winding. There is a DC component that may saturate the iron core and result in a poor utilization of the transformer, which has to be correspondingly oversized. The best Transformer Utilization Factor ($TUF$) that can be achieved with a single-way connection is $TUF = 0.79$ while with a bridge configuration it is possible to reach higher values, up to $TUF = 0.955$ [8].

3.3.2 Six-pulse bridge configurations

In a bridge configuration, the number of pulses is twice the number of phases ($p = 2m$). It is possible to obtain the same values for the rectified voltage and ripple factor using fewer phases, i.e., simpler transformers with fewer windings and better utilization factor (fewer oversized transformers).

Starting from the basic 6-pulse structure shown in Fig. 11 it is possible to combine two bridges in order to obtain 12 or more pulse rectifiers.

The $PIV$ on the diodes in a bridge rectifier is half the $PIV$ in an equivalent star rectifier: it is possible to use components with a lower $V_{RRM}$. 

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In Fig. 11 the secondary of the transformer is connected as a ‘Y’. Starting from a three-phase mains distribution there are four possible combinations for the connections at the primary and the secondary of the transformer: delta–delta, delta–Y, Y–delta and Y–Y (Fig. 12). They are not equivalent. A delta primary requires three mains lines, without neutral, and avoids the so-called excitation unbalance. With this connection, each winding is tied between two lines, the nonsinusoidal exciting currents can be taken from the supply system so that there is a complete ampere-turn balance and the excitation unbalance is avoided [10].

The Y secondary has some advantages compared to a delta one with the same turn ratio between primary and secondary: the rectified voltage is $\sqrt{3}$ times higher; the current in the windings is the same as in the load; there is an easily accessible common zero-point in case one wants to get two voltages with opposite sign (each side of the bridge acts as a single-way rectifier with $m = 3$).

The bridge structure is a double-way configuration; the secondary windings do not carry any DC component and the currents are well balanced. The power ratings at primary and at secondary are equal.

From the definitions presented in Section 2 and taking into account the symmetries given by the presence of $p = 6$ pulses in the period, we get

$$V_{DC} = \frac{6}{2\pi} \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sqrt{3} \cdot V_S \cdot \sin(\omega t)\,dt = \frac{3 \cdot \sqrt{3}}{\pi} V_S = 1.654 \cdot V_S \quad (41)$$

$$V_L = \sqrt{\frac{9}{4\pi} \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} V_S^2 \cdot \sin^2(\omega t)\,dt} = V_s \cdot \sqrt{\frac{3}{2} + \frac{9 \cdot \sqrt{3}}{4 \cdot \pi}} = 1.655 \cdot V_S \quad (42)$$
As for the rectified voltage, the bridge acts as a single-way system with \( p = 2m \) pulses. By putting \( 2m \) in Eqs. (37) and (38) instead of \( m \), we obtain the same results.

Calculating the other performance parameters of Section 2, we get

\[
FF = 1.009 \quad \eta = 0.998 \quad RF = 0.042.
\]  

(43)

The r.m.s. current in each secondary winding is given by

\[
I_S = \frac{\sqrt{3} \cdot V_S}{R_L} \sqrt{\frac{2}{\pi} \left( \frac{\pi}{6} + \frac{\sqrt{3}}{4} \right)},
\]

(44)

and the r.m.s. current through a diode is

\[
I_D = \frac{\sqrt{3} \cdot V_S}{R_L} \sqrt{\frac{1}{\pi} \left( \frac{\pi}{6} + \frac{\sqrt{3}}{4} \right)}.
\]

(45)

The \( TUF \) is calculated from definition (14) and is

\[
TUF = \frac{P_{DC}}{V_{Srms} \cdot I_{Srms}} = 0.955.
\]

(46)

3.3.3 Twelve-pulse bridge configurations

As shown in Table 2, a 12-pulse system performs much better in terms of rectification efficiency and ripple content, both in amplitude and frequency, than a 6-pulse one. The three-phase bridge, along with the possibility to use indifferently delta or Y secondary connections without affecting the performance of the rectifier, makes it possible to build 12-pulse structures quite easily, avoiding complex transformer configurations.

Figure 13 shows two 6-pulse bridges connected in series with the associated waveforms.

![Fig. 13: Structure and voltage waveforms for two six-pulse bridges in series](image_url)
In order to achieve a proper 12-pulse operation, as shown in the plots on the left of Fig. 13, a phase displacement of 30 degrees has to be introduced between the corresponding phase-to-phase voltages of the two 6-pulse units. This is easily achieved by connecting one secondary as delta and the other as Y.

The primary connection is normally delta to avoid excitation unbalance.

In order to obtain equal secondary voltages, the number of turns of the two secondary windings must be in a ratio of $1:\sqrt{3}$. Since $\sqrt{3}$ is an irrational number, the turn ratio of the two secondary windings can only be approximated. Good ratios are 4:7 (1/1.75, i.e., +1% off) or 7:12 (1/1.71, i.e., −1% off) [10].

The values of the rectified voltage and of the other parameters are summarized in Table 3 (extracted from Ref. [7]).

It is also possible to connect two 6-pulse bridges in parallel, as shown in Fig. 14 (the voltages are the same of Fig. 13). In this case, it is necessary to insert an interphase reactance between the bridges in order to adjust the instantaneous voltage difference.

The load voltage, $v_L(t)$, is the average of the two output voltages from the bridges, $v_{B1}(t)$ and $v_{B2}(t)$.

### 3.3.4 Summary

As reported also in Ref. [8], for multi-phase systems we can make the following observations:

- The higher the number of pulses, the better the utilization of the rectifier, the lesser the ripple amplitude and the higher the ripple frequency — this implies that filtering the ripple is easier. Nevertheless, systems with a number of pulses higher than 12 (normally obtained by combining two three-phase bridges) are not often used since their advantages are compensated by their growing complexity.

- Bridge structures are the most convenient in terms of TUF and PIV on diodes.
Single-way structures may become convenient for those applications where the output voltage is so low that the voltage drop on diodes is no longer negligible. In a bridge there are two diodes conducting and the voltage drop is double.

Table 3 (extracted from Ref. [7]) summarizes the main performance parameters for the three-phase topologies described here. As in Table 1, the parameters are expressed in terms of DC output (designer’s view). As already stated at the beginning of this section, in the literature and in practice there are many other possible topologies, mainly based on particular arrangements of the transformer windings or using more transformers connected via interphase reactors ([10], [11]).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>3-ph star (single-way)</th>
<th>6-ph star (single-way)</th>
<th>6-pulse bridge</th>
<th>12-pulse series br.</th>
<th>12-pulse parallel br.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak reverse voltage $V_{RRM}$</td>
<td>2.092 $V_{DC}$</td>
<td>2.092 $V_{DC}$</td>
<td>1.05 $V_{DC}$</td>
<td>0.524 $V_{DC}$</td>
<td>1.05 $V_{DC}$</td>
</tr>
<tr>
<td>r.m.s. input voltage $V_{Srms}$</td>
<td>0.855 $V_{DC}$</td>
<td>0.74 $V_{DC}$</td>
<td>0.428 $V_{DC}$</td>
<td>0.37 $V_{DC}$</td>
<td>0.715 $V_{DC}$</td>
</tr>
<tr>
<td>Diode average current $I_{F(AV)}$</td>
<td>0.333 $I_{DC}$</td>
<td>0.167 $I_{DC}$</td>
<td>0.333 $I_{DC}$</td>
<td>0.333 $I_{DC}$</td>
<td>0.167 $I_{DC}$</td>
</tr>
<tr>
<td>Diode forward current $I_{FRM}$</td>
<td>3.63 $I_{F(AV)}$</td>
<td>6.28 $I_{F(AV)}$</td>
<td>3.14 $I_{F(AV)}$</td>
<td>3.033 $I_{F(AV)}$</td>
<td>3.14 $I_{F(AV)}$</td>
</tr>
<tr>
<td>Diode r.m.s. current $I_{F rms}$</td>
<td>0.587 $I_{DC}$</td>
<td>0.409 $I_{DC}$</td>
<td>0.579 $I_{DC}$</td>
<td>0.576 $I_{DC}$</td>
<td>0.409 $I_{DC}$</td>
</tr>
<tr>
<td>Curr. form factor – $I_{F rms}/I_{F(AV)}$</td>
<td>1.76</td>
<td>2.45</td>
<td>1.74</td>
<td>1.73</td>
<td>2.45</td>
</tr>
<tr>
<td>Form factor – $FF$</td>
<td>1.0165</td>
<td>1.0009</td>
<td>1.0009</td>
<td>1.00005</td>
<td>1.00005</td>
</tr>
<tr>
<td>Rectification ratio – $\eta$</td>
<td>0.968</td>
<td>0.998</td>
<td>0.998</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Ripple factor – $RF$</td>
<td>0.182</td>
<td>0.042</td>
<td>0.042</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Transf. rating primary $VA$</td>
<td>1.23 $P_{DC}$</td>
<td>1.28 $P_{DC}$</td>
<td>1.05 $P_{DC}$</td>
<td>1.01 $P_{DC}$</td>
<td>1.01 $P_{DC}$</td>
</tr>
<tr>
<td>Transf. rating secondary $VA$</td>
<td>1.51 $P_{DC}$</td>
<td>1.81 $P_{DC}$</td>
<td>1.05 $P_{DC}$</td>
<td>1.05 $P_{DC}$</td>
<td>1.05 $P_{DC}$</td>
</tr>
<tr>
<td>Transf. Utilization Factor – $TUF$</td>
<td>0.73</td>
<td>0.647</td>
<td>0.952</td>
<td>0.971</td>
<td>0.971</td>
</tr>
<tr>
<td>Output ripple freq. $f_{R}$</td>
<td>3 $f_{mains}$</td>
<td>6 $f_{mains}$</td>
<td>6 $f_{mains}$</td>
<td>12 $f_{mains}$</td>
<td>12 $f_{mains}$</td>
</tr>
</tbody>
</table>

4 Three-phase controlled rectifiers

4.1 Introduction

In the first section we said that it is necessary to vary the output voltage of the rectifier. The structures seen in Section 3 provide output voltages that are in a fixed ratio with the input AC voltages. The diodes alone cannot satisfy our requirements. The next step is to substitute the diodes (all or only some of them — creating the so-called full- or semi-controlled bridges) with thyristors.

The thyristor is a device whose transition from the blocking to the conducting state depends not only on the polarity of the anode–cathode voltage (as for diodes, which are naturally commutating devices) but is also controlled via the application of an adequate current pulse. Thyristors have three terminals: the trigger pulse is applied to the gate while the anode–cathode voltage is positive. The name thyristor derives from the Greek word $\text{thy}-$, meaning ‘switch’, and the suffix -istor, which derives from transistor (trans-fer res-istor) to indicate that the device belongs to the semiconductor family [12]. Sometimes it is called Silicon Controlled Rectifier (SCR) to distinguish it from similar devices like the Gate Turn-Off thyristor (GTO) or the TRIode to control AC (Triac) or others, much
less capable of handling high power, that are often used in the circuitry generating the trigger pulses for the SCR. In the rest of this paper, thyristor means SCR.

In this section we still assume that we have ideal switches and purely resistive loads.

Here we shall only consider three-phase systems and those topologies that are more commonly used to supply the load with variable voltage and, consequently, variable current.

4.2 Three-phase fully controlled bridge

Figure 15 is equivalent to Fig. 11: here thyristors have replaced diodes.

Similarly Fig. 16 shows the voltage waveforms with a delay angle \( \alpha = 45 \) degrees.

The delay angle or firing angle, indicated as \( \alpha \), is defined as that angle in electrical radians or electrical degrees comprised between the instant at which the thyristor would naturally switch on if it were a diode and the instant at which the trigger pulse is applied and the thyristor starts to conduct (assuming ideal devices with instantaneous turning on/off). In a bridge structure, two switches are
conducting at the same time, i.e., two trigger pulses must be applied simultaneously to the couples of thyristors that must conduct.

In order to calculate the rectified voltage as a function of the delay angle $\alpha$, starting from definition (1) and considering the symmetries, one should consider the two cases:

$$V_{\text{DC}}(\alpha) = \frac{2}{2\pi} \int_{\alpha}^{\frac{\pi + \alpha}{3}} \sqrt{3} \cdot V_s \cdot \sin(\alpha t + \frac{\pi}{3}) \, dt = \frac{3}{\pi} \cdot \sqrt{3} \cdot V_s \cdot \cos(\alpha) \quad 0 \leq \alpha \leq \frac{\pi}{3}$$

$$V_{\text{DC}}(\alpha) = V_{\text{DC0}} \cdot \cos(\alpha)$$

$$V_{\text{DC}}(\alpha) = \frac{6}{2\pi} \int_{\alpha}^{\frac{3\pi}{3}} \sqrt{3} \cdot V_s \cdot \sin(\alpha t + \frac{\pi}{3}) \, dt = \frac{3}{\pi} \cdot \sqrt{3} \cdot V_s \left[ 1 + \cos(\alpha + \frac{\pi}{3}) \right] \quad \frac{\pi}{3} < \alpha \leq \frac{2 \cdot \pi}{3}$$

Equation (47) is valid when the condition of continuous conduction (i.e., the instantaneous voltage at the DC terminals is at all times positive) is satisfied. For delay angles beyond 60 degrees the instantaneous voltage at the DC terminals goes to zero (negative if the load has an inductive component, as will be seen later) for a while and the current does not flow continuously anymore. Figure 17 shows the load voltage waveforms at four different values of $\alpha$.

![Waveforms of a three-phase fully controlled bridge rectifier at different values of $\alpha$](image)

**Fig. 17:** Waveforms of a three-phase fully controlled bridge rectifier at different values of $\alpha$

All performance parameters defined in Section 2 can be recalculated as function of $\alpha$ and there are different equations depending on the type of conduction. They are reported in Appendix A.

### 4.3 Three-phase current regulator and diode rectifier

Thyristors are available with $V_{\text{RRM}}$ and $V_{\text{DRM}}$ higher than 6500 V (or more) and, at the same time, on-state average currents exceeding 1200 A or even 3000 A (e.g. Powerex TBK0 or FT1500AU-240 or EUPEC T2871N or T2563N). Nevertheless there are applications, like RF klystrons, requiring much higher voltages, up to 100 kV [13] and [14].

Connecting thyristors in series, in order to handle the high voltage, introduces the additional problem of their simultaneous firing: a very good equalization of the trigger pulses as well as of the voltage drop on the stack must be guaranteed. If some thyristors of the stack are already conducting while others are still turning on, the voltage on the components may reach destructive levels.
For this reason it is preferable to use stacks of diodes, which are naturally commutating devices that require no additional trigger. In order to keep the possibility of controlling the output voltage, a pre-regulation with thyristors on the AC side is used, see Fig. 18.

![Fig. 18: Three-phase current regulator and diode bridge rectifier](image)

The rectified output voltage as a function of the delay angle $\alpha$ is expressed by [15]

$$V_{DC}(\alpha) = \frac{V_{DC0}}{2} \cdot \left[ 1 + \cos(\alpha + \frac{\pi}{3}) \right] \quad 0 \leq \alpha \leq \frac{\pi}{3}$$

$$V_{DC}(\alpha) = \frac{V_{DC0}}{2} \cdot \sqrt{3} \cdot \sin(\alpha + \frac{\pi}{3}) \quad \frac{\pi}{3} < \alpha \leq \frac{\pi}{2}$$

$$V_{DC}(\alpha) = \frac{V_{DC0}}{2} \cdot \sqrt{3} \cdot \left[ 1 - \cos(\alpha - \frac{5 \cdot \pi}{6}) \right] \quad \frac{\pi}{2} < \alpha \leq \frac{5 \cdot \pi}{6}$$

For additional details, see the bibliography.

### 4.4 Three-phase uncontrolled or controlled bridge and linear regulator

Quite often there is a need to supply many low power loads (up to some kilowatts) or loads that require a dynamics higher than that directly achievable with a plain line commutated rectifier, or there is a need to supply loads having different characteristics with the same converter (this is the typical case of multi-purpose spare power supplies). Mixed structures can be applied, as shown in Fig. 19.

![Fig. 19: Multi-channel linear transistor regulators with common diode rectifier (left) and fully controlled rectifier with single linear transistor regulator (right)](image)
and using as return path the common point, provide positive and negative DC voltage to power 4-
quadrant (bipolar in voltage and current) transistor regulators.

The use of a thyristor bridge alone is not the best solution to supply loads that require variable
currents or loads with different characteristics (as in the case of spare power supplies), because at low
currents the delay angle is big and the ripple on the DC side is high. Neither is a diode rectifier really
suitable, since it provides a fixed DC voltage, and at low currents most of the output voltage drops on
the transistors of the regulator, increasing the losses. A fully controlled rectifier, followed by a filter
with a big reservoir capacitor and a series linear regulator, provides a high dynamics with high
efficiency and low ripple. Such power supplies are currently available on the market as standard
products.

5 The real converter: effects of the load and mains

5.1 Introduction

Until now, we have assumed we were working with ideal devices feeding pure resistive loads. The
real world is quite different.

First of all, the ripple on the DC side usually exceeds the tight specifications required for
particle accelerator applications. Even if 12-pulse structures are used, passive LC and often active
filters are needed to satisfy the ripple specifications for the DC.

The load usually has an inductive component. The time constant of the load, \( \tau = L/R \), can be
very high (hours in case of superconducting magnets), and, as we shall see, this is of great importance
for the operation of the converter.

The status of diodes and thyristors blocking or conducting changes in a finite, non-zero time
and they have a non-zero on-resistance that introduces losses during the conduction phase.

Last but not least, there is an inductance on the AC side of the rectifier (the inductance of the
secondary of the transformer and the stray inductance of the connecting lines).

The real-world circuit, for example a 3-phases thyristor bridge rectifier, can be represented as in
Fig. 20.

Fig. 20: Six-pulse fully controlled bridge with inductive load
5.2 DC side harmonics filtering

From Tables 1 and 3, it has been seen that the fundamental ripple frequency is given by

\[ f_i = p \cdot f_{\text{mains}}, \tag{50} \]

where \( p \) is the number of pulses.

The ripple voltage at the output of the rectifier can be represented as an independent time-varying voltage over/imposed on the average value of the rectified voltage (\( V_{\text{DC}} \)). In order to reduce the ripple amplitude, a passive \( LC \) filter is usually connected between the rectifier and the load. The time-varying voltage can thus be interpreted as a composition of harmonics that are individually weakened by the filter.

Assuming a \( p \)-pulse rectifier, the harmonics, whose number will be indicated with \( n \), are whole multiples of \( p \). Since all currents and voltages are sinusoidal functions, by properly choosing the origin of the reference system and using the symmetries, it is possible to represent the ripple voltage as a Fourier series composed of cosine terms only whose amplitudes are described by [10]

\[
\begin{align*}
 b_n &= V_S \cdot \left[ \sin \left( \frac{\pi}{p} \right) \right] \cdot \frac{2}{n^2-1} \left[ -\cos \left( \frac{n \cdot \pi}{p} \right) \right] \quad p \neq 1 \quad n = k \cdot p \quad k \in N^+, \\
 b_n &= V_{\text{DC0}} \cdot \frac{2}{n^2-1} \left[ -\cos \left( \frac{n \cdot \pi}{p} \right) \right].
\end{align*} \tag{51a}
\]

that is,

\[
 b_n = V_{\text{DC0}} \cdot \frac{2}{n^2-1} \left[ -\cos \left( \frac{n \cdot \pi}{p} \right) \right]. \tag{51b}
\]

Equation 37 was used, replacing the number of phases \( m \) with the number of pulses \( p \). This result shows that the amplitude of the harmonics is independent of the number of pulses and scales the maximum rectified voltage \( V_{\text{DC0}} \) with its harmonic number \( n \).

It should be noted that Eqs. (51) are valid only for ideal devices (instantaneous commutation) and delay angle \( \alpha = 0 \). When taking into account real devices and phase control, the calculations become complex and, therefore, only the fundamental harmonic is usually computed. This is normally sufficient, since the higher harmonics are usually smaller and are weakened by the \( LC \) filter at a higher ratio. For more details on the voltage and current ripples, from rectifiers, see the bibliography (e.g., Refs. [9] or [10]).

As already mentioned, passive \( LC \) filters are used to reduce the harmonics content on the output of the rectifiers: “a combination of \( L \) and \( C \) produces a lower ripple with normal components values than is possible with either \( L \) or \( C \) alone” [9]. The inductance smooths the oscillations in the current and the capacitance those in the voltage. The resonance frequency is given by

\[ f_0 = \frac{1}{2 \cdot \pi \cdot \sqrt{LC}} \tag{52} \]

and it has to be chosen so as to satisfy the condition \( f_0 \ll f_i \) furthermore, for a given \( f_0 \) a degree of freedom remains in the choice of \( L \) or \( C \).

Some authors, e.g., Ref. [16], use economic criteria in choosing the proper compromise between \( L \) and \( C \). A purely technical approach consists in taking an inductance higher than the critical one. If the load inductance is infinite, the current that flows through it is perfectly constant. As soon as the
value of the inductance decreases, the attenuation of the current ripple also decreases. It was shown in Section 4.2 — and in particular in Fig. 17 — that with a resistive (with a very small time constant) load, at high delay angles (i.e., at low $I_{DC}$), the instantaneous current in the load, $i_i(t)$, becomes zero, entering the discontinuous mode. This should be avoided. The critical inductance is the minimum inductance that guarantees a continuous current flow at the minimum $I_{DC}$ foreseen for the operation of the rectifier with that particular load.

The critical inductance is calculated with the condition that the peak of the fundamental harmonics of the current ripple through the filter inductance is equal to the minimum $I_{DC}$ in the load.

\[
I_{DC_{\text{min}}} = \frac{V_{\text{ripple,1}}}{2 \cdot \pi \cdot f_{\text{r,1}} \cdot L_c} \quad \Rightarrow \quad L_c = \frac{V_{\text{ripple,1}}}{2 \cdot \pi \cdot f_{\text{r,1}} \cdot I_{DC_{\text{min}}}}
\]

(53)

where $V_{\text{ripple,1}}$ is the amplitude of the fundamental harmonic of the ripple at frequency $f_{\text{r,1}}$.

In order to avoid resonances, a damping resistor is normally added in the filter structure. A typical scheme for a damped $LC$ filter is given in Fig. 21.

\[ f_0 = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot (C_1 + C_2)}} \]

$L > L_c$ \quad $C_1 = 4 \cdot C_2$ \quad (or \quad $C_1 = 5 \cdot C_2$)

\[
\delta = \frac{R}{2 \sqrt{\frac{L}{C_1 + C_2}}} \]

(54)

\[
\delta = 0.2 \quad \Rightarrow \quad R = 0.4 \cdot \sqrt{\frac{L}{C_1 + C_2}}.
\]
5.3 Effect of the inductance of the load

Owing to the presence of an inductive component in the load (Fig. 20)—including the inductance of the passive filter—the current does not follow the voltage waveform but it is smoother and extends further depending upon the value of the inductance and of the firing angle (Fig. 23 shows as an example the load voltage and current along with their average values for the circuit of Fig. 20 with $\alpha = 50^\circ$).

This fact keeps the thyristor in the conducting state for a longer time, even if the anode–cathode voltage reverses and becomes negative (a thyristor, once it has been turned on, will remain on until the anode current has been reduced to near zero, below the threshold value of the device).

For delay angles greater than 60 degrees, the output current remains continuous and positive even if part of the load voltage waveform is negative. It should be noted that the load voltage is also continuous, see Fig. 24, to be compared with the top-right plot in Fig. 17. The DC voltage, i.e., the average of the rectified voltage ($V_{DC}$), is also positive.
Fig. 24: Output voltage and current waveforms for $\alpha = 70^\circ$; the load voltage is negative during part of the period (its average is still positive) while the current is always positive (left axis: voltages; right axis: current); two phase–phase voltages have also been plotted.

The $V_{DC}$ becomes zero when the delay angle is 90 degrees and negative when the delay angle is within 90 to 180 degrees. If the current is still positive (i.e., the inductance is sufficiently large), while the $V_{DC}$ is negative, the power is fed back to the AC supply from the load. This operating mode is called inverter or inverting mode. We shall come back to this topic later.

For a rectifier that operates in 1-quadrant mode (positive voltage and positive current), the negative portion of the output voltage is a nuisance. The average value $V_{DC}$ decreases faster and the delay angle range is in any case limited to 90 degrees. To prevent the output voltage from going negative and, at the same time, let the current flow in the load even when the voltage is zero, a so-called freewheeling diode is placed in parallel to the bridge (Fig. 25).

Fig. 25: Six-pulse fully controlled bridge with freewheeling diode and inductive load

When the output voltage becomes negative, the diode starts to conduct and the current of the load flows through it. Another positive effect of the freewheeling diode is to reduce the voltage ripple and the reactive power (the effects of the freewheeling diode on the bridge operations are particularly well explained by J. Schaeffer and B.R. Pelly, see bibliography). The freewheeling diode is also an effective protection against overvoltages when all firing pulses of the thyristors are stopped or the main contactor on the primary of the transformer is opened (see Section 6.3).
Figures 26 and 27 are useful to compare the effects of the freewheeling diode on the operation of the rectifier for delay angles greater than 60 degrees. Two phase-to-phase voltages are also plotted.

**Fig. 26:** Output voltage and current waveforms of the circuit of Fig. 20 ($\alpha = 88^\circ$); the load voltage is negative during part of the period (its average is almost zero) while the current is still positive but with high ripple (left axis: voltages; right axis: current)

**Fig. 27:** Output voltage and current waveforms of the circuit of Fig. 25 ($\alpha = 88^\circ$); the effect of the FW diode on the load voltage is evident (its average is also higher than that in Fig. 26; the scale of the axis are the same in both plots) and the current is correspondingly higher and smoother (left axis: voltages; right axis: current)

Figure 28 shows the voltage and current waveform for the circuit of Fig. 25 for a very large delay angle ($\alpha = 100^\circ$).
Before closing the topic of the effects of the load inductance, I shall briefly describe the inverting mode of operation. As was seen, with a highly inductive load and delay angles greater than 90 degrees and no freewheeling diode, the $V_{DC}$ is negative. If the $I_{DC}$ is still positive, power is flowing from the load to the AC supply. The energy is transferred from the DC side to the AC one. This operating mode can be used to extract energy from a large inductance (e.g., superconducting magnet) when a rapid variation of the load current is needed.

In theory an inverter could work with delay angles up to 180 degrees. In practice this is not possible. The overlapping angle $\mu$ (see Section 5.4), during which two phases are almost short-circuited, reduces the angular operating range for the inverter. Moreover, the extinction angle has also to be taken into consideration. It is defined as the minimum angle required by the thyristor to reach its blocking state after commutation. It has to be

$$\gamma > 2 \cdot \pi \cdot f_{\text{mains}} \cdot t_q$$

(55)

where $f_{\text{mains}}$ is the frequency of the AC mains and $t_q$ is the thyristor turn-off time.

The maximum delay angle is then given by:

$$\alpha_{\text{max}} = \pi - \mu - \gamma \quad \text{(usually, } \alpha_{\text{max}} = 150^\circ \text{)}.$$

(56)

If this condition is not satisfied, the commutation of the thyristor is not fully completed and overcurrents may occur, resulting in the destruction of the device.

5.4 Thyristor commutation — effect of the AC input reactance

The commutation of real devices is not instantaneous. Turning on and off a diode or a thyristor takes a finite time that may be as long as some tens or hundreds of microseconds (diodes turn off faster: some tens of nanoseconds to some microseconds). Taking into consideration the thyristors only (we need to control the DC level of the rectified voltage), turning off occurs when the anode current goes below the so-called holding current $I_h$. 

![Fig. 28: Output voltage and current waveforms of the circuit of Fig. 26 (\(\alpha = 100^\circ\)); the load current is still smooth even if the voltage is quite distorted (left axis: voltages; right axis: current)](image-url)
In addition to the finite turn off time, the inductance of the line and of the secondary of the transformer plays an important role in rectifier operation. The commutation between the phases takes a finite time, during which the two phases involved are almost shorted. This time (electrical angle) is called commutating or overlapping time (angle), and it is usually indicated as $\mu$.

Referring to Figs. 29 and 30 (taken from Ref. [7]), during the transition from phase B to phase A, both thyristors T2 (which is turning off) and T1 (which is turning on) are conducting at the same time. The current $i_{sc}$ is limited, in practice, by the impedance seen at the AC input of the rectifier, here indicated as $L_S$.

The overlapping time $\mu$ depends both on the phase-to-phase r.m.s. voltage $V_{ef}$ and on the line/transformer secondary inductance (here indicated as $L_S$). For a given direct current $I_D$ and the corresponding delay angle $\alpha$, it can be calculated from the following equation:

$$I_D = \frac{V_{ef}}{\sqrt{2} \cdot \omega \cdot L_S} \cdot \left[ \cos(\alpha) - \cos(\alpha + \mu) \right]. \quad (57)$$

The fact that during the commutation time two switches are conducting at the same time creates a sort of short circuit and a reduction in the rectified voltage ($v_D$) and in its average ($V_D$). The reduction in the DC voltage, indicated as $\Delta V_{med}$ in Fig. 30, is given by

$$\Delta V_{med} = \frac{3 \cdot V_{ef}}{\pi \cdot \sqrt{2}} \cdot \left[ \cos(\alpha) - \cos(\alpha + \mu) \right] \quad (58)$$

or, referring to the average of the ideal rectified voltage [9, 10], it is possible to write

$$V_{DC} = V_{DC0} \cdot \frac{\cos(\alpha) + \cos(\alpha + \mu)}{2}. \quad (59)$$
As can be seen in Fig. 30, the waveform of the rectified voltage $v_D$ is additionally distorted during the overlapping angle, worsening the output ripple of the rectifier.

5.5 Effect of the rectifier on the AC mains

5.5.1 Introduction

The switching action of the rectifying device inevitably results in a non-sinusoidal current being drawn from the AC supply system. This non-sinusoidal current can be expressed as a fundamental current at the mains frequency with harmonics superimposed on it. It can be demonstrated that to each $n$th-order harmonic in the output voltage of the rectifier ($n$ being an integer multiple of the number of pulses $p$), there are two harmonics in the AC supply, whose orders are $(n-1)$ and $(n+1)$. In a first approximation the amplitude of these harmonics, referred to the fundamental, decrease as $1/(n-1)$ and $1/(n+1)^1$. Considering 3-phase rectifiers, Table 4 reports the first four harmonic components for a 6-pulse and a 12-pulse rectifier (e.g., a single and a double bridge—no matter if in series or parallel), with a mains frequency of 50 Hz. Figure 31 shows the waveforms of the line current compared to its fundamental harmonic component [17].

<table>
<thead>
<tr>
<th>Harmonic no.</th>
<th>Frequency [Hz]</th>
<th>Harmonic no.</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>250</td>
<td>11</td>
<td>550</td>
</tr>
<tr>
<td>7</td>
<td>350</td>
<td>13</td>
<td>650</td>
</tr>
<tr>
<td>11</td>
<td>550</td>
<td>23</td>
<td>1150</td>
</tr>
<tr>
<td>13</td>
<td>650</td>
<td>25</td>
<td>1250</td>
</tr>
</tbody>
</table>

1 A more accurate calculation should take into consideration the overlap during commutation and the fact that the load inductance is finite (this means that the current drawn from each phase during the conduction intervals is not constant).
From Fig. 31 it is quite evident that a 12-pulse rectifier introduces less disturbance on the mains current than a 6-pulse one.

![Fig. 31: Line current and fundamental harmonic component for the 6-pulse (left) and 12-pulse (right) rectifier](image)

In the following paragraphs I shall mention some of the effects on the network of the operation of rectifiers. Refer to the bibliography (most of the mentioned books have dedicated chapters, but in particular see Refs. [17] and [18]) for more details.

### 5.5.2 Power displacement factor and power factor [19]

Starting from the circuit of Fig. 32, let us consider the fundamental of the current only. We can define as input displacement angle, $\Phi_1$, the angular displacement between the fundamental components of the AC line current and the associated line to neutral voltage (e.g., phase R, Figs. 27 and 28). The Displacement Power Factor is defined as

$$DPF = \cos(\Phi_1).$$

![Fig. 32: Three-phase bridge. The load inductance $L_D$ is assumed to be large enough that the rectified current $i_D$ is almost equal to the direct current $I_D$.](image)

Referring to the circuit of Fig. 32 and assuming a balanced steady-state operation, it is possible to consider only one of the phases. The quantities for the other two phases will be identical except for the ±120° displacement. The following plots show the waveforms of the line voltage $v_R$ and of the line current $i_R$ for phase R in the ideal case ($L_S = 0$, i.e., no overlapping) for two delay angles, $\alpha = 0°$ (Fig. 33) and $\alpha = 30°$ (Fig. 34).
When $\alpha = 0^\circ$ the waveforms of the line voltage and of the fundamental of the line current are in phase, i.e., $\Phi_1 = \alpha = 0^\circ$. As soon as a positive delay angle is applied, $i_R$ (and hence $i_{R1}$) start to lag behind the $v_R$ by the delay angle $\alpha$. Also in this case $\Phi_1 = \alpha > 0$.

The r.m.s. values of the Fourier components for the current waveforms are

$$I_{R1} = \frac{2 \cdot \sqrt{3}}{\sqrt{2} \cdot \pi} \cdot I_D$$

$$I_{Rn} = \frac{I_{R1}}{n} \quad n = k \cdot p \pm 1 \quad k = 1, 2, \ldots.$$ (61)

The total r.m.s. value of the phase current is given by

$$I_R = \sqrt{I_{R1}^2 + \sum_{n=1}^{\infty} I_{Rn}^2}.$$ (62)

In this case the r.m.s. value of the phase current can be calculated to be

$$I_R = \sqrt{\frac{2}{3}} \cdot I_D.$$ (63)

The average power flowing through the rectifier is ($V_R$ and $I_{R1}$ are r.m.s. values)

$$P = V_R \cdot I_{R1} \cdot \cos(\Phi_1).$$ (64)

The apparent power is ($I_R$ is the r.m.s. value of the line current, which includes all harmonics)

$$S = V_R \cdot I_R.$$ (65)
The power factor is therefore defined as

\[ PF = \frac{P}{S} = \frac{I_{R1}}{I_R} \cdot \cos(\Phi_1) = \frac{I_{R1}}{I_R} \cdot \cos(\alpha) = \frac{I_{R1}}{I_R} \cdot DPF . \]  

(66)

Substituting (61) and (63) in (66) we get (for the circuit of Fig. 26)

\[ PF = \frac{3}{\pi} \cdot \cos(\alpha) = 0.955 \cdot \cos(\alpha) . \]  

(67)

This is an important result, showing that the power factor for a thyristor rectifier depends on the firing angle \( \alpha \).

Considering now the r.m.s. value of the distortion component in the line current,

\[ I_{\text{dis}} = \sqrt{I_R^2 - I_{R1}^2} , \]  

(68)

we define the total harmonic distortion as

\[ THD = \frac{I_{\text{dis}}}{I_{R1}} \cdot \sqrt{I_R^2 - I_{R1}^2} . \]  

(69)

Up to now we have considered the ideal case of \( L_S = 0 \), i.e., no overlapping. In the real world, \( L_S > 0 \) and there is a positive angle \( \mu > 0^\circ \) to be added to the firing angle \( \alpha \). It is possible to approximate the displacement power factor as:

\[ DPF \equiv \frac{\cos(\alpha) + \cos(\alpha + \mu)}{2} . \]  

(70)

The presence of the line inductance has, therefore, the effect of further reducing the power factor of the rectifier.

5.5.3 Effects on the AC mains voltage [19]

As was seen in Section 5.4, during the commutation of the thyristors the two phases involved are almost shorted through the line/transformer secondary reactance. This causes notches on the AC voltage. It can be demonstrated that these notches have a maximum depth and width depending on the delay angle \( \alpha \), the line/transformer secondary reactance \( L_S \), the phase-to-phase voltage \( V_{f-f} \), and the average value of the rectified current \( I_D \).

\[ \text{Notch Depth} \equiv \sqrt{2 \cdot V_{f-f} \cdot \sin(\alpha)} \]

\[ \text{Notch Width} \equiv \frac{2 \cdot \pi \cdot f \cdot 2 \cdot L_S \cdot I_D}{\sqrt{2 \cdot V_{f-f} \cdot \sin(\alpha)}} . \]  

(71)

The total harmonic distortion of the mains AC voltage can be calculated from the impedance of the AC source (the line feeding the primary of the rectifier’s transformer) \( L_{\text{Sline}} \) and the harmonics of the converter’s input current. Using the notation of Eqs. 61 and 62, it is possible to write the following equation:

\[ THD_V = \frac{\sum_{k=1}^{\infty} (I_{Ra} \cdot n \cdot 2 \cdot \pi \cdot f \cdot L_{\text{Sline}})^2}{V_{\text{phase}}} . \]  

(72)
5.5.4  How to reduce the harmonics on the AC mains

According to Ref. [17], there are two types of solutions to mitigate the harmonics on the mains current. They can be preventive or remedial ones. The former include the use of converters with a high number of pulses (the total harmonic distortion in the line current for a 6-pulse and for a 12-pulse rectifier are $THD_{6} = 28.45\%$ and $THD_{12} = 9.14\%)$ or with the proper choice of transformer connections (delta-connected primary transformers are preferable). These make use of filters to damp specific harmonic frequencies. The filters can be passive: a combination of capacitors and reactors — and resistors for the damped type — tuned to the specific harmonic to be suppressed. There are also active filters. They consist of a switched-mode power supply injecting into the line a current whose harmonic spectrum is equal in amplitude and opposite in phase to that of the distorted harmonic current. Harmonics are thus cancelled and the result is a non-distorted sinusoidal current.

5.5.5  Unity power factor rectifiers

In order to improve the harmonics content of the mains and, at the same time, to improve the power factor of controlled rectifiers (which, as was seen, depends on the delay angle $\alpha$), the so-called High Power Factor or Unity Power Factor rectifiers are more and more studied (see, for example, Ref. [6]) and increasingly used. In principle they consist of a combination of conventional rectifier and PWM techniques. Using an appropriate firing pattern for the PWM part, the waveform of the current drawn from the AC line can be controlled to approximate a sinusoid in phase with the voltage waveform.

6  Protection and interlocks

6.1  Introduction

The topics of protection and interlock for power converters are covered by Steve Griffiths in another paper of these proceedings [20]. In this section I just want to pinpoint some aspects of the problem, and I shall briefly mention some precautions that should be adopted at the component level and for whole converters. For more details, in addition to the above-mentioned paper, refer also to Refs. [16] and [21].

6.2  Device protection

According to some, diodes are less fragile than thyristors. They do not include low-power gate circuits and their simpler structure — a $pn$ junction — makes them less sensitive than thyristors to overcurrents, overvoltages and transients. Nonetheless they have to be protected and most of what is mentioned in the following paragraphs about thyristor protection can be applied to diodes as well.

6.2.1  Overcurrent

The current rating of a device is the current which raises the temperature of the junction to its top limit (normally around 125°C). An overcurrent will raise the temperature of the junction excessively and cause malfunctions or the destruction of the device.

The simplest way to protect a thyristor is using adequate fuses. They must be fast acting fuses preventing the rise to high arc voltages (less than 1.5 times the peak voltage in circuit). The $I^2t$ parameter normally characterizes fuses: this value must be lower than the $I^2t$ that would damage the thyristor (the semiconductor manufacturer usually indicates the maximum $I^2t$ for the protection fuses).

A more sophisticated method consists in monitoring the current through the device and increasing the delay angle $\alpha$ as soon as the anode current exceeds a threshold. This system must be able to bypass the normal control of the firing angle and must take into account the delay of the protective action which, in fact, occurs only after the overcurrent is detected.
Normal practice suggests choosing thyristors with peak current limits higher than the foreseen operating conditions accepting a reasonable oversizing (between 30% and 50%).

6.2.2 Overvoltage

Withstanding the estimated reverse voltage for the application where it is used is one of the main parameters for the design of the converter. If the thyristor is submitted to a reverse voltage greater than its rated value, it will break down. Choosing oversized devices ($V_{RRM}$ 30% or 50% higher than that one expected) is also a good solution in this case.

Unlike diodes, thyristors have to be able to resist a forward voltage without turning on until the gate trigger is applied. If an overvoltage exceeds the forward withstanding value, it turns the device incorrectly on and can damage it. Again, an adequate oversizing of the $V_{DRM}$ (30% or 50% higher than what is expected) solves the problem.

6.2.3 Transients

Voltage transients or voltage surges, i.e., an excessive slew rate of voltage ($\frac{dv}{dt}$) are another source of overvoltages that may damage the thyristors. Transients may originate from sources that are either internal or external to the device. The general approach to protect thyristors from voltage surges is to quickly store the surge energy in a capacitor, and then to dissipate it slowly in a resistor.

Let us consider first transients internal to the circuit. Each thyristor commutation causes some transient voltage peaks, in particular at turn off. Owing to the presence of an inductance (line, transformer winding, etc.) in series during its conduction phase, a high peak reverse voltage is generated when the thyristor is turned off. In order to mitigate this voltage surge, a $RC$ combination, called snubber circuit, is connected in parallel to each device.

Figure 35 presents two versions of the snubber circuit. The principle is the same. Capacitor $C_1$ suppresses the voltage surge $\frac{dv}{dt}$ that appears when the thyristor $Th$ goes into the blocking state. Resistor $R$ ($R_1$) is used to damp possible oscillations in the $LC$ circuit ($L$ is the inductance of the AC connection seen by the thyristor). The same resistance $R$ (or, in the circuit on the right, $R_2$) has to limit the discharge current from the capacitor through the thyristor when it starts to conduct again. The diode $D$ in series with $R_1$ is used to separate the action of the $\frac{dv}{dt}$ protection resistor $R_1$ ($< R_2$) from that of the discharge resistor $R_2$.

Typical values for $C$ are 0.1–1 μF and for $R$ are 10–1000 Ω. More details on the dimensioning of the snubber parameters are reported in the literature [8], [12] and in particular Refs. [9] and [22].

![Fig. 35: Protecting thyristors from internal voltage surges: snubber circuits](image-url)
the thyristors and the load. When the contactor closes, a voltage overshoot may occur in the oscillating circuit constituted by the inductance of the secondary windings and a capacitance, either stray or physically present. Also in this case a $RC$ combination is used. The capacitance must be able to store the energy of the transformer. Sometimes the $RC$ groups are connected directly between the phases immediately before the rectifier. In order to decouple the capacitance from the inductance of the connection, a bucket circuit is often preferred. In practice the $RC$ group is connected to the line through a diode rectifier (Fig. 36). Resistance $R_1$ is the damping resistance, calculated from the inductance of the line and of the transformer and the capacitance $C$. Resistance $R_2$ is the discharge resistor of the capacitor; it is sized in order to have a time constant of about $100$ ms.

**Fig. 36:** Protecting thyristors from external voltage surges: bucket circuit

Some examples on how to calculate the bucket circuit parameters are presented in Refs. [8], [16], [22], and [23].

### 6.3 Converter protection

Referring to the unifilar schematic of Fig. 37, I shall quickly present some of the main aspects to be considered in the protection of a complete converter.

**Fig. 37:** Diagram of the generic converter
6.3.1 Circuit breaker and contactor

The circuit breaker’s duty is to disconnect the converter from the AC mains both under normal operations (e.g., during maintenance) and in case of internal fault of the converter/load system. It has to be chosen according to the short-circuit power rating of the AC line and has to switch off in case of converter overload. Usually, for converters fed by a low-voltage line (380 V), there is a contactor between the circuit breaker and the transformer for the normal On/Off operations.

![Circuit Breaker and Contactors Diagram](image)

**Fig. 38:** Circuit breaker with ‘soft start’ connection on the main contactor

In order to limit the transformer’s magnetizing currents at the switch-on of the converter, a soft start procedure is normally adopted (Fig. 38). The turn-on sequence is the following: command switch on the first closes the secondary contactor $S$ — the circuit breaker has already been closed — in order to limit the inrush current through the resistor $R$; then, with $S$ closed, after some hundred milliseconds, the main contactor $M$ is closed and, finally, $S$ is opened. This procedure should include automatic cross-checks on the status of $S$ and $M$. If $S$ is closed for too long while $M$ is not, an interlock must be triggered to open $S$ and signal the presence of a problem in order to protect the limitation resistor.

6.3.2 Transformer

Assuming that the transformer has been correctly dimensioned to be used with a rectifier that generates high harmonics in the secondary windings, the major risk for the transformer is high temperature. Adequate cooling systems — oil, water, or air — must be provided and monitored. Thermal sensors have to be mounted on the coils and connected to an interlock that switches off the converter. The temperature of the coolant must also be monitored.

6.3.3 Rectifier structure

In Section 6.2 we discussed how to protect the devices (diodes or thyristors). Here I am considering the rectifier structure. Adequate cooling is a very important issue. Thermal switches (connected to an interlock) have to trip if the temperature of the heat-sinks is too high. Flow switches should monitor the flow of water or of forced air used to cool the heat-sinks.

Normally, there is a passive filter cascaded to the rectifier, which, as presented in Section 5.2, usually includes an inductance. To avoid overvoltages on the output of the rectifier structure when the rectified current is suddenly interrupted (opening of the main switch, thyristors’ trigger pulses disabled, etc.) a so-called ‘freewheeling diode’ is connected in anti-parallel to the rectifier structure.

6.3.4 Passive filter

A malfunction in the rectifier structure—for example a broken thyristor that does not turn on— increases the ripple content in the rectified voltage. This leads to a great increase in the ripple current through the capacitors of the passive filter. The capacitors have to be protected with properly sized fuses or thermal contactors, or both. The magnetic components—the inductance—have to sustain the whole rectified current (i.e., the DC component with the ripple current on it). Besides being
adequately dimensioned in order to avoid saturation, they have to be properly cooled. Thermal switches and—if water or forced air is used—flow switches have to be planned for.

6.3.5 Load

Protections for the load are also included in the converter’s structure. If the load has a high inductive component, a freewheeling diode is normally connected at the output of the converter in anti-parallel to the load. The freewheeling diode has to be able to withstand the peak load current.

It is not good practice to let float the load and the output of the power supply. In case of fault or accident, if the power supply or the load goes to earth potential, very high currents could be generated. Normally, the low side of the converter output is connected to earth through a resistor and the current that flows through it is monitored. If this earth current exceeds a certain threshold, this may indicate a problem of the load or of the converter.

The load itself has to be adequately cooled and there should exist individual protections (e.g., if the load is a string of magnets) against over temperature and other parameters (e.g., the flow of cooling water or the quench detection in superconducting magnets) that act as external interlocks, switching off the associated power supply.

6.3.6 Additional protections/interlocks

Under this category, one could include interlocks related to a malfunction of the converter (e.g., one or more missing phases in the AC mains—including voltage sags—or error signal on the output current reading device, absence of remote control, etc.), or those related to the safety of the personnel, like door switches on the cabinet that contains the converter, emergency turnoff push buttons, etc.

7 Example of dimensioning

7.1 Introduction

In this section I present the dimensioning, based on Refs. [24] and [25], of a DC magnet power supply that has been operating since 1993 at the Elettra Synchrotron Light Laboratory. The calculations have been reorganized in a Mathcad© worksheet. The names of the constants and of the variables are those used in the calculations. The Mathcad© notation is used in the following paragraphs.

7.2 System requirements and technical specifications

The load consists of two quadrupole magnets connected in series. Its characteristics are

- Load inductance: \( L_m = 27 \text{ mH} \)
- Load resistance (including cables): \( R_m = 105 \text{ m\Omega} \)
- Maximum DC output voltage (incl. safety margin): \( V_{DC} = 40 \text{ V} \)
- Maximum output current (incl. 10% safety margin): \( I_{DC} = 385 \text{ A} \)
- Minimum output current (10% of nominal one): \( I_{DC\text{min}} = 35 \text{ A} \)
- Power supply output power: \( P_{DC} = V_{DC} I_{DC} = 15.4 \text{ kW} \)

For test purposes, the power supply has to operate at full current on a load 25% of nominal.

- Maximum output current ripple (\( \Delta I/I_{DC} \)): \( \gamma_{\text{ripple}} = 2 \times 10^{-5} \)

A step-down transformer provides the AC voltage. The characteristics of the transformer are as follows:
Transformer ratio: 20/0.4 kV  
Nominal mains voltage: $V_{\text{mains rms}} = 380$ V  
Maximum mains voltage variation: $\Delta V_{\text{mains}} = 10\%$  
Short-circuit power of the transformer: $P_{\text{SC trafo}} = 1.6$ MVA  
Short-circuit voltage of the transformer: $V_{\text{SC trafo}} = 7.5\%$  
Mains frequency: $f_{\text{mains}} = 50$ Hz

7.3 Dimensioning of the components

The general scheme of the power supply is shown in Fig. 39. In the following paragraphs the main components of the power supply will be calculated by means of the definitions and formulas reported in the previous sections with some additional details when needed.

Fig. 39: Summary of protections and interlocks on a converter

7.3.1 Transformer

In order to dimension the transformer, the no-load DC voltage has to be obtained. This voltage includes all losses.

Voltage drop on thyristors (2 switches: it is a bridge structure): $V_{\text{Th}} = 1.5$ V  
Voltage drop on filter inductance and inner connections: $V_{\text{L conn}} = 0.5$ V  
Voltage drop on transformer: $\Delta V_{\text{trafo}} = 3\%$  
Voltage drop due to initial end stop delay angle: $\Delta V_{\text{end stop}} = 3\%$

The rectification structure is a fully controlled 3-phase bridge. The rectification ratio between the r.m.s. secondary interphase voltage and the average rectified voltage is

$$r_{v_{\text{Br}}} = \frac{3\sqrt{2}}{\pi}$$

The required no-load voltage is given by

$$V_{\text{DC0}} := (V_{\text{DC}} + 2V_{\text{Th}} + V_{\text{L conn}}) \cdot (1 + \Delta V_{\text{trafo}}) \cdot (1 + \Delta V_{\text{end stop}}) \cdot (1 + \Delta V_{\text{mains}})$$

$V_{\text{DC0}} = 50.764$ V.

The r.m.s. secondary phase-phase voltage is
\[ V_s := \frac{V_{DC0}}{r_{V,Br}} \]

\[ V_s = 37.6 \text{ V} \]

From the ratio between the average rectified current and the r.m.s. value of the phase current, it is possible to calculate the r.m.s. secondary phase current:

\[ r_{1,Br} = \sqrt[3]{2} \]

\[ r_{1,Br} = 0.816 \]

\[ I_s := I_{DC} \cdot r_{1,Br} \]

\[ I_s = 314.4 \text{ A} \]

The dimensioning power of the transformer is (for a bridge rectifier the power at the secondary is equal to the power at the primary)

\[ P_{Tr} := \sqrt{3} \cdot V_s \cdot I_s \]

\[ P_{Tr} = 20.467 \text{ kVA} \]

The r.m.s. primary phase current (including a 5% safety margin for the magnetizing current and other losses) is

\[ I_p := 1.05 \cdot \frac{P_{Tr}}{\sqrt{3} \cdot V_{\text{mains rms}}} \]

\[ I_p = 32.7 \text{ A} \]

Since the transformer has to be installed inside the cabinet containing the power supply, a demineralized-water-cooled type is chosen.

### 7.3.2 Circuit breaker and main contactor

The short-circuit current of the upstream transformer (the one that feeds the mains line) defines the breaking capacity of the circuit breaker for the converter:

\[ I_{SC} := \frac{P_{PC,trafo}}{\sqrt{3} \cdot V_{\text{mains rms}} \cdot V_{SC,trafo}} \]

\[ I_{SC} = 32.413 \text{ kA} \]

The circuit breaker has to have a breaking capacity higher than \( I_{SC} = 32.4 \text{ kA} \) and its size has to be greater than \( I_p = 32.7 \text{ A} \). A commercially available size could be \( I_{CB} = 35 \text{ A} \). This is also the size of the main contactor.

When a transformer is initially connected to the mains, there may be a substantial surge of current through the primary winding (inrush current). This current surge may be quite high if the core saturates (this is possible since transformers’ cores are usually dimensioned to sustain the magnetic flux during normal operations) and should be limited. After a few periods the current surge is reduced to the normal value of the magnetizing current. The limitation of the current surge is achieved by using an additional contactor in parallel to the main one that connects the primary of the transformer to the mains line through resistors.

The secondary contactor with its thermal protection and the resistors has to sustain the magnetizing current of the transformer:

\[ I_\mu := I_p \cdot 5\% \]

\[ I_\mu = 1.6 \text{ A} \]

The chosen resistors (and their power ratings) are

\[ R_\mu = 4.7 \Omega \]

\[ R_\mu \cdot I_\mu^2 = 12.5 \text{ W} \]
Since the price difference is not too high, for safety reasons let us take 50 W resistors. The commercially available one, for example, a 5 A secondary contactor should feature a thermal relay set to $\sqrt{50 \, \text{W}/R_{\mu}} = 3 \, \text{A}$.

### 7.3.3 Bridge thyristors, snubber and bucket circuits, freewheeling diodes

The choice of the thyristors depends on the average and r.m.s. currents, on the peak voltages (forward and reverse) and on the peak current it may experience in case of short circuit of the load. The currents and peak voltages are (see Table 3)

$$I_{\text{Th,avg}} := \frac{I_{\text{DC}}}{3}, \quad I_{\text{Th,rms}} := 0.579 \cdot I_{\text{DC}}, \quad V_{\text{Th,peak}} := 1.05 \cdot V_{\text{DC0}}.$$

Therefore the $I_{f(AV)}$ and $I_{f(rms)}$ and $V_{\text{RMS}}$ of the thyristor have to be higher than

$$I_{\text{Th,avg}} = 128 \, \text{A}, \quad I_{\text{Th,rms}} = 222.9 \, \text{A}, \quad V_{\text{Th,peak}} = 53.3 \, \text{V}.$$

The peak current in the thyristors depends also on the short-circuit voltage of the transformer. This short-circuit voltage can be assumed to be $V_{\text{Tr,SC}} = 6\%$. Additionally, the current flow in the load depends also on the impedance of the transformer’s secondary and of the connection to the thyristors.

Since $X_{\text{Tr}} = \omega L_{\text{Tr}}$ the reactive component and $R_{\text{Tr}}$ the resistive component, according to Ref. [10], Chapter 11, from which Fig. 40 has been taken, the peak current depends on the crest transient factor, which is a function of the ratio $X_{\text{Tr}}/R_{\text{Tr}}$. From Fig. 40, assuming $X_{\text{Tr}}/R_{\text{Tr}} = 6$, the crest factor is $f_{\text{crest}} = 1.6$.  

![Fig. 40: Transient factors](image)
The peak of the short circuit current is:

\[ I_{\text{Th,SC,Peeak}} := \sqrt{2} \cdot \frac{I_s}{V_{\text{Tr,SC}}} \cdot f_{\text{crest}} \quad \text{for} \quad I_{\text{Th,SC,Peeak}} = 11.9 \text{kA} . \]

Assuming 10 ms (half-period) time span before the intervention of the protections, the resulting \( \dot{F}t \) is

\[ I^2 t := \int_0^{10\text{ms}} \left[ I_{\text{Th,SC,Peeak}} \cdot \sin \left( 2 \cdot \pi \cdot f_{\text{mains}} \cdot t \right) \right]^2 \text{d}(t) \quad 12\tau = 7.027 \times 10^5 \text{A}^2\text{s} . \]

The thyristors have to have a \( \dot{F}t \) characteristic higher than that calculated above.

To calculate the snubber network, \( R_{\text{sn}} \) and \( C_{\text{sn}} \), to be placed individually on each thyristor to suppress the reverse recovery voltage, it is necessary to know the stored charge in the thyristor. The stored charge depends on the rate of commutation, which is related to the reactance of the secondary of the transformer. The leakage inductance at the secondary of the transformer can be calculated as follows:

\[ L_{\text{Tr,S}} := \frac{V_s}{2 \cdot \pi \cdot f_{\text{mains}} \cdot I_s} \cdot V_{\text{Tr,SC}} \quad L_{\text{Tr,S}} = 22.8 \mu\text{H} . \]

The peak secondary voltage and the rate of commutation, \( \frac{\text{d}i}{\text{d}t} \), are given by

\[ V_{\text{s,peak}} := \sqrt{2} \cdot V_s \quad V_{\text{s,peak}} = 53.16 \text{V} \quad \frac{V_{\text{s,peak}}}{2 \cdot L_{\text{Tr,S}}} = 1.2 \times 10^6 \text{A}\text{s} . \]

For the chosen thyristor, at the on-state current of \( I_{\text{DC}} = 385 \text{A} \), the recovered charge can be as high as

\[ Q_{\text{tr}} = 130 \mu\text{C} . \]

The capacitor and the resistor can be calculated as follows [22]:

\[ C_{\text{sn,t}} := \frac{Q_{\text{tr}}}{V_{\text{s,peak}}} \quad C_{\text{sn,t}} = 2.45 \mu\text{F} \quad \text{standard value:} \quad C_{\text{sn}} := 2.7 \mu\text{F} . \]

To limit the overvoltage to 15%, the damping factor should be \( \xi = 0.5 \) and, consequently, the damping resistance has to be

\[ R_{\text{sn,t}} := 2 \cdot \xi \cdot \sqrt{\frac{2 \cdot L_{\text{Tr,S}}}{C_{\text{sn}}}} \quad R_{\text{sn,t}} = 4.11 \Omega \quad \text{standard value:} \quad R_{\text{sn}} = 4.3 \Omega . \]

We use a bucket circuit to protect the bridge from overvoltages generated by the opening of the main switch while the thyristors are conducting. From Ref. [8] or Ref. [9], the capacitor and the resistance are calculated as follows. The magnetizing current of the transformer transferred to the secondary is

\[ I_{\mu,s} := 5\% \cdot I_s \quad I_{\mu,s} = 15.7 \text{A} . \]
Assuming an overvoltage \( k = 1.55 \) (between the overvoltage and the peak voltage at the secondary) we have

\[
C_{BC_I} := \frac{I_{\mu_S}}{2 \cdot \pi \cdot f_{\text{mains}} \cdot \left(k^2 - 1\right)} \cdot V_{S\text{peak}}
\]

\[C_{BC_I} := 671.038 \, \mu F \quad \text{standard value: } C_{BC} := 680 \, \mu F.\]

Accepting a 15\% overvoltage, \( \xi = 0.5 \), the damping resistance is

\[
R_{BC_I} := 2 \cdot \xi \cdot \frac{2L_{I\mu_S}}{C_{CB}} \quad R_{BC_I} = 0.259 \, \Omega \quad \text{standard value: } R_{BC} = 0.27 \, \Omega.
\]

The diode bridge of the bucket circuit has to be chosen in order to sustain the peak current, i.e.,

\[
I_{BC\text{peak}} := \frac{V_{S\text{peak}}}{R_{BC}} \quad I_{BC\text{peak}} := 196.9 \, \text{A}.
\]

In parallel to the capacitor there is a discharging resistor. It can be chosen assuming a time constant \( \tau_{BC} = 0.25 \, \text{s} \):

\[
R_{BC\text{disc}} := \frac{\tau_{BC}}{C_{BC}} \quad R_{BC\text{disc}} = 368 \, \Omega \quad \text{standard value: } R_{BC\text{disc}} := 360 \, \Omega.
\]

maximum dissipated power:

\[
\frac{V_{S\text{peak}}^2}{R_{BC\text{disk}}} = 8 \, \text{W}.
\]

In parallel to the thyristor bridge and the load, we install freewheeling diodes. Let us calculate their parameters. According to Ref. [10], Chapter 13, the average and r.m.s. currents carried by the freewheeling diodes are a function of the ratio \( A/B \) between the real output voltage and the maximum possible rectified voltage. Neglecting the commutation angle \( \mu \), and referring to Fig. 41 (taken from Ref. [10]), it is possible to write:

\[
I_{FW\text{avg}} := \frac{A}{B} \cdot I_{DC} \quad I_{FW\text{avg}} := \sqrt[4]{\frac{A}{B}} \cdot I_{DC} \quad E_d = E_{dh} - E_a.
\]

\[\text{Fig. 41: Chart for calculating the freewheeling diode currents}\]
It is possible to see that the worst case is when the load resistance is 25% of the nominal one, where we have

\[
E_d := \frac{V_{DC}}{4}, \quad E_d := 10 \text{ V}, \quad E_{d0} := V_{DC0}, \quad \frac{E_d}{E_{d0}} = 0.197.
\]

From the curve for \( q = 6 \) (\( q \) is the number of pulses), we get \( A/B = 0.4 \) and therefore the maximum average and r.m.s. currents in the freewheeling diodes are

\[
I_{FW_{avg}} := 0.4 \cdot I_{DC}, \quad I_{FW_{avg}} = 154 \text{ A}, \quad I_{FW_{rms}} := \sqrt{0.4 \cdot I_{DC}}, \quad I_{FW_{rms}} = 243.5 \text{ A}.
\]

Thyrstors and diodes have to be mounted on heat-sinks properly dimensioned to dissipate the heat and keep their junction temperature <125°.

### 7.3.4 Passive filter

Using a 3-phase bridge, the frequency of the first harmonic of the ripple is

\[
f_{\text{ripple}_1} := 6 \cdot f_{\text{mains}}, \quad f_{\text{ripple}_1} = 300 \text{ Hz}.
\]

In order to dimension the filter, let us consider the first harmonic only. A filter with a good attenuation at that frequency will provide an even better attenuation for the higher frequencies. According to Ref. [11] (see also Fig. 42), the maximum ripple voltage is given by

\[
V_{\text{ripple}_1_{pk}} := 0.33 \cdot 1.35 \cdot V_S, \quad V_{\text{ripple}_1_{pk}} = 16.7 \text{ V},
\]

\[
V_{\text{ripple}_1_{rms}} := \frac{V_{\text{ripple}_1_{pk}}}{\sqrt{2}}, \quad V_{\text{ripple}_1_{rms}} = 11.8 \text{ V}.
\]

![Fig. 42: Peak amplitude of first harmonic (x-axis) vs. DC voltage ratio](image)

The nominal load impedance as a function of the frequency \([\omega(f) = 2\pi f]\) is

\[
Z_m(f) := R_m + i \cdot \omega(f) \cdot L_m.
\]
At the fundamental ripple frequency, the amplitude of the ripple current in the load is

\[ I_{\text{ripple,1.pk}} := \frac{V_{\text{ripple,1.pk}}}{Z_m(f_{\text{ripple,1}})} \]

\[ I_{\text{ripple,1.pk}} = 0.329 \text{ A} \quad \frac{I_{\text{ripple,1.pk}}}{I_{\text{DC}}} = 8.5 \times 10^{-4}. \]

The filter has to provide an attenuation of

\[ \gamma_{\text{filter}} := \frac{\gamma_{\text{ripple}}}{I_{\text{ripple,1.pk}} / I_{\text{DC}}} \quad \gamma_{\text{filter}} = 0.023. \]

This corresponds to an attenuation of –32.8 dB.

According to Ref. [8], the resonance frequency of the passive LC filter, \( f_0 \), is given by

\[ f_0 := f_{\text{ripple,1}} \sqrt{\gamma_{\text{filter}}} \quad f_0 = 45.9 \text{ Hz}. \]

The calculated frequency is too close to the mains frequency, we can choose \( f_{\text{of}} = 30 \text{ Hz} \).

The minimum inductance value is calculated assuming continuous conduction at the minimum rectified current. As shown in formula (5.4), the critical inductance is given by

\[ L_c := \frac{V_{\text{ripple,1.pk}}}{2 \cdot \pi \cdot f_{\text{ripple,1}} \cdot I_{\text{DCmin}}} \quad L_c = 230.8 \text{ \mu H}. \]

Let us take \( L_t = 250 \text{ mH} \). The r.m.s. current of first ripple harmonic is

\[ I_{\text{ripple,1 rms}} := \frac{V_{\text{ripple,1 rms}}}{2 \cdot \pi \cdot f_{\text{ripple,1}} \cdot L_t} \quad I_{\text{ripple,1 rms}} = 25.1 \text{ A}. \]

Let us assume a 25% margin on the current in order to include all ripple harmonics:

\[ I_{\text{ripple rms}} := 1.25 \cdot I_{\text{ripple,1 rms}} \quad I_{\text{ripple rms}} = 31.41 \text{ A}. \]

From the cut-off frequency equation, it is possible to calculate the value of the capacitor:

\[ f_{\text{of}} = \frac{1}{2 \cdot \pi \cdot \sqrt{L_t \cdot C_{\text{f,t}}}} \quad C_{\text{f,t}} := \frac{1}{4 \cdot L_t \cdot f_{\text{of}}^2 \cdot \pi^2} \quad C_{\text{f,t}} = 112.6 \text{ mF}. \]

By taking a standard value for the capacitor we slightly change the cut-off frequency:

\[ C_t := 150 \text{ mF} \quad \frac{1}{2 \cdot \pi \cdot \sqrt{L_t \cdot C_t}} = 26 \text{ Hz}. \]

It is better to insert a damping resistor in order to limit the overvoltage at the cut-off frequency. The structure of the filter is shown in Fig. 43 (Section 5.2):
considering a 25% margin on the maximum voltage on the capacitors, their voltage rating (including a 10% fluctuation on the mains) has to be higher than

\[ V_{\text{peak}} \cdot 1.1 \cdot 1.25 \cdot V_s \text{peak} = 73.1 \text{ V}. \]

The nearest commercial size is 100 V.

To calculate the damping resistor, we can impose a critical damping (no overshoot) of the step response, that is \( \delta = 1 \).

\[ R_t := 2 \cdot \delta_t \cdot \frac{L}{(C_1 + C_2)} \]

\[ R_T = 81.6 \text{ m} \Omega. \]

Using five standard resistors in parallel,

\[ R_T := \frac{0.43}{5} \text{ } \Omega \]

\[ R_T = 86 \text{ m} \Omega. \]

Calculating the impedances of the filter and of the load at the first harmonic frequency of the ripple, it is reasonable to assume that most of the ripple current flows through the two parallel capacitor branches. Considering a 25% margin to also take into account the higher-order harmonics, the currents in \( C_1 \) and \( C_2 + R_f \) are

\[ I_{\text{ripple, rms, 1}} = 26.1 \text{ A} \]

\[ I_{\text{ripple, rms, 2}} = 5.4 \text{ A}. \]

The power dissipated by \( R_T \) is

\[ P_{R_T} := R_T I_{\text{ripple, rms, 2}}^2 \]

\[ P_{R_T} = 2.5 \text{ W}. \]

The peak currents in the resistor at turn-on or in case of accidental oscillation of the bridge regulation are much higher than 5.4 A\(_{\text{rms}}\). It is better to take 25 W resistors.

At this stage it is possible to calculate the transfer function of the filter \( F(f) = (V_{\text{out}}/V_{\text{in}}) \) and plot its Bode diagram (Figs. 44 and 45).
Zooming in the two bands 20–40 Hz and 64–74 Hz, it is possible to see that the actual resonance peak is at ~32 Hz and the −3 dB frequency is ~70 Hz. This is the effect of inserting the damping resistor.

The filter attenuation at the fundamental ripple frequency is −28 dB. This is far from the original requirement of −32.8 dB. To stay within the ripple specifications, we should change some parameters of the filter or start to consider adding an active filter at the output of the power supply in order to attenuate the remaining ripple.

As an example, let us accept a 50% overvoltage in the step response of the filter. This means that the damping factor is $\delta_0 = 0.2$ and the damping resistor is $R_{f0} = 0.2$ Ω. The Bode plot is reported in Fig. 46 and the attenuation at 300 Hz is −31.6 dB.
7.3.5 Current reading devices and regulation

To complete the design of the power supply—at least at this level of detail—we should choose the current reading devices in the regulation for the feedback loop. A zero-flux DCCT is perfectly suited for the job. It provides a galvanic isolation between the output of the power supply and the regulation electronics. The DCCT generates a current (or a voltage, on a burden resistor) that is proportional to the measured current. It is very stable: it has practically zero thermal coefficient (<0.3 ppm/°C) and long-term drift (<1 ppm/month).

8 Conclusions and future developments

In this paper I have briefly presented some issues related to the theory and practice of line commutated rectification. Most of the sections would require a dedicated chapter and this is far beyond the goals of this paper. For that, refer to the bibliography.

As seen in the previous sections of this paper, the bridge structures are the most efficient. It is difficult to find in the literature examples of new topologies based on diodes and thyristors only.

Most studies either deal with devices — diodes or thyristors — simultaneously handling high currents and high voltages (up to some kA/kV, see for example, Refs. [26], [27], [28]) or with their control, for example the use of microcontrollers in a new design for firing pulse generation (see Ref. [29]).

The use of mixed structures, a combination of diode rectifiers and PWM techniques, decreases the line-current harmonics and improves the power factor by absorbing a quasi-sinusoidal line-current waveform with minimum lag behind the line voltage. These high power factor converters are widely used, in particular in installations where a large number of converters can severely pollute the AC mains.
Appendix A — Performance parameters for a three-phase FC bridge

Referring to Fig. A.1, to the definitions of Section 2, assuming ideal switches and a pure resistive load, it is possible to calculate the expressions for the performance parameters of a three-phase fully controlled bridge in the two conditions of continuous and discontinuous conduction [9].

![Three-phase fully controlled bridge rectifier](image)

**Fig. A.1:** Three-phase fully controlled bridge rectifier

### A.1 Continuous conduction

The continuous conduction condition is defined by

\[ 0 \leq \alpha \leq \frac{\pi}{3} \]  \hspace{1cm} (A.1)

\[ V_{DC}(\alpha) = \frac{6}{2\pi} \int_{0}^{\alpha} \sqrt{3} \cdot V_s \cdot \sin(\omega t + \frac{\pi}{3}) dt = \frac{3}{\pi} \sqrt{3} \cdot V_s \cdot \cos(\alpha) = V_{DC0} \cdot \cos(\alpha) \]  \hspace{1cm} (A.2)

\[ V_L(\alpha) = \frac{6}{2\pi} \int_{0}^{\alpha} 3 \cdot V_s^2 \cdot \sin^2(\omega t + \frac{\pi}{3}) dt = \sqrt{3} \cdot V_s \cdot \sqrt{1 + \frac{3\sqrt{3}}{2\pi} \cos(2\alpha)} \]  \hspace{1cm} (A.3)

\[ FF(\alpha) = \frac{V_L(\alpha)}{V_{DC}(\alpha)} = \frac{\pi}{3\cos(\alpha)} \cdot \sqrt{1 + \frac{3\sqrt{3}}{2\pi} \cos(2\alpha)} \]  \hspace{1cm} (A.4)

\[ I_{DC}(\alpha) = \frac{V_{DC}(\alpha)}{R_L} = \frac{3}{\pi} \cdot \sqrt{3} \cdot \frac{V_s}{R_L} \cdot \cos(\alpha) = I_{DC0} \cdot \cos(\alpha) \]  \hspace{1cm} (A.5)

\[ I_L(\alpha) = \frac{V_L(\alpha)}{R_L} = \sqrt{3} \cdot \frac{V_s}{R_L} \cdot \sqrt{1 + \frac{3\sqrt{3}}{2\pi} \cos(2\alpha)} \]  \hspace{1cm} (A.6)
\[ \eta(\alpha) = \left( \frac{1}{FF(\alpha)} \right)^2 = \frac{9}{\pi^2} \cos^2(\alpha) \cdot \frac{1}{1 + \frac{3\sqrt{3}}{2 \cdot \pi} \cos(2\alpha)} \]  
(A.7)

\[ RF(\alpha) = \sqrt{FF^2(\alpha) - 1} = \sqrt{\frac{\pi^2}{9\cos^2(\alpha)} \left[ 1 + \frac{3\sqrt{3}}{2 \cdot \pi} \cos(\alpha) \right]} - 1. \]  
(A.8)

Average and r.m.s. thyristor currents:

\[ I_{F(AV)} = \frac{I_{DC}}{3} \quad I_{F(rms)} = \frac{I_{L}}{\sqrt{3}}. \]  
(A.9)

### A.2 Discontinuous conduction

The discontinuous conduction condition is defined by

\[ \frac{\pi}{3} < \alpha \leq \frac{2 \cdot \pi}{3} \]  
(A.10)

\[ V_{DC}(\alpha) = \frac{6}{2\pi} \int_{\alpha}^{\frac{2\pi}{3}} \sqrt{3} \cdot V_s \sin(\alpha t + \frac{\pi}{3}) dt = \frac{3}{\pi} \sqrt{3} \cdot V_s \left[ 1 + \cos \left( \alpha + \frac{\pi}{3} \right) \right] \]  
(A.11)

\[ V_L(\alpha) = \frac{6}{2\pi} \int_{\alpha}^{\frac{2\pi}{3}} \sqrt{3} \cdot V_s^2 \cdot \sin^2 \left( \alpha t + \frac{\pi}{3} \right) dt = \sqrt{3} \cdot V_s \sqrt{\frac{\pi}{3} - \frac{\alpha}{2} + \frac{1}{4} \sin^2 \left( \frac{\pi}{3} + \alpha \right)} \]  
(A.12)

\[ FF(\alpha) = \frac{V_L(\alpha)}{V_{DC}(\alpha)} = \frac{\pi \cdot \sqrt{\frac{6}{\pi} \left[ \frac{\pi}{3} - \frac{\alpha}{2} + \frac{1}{4} \sin^2 \left( \frac{\pi}{3} + \alpha \right) \right]}}{3 \cdot \sqrt{2} \left[ 1 + \cos \left( \alpha + \frac{\pi}{3} \right) \right]} \]  
(A.13)

\[ I_{DC}(\alpha) = \frac{V_{DC}(\alpha)}{R_l} = \frac{3}{\pi} \sqrt{3} \cdot V_s \frac{V_L}{R_L} \left[ 1 + \cos \left( \alpha + \frac{\pi}{3} \right) \right] \]  
(A.14)

\[ I_L(\alpha) = \frac{V_L(\alpha)}{R_L} = \sqrt{3} \cdot V_s \sqrt{\frac{6}{\pi} \left[ \frac{\pi}{3} - \frac{\alpha}{2} + \frac{1}{4} \sin^2 \left( \frac{\pi}{3} + \alpha \right) \right]} \]  
(A.15)

\[ \eta(\alpha) = \left( \frac{1}{FF(\alpha)} \right)^2 \]  
(A.16)

\[ RF(\alpha) = \sqrt{FF^2(\alpha) - 1}. \]  
(A.17)
Average and r.m.s. thyristor currents:

\[ I_{F(AV)} = \frac{I_{DC}}{3} \quad I_{F(mms)} = \frac{I_{F}}{\sqrt{3}} \quad \text{(A.18)} \]

**Acknowledgements**

I would like to thank my colleagues R. Fabris, M. Zaccaria and D. Zangrando for the helpful discussions we had and for their comments on the topics presented in or related to this paper.

**References**


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Bibliography

Most of the books reported as References cover almost all the topics of this lecture well beyond the specific citation. I prefer to cite them again along with additional sources. As useful integration to printed texts, it is possible to find on the Internet quite good educational sites dedicated to power electronics applications. I have therefore included also some relevant URLs.

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